

# Schwinger-Dyson equation for a dressed fermion propagator

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# Table of Contents

- 1 Motivation
- 2 Schwinger-Dyson equation
- 3 Self-energy
- 4 Dynamical mass generation
- 5 Results
- 6 Summary

# Motivation

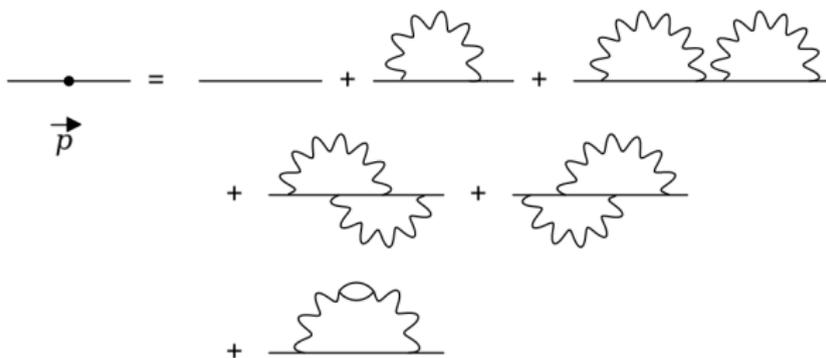
# Motivation

The study of physics at the level of the fundamental constituents of the Universe has represented a challenge to the intellect. The classification of these objects by their properties makes it easier for us to understand the interactions between them. Of these properties, its mass remains one of the most intriguing, since, up to now, there is no theory capable of to explain its origin. The way we conceive of interactions at the fundamental level is through the exchange of mediator particles in the so-called standard model (ME) of elementary particles

# Motivation

The quarks  $u$  and  $d$  have current masses are almost 0, but in hadron mass is around 300-350 MeV.

In perturbation theory, mass function is



$$\mathcal{M}(p) = m_0 \left( 1 + c_1 \alpha \ln \left( \frac{p^2}{\mu^2} \right) + c_2 \alpha^2 \ln^2 \left( \frac{p^2}{\mu^2} \right) + \dots \right)$$

# Schwinger-Dyson equation

# Schwinger-Dyson equation

DSE's are a set infinity of integral relations between Green's functions of a quantum field theory and provide the structure analytic root that these functions possess.

In general the Green functions for a given number  $n$  of points, there are correlation functions of type

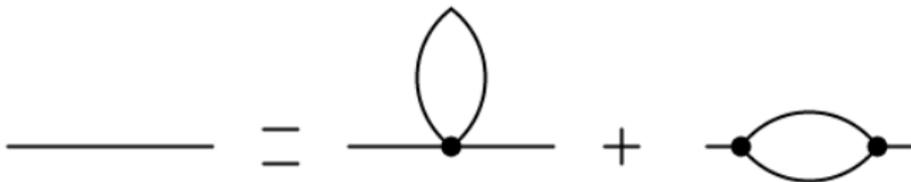
$$G(x_1 \cdots x_n) = \langle 0 | T(\phi(x_1) \cdots \phi(x_n)) | 0 \rangle$$

Which are related to Feynmann diagrams, for  $n = 2$ , it is related to the propagator; it is likely that at  $n=3$  (tree level) a-loops may appear, and this would generate an anomaly.

# Self-energy

# Self-energy

In QFT, the energy that a particle has as a result of changes that it causes in its environment defines self-energy  $\Sigma$ , and represents the contribution to the particle's energy, or effective mass, due to interactions between the particle and its environment.



The term of interaction will be

$$\mathcal{L}_I = \frac{\lambda}{4!} \phi^4 + \frac{\mu}{3!} \phi^3$$

# Self-energy

If we extract the external legs, amputating each of these, we obtain the diagrams

$$\text{---} = \text{---} \left( \text{---} \begin{array}{c} \text{loop} \\ \bullet \end{array} \text{---} + \text{---} \begin{array}{c} \text{loop} \\ \bullet \quad \bullet \end{array} \text{---} \right) \text{---}$$

# Dynamical mass generation

# Dynamical mass generation

Using the definition of self-energy, the full fermion propagator is given by the summation of all diagrams 1PI. The solid dots indicate the Green's functions are fully dressed.

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma S_F^0(p) + S_F^0(p)\Sigma S_F^0(p)\Sigma S_F^0(p) + \dots$$

This can equally well be written as

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma S_F(p)$$

or also

$$S_F(p) = S_F^0(p) + S_F^0(p)\Sigma S_F(p)$$

# Dynamical mass generation

So finally, we obtain the field equation for the inverse fermion propagator

$$S_F(p)^{-1} = S_F^0(p)^{-1} - \Sigma$$

$$\text{---}\overset{\bullet}{\text{---}}\xrightarrow{p} = \text{---}\overset{\bullet}{\text{---}}\xrightarrow{p}^{-1} - \text{---}\overset{\bullet}{\text{---}}\xrightarrow{k} \text{---}\overset{\bullet}{\text{---}}\xrightarrow{q=k-p}$$

whose integral equation has the form

$$S_F(p)^{-1} = S_F^0(p)^{-1} - \frac{\alpha}{4\pi} (3 + \xi) \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta(q)$$

# Dynamical mass generation

Where  $S_F^0(p)^{-1} = p - m_0$  is the inverse bare propagator; the photon propagator in a covariant gauge  $\Delta^{\mu\nu}(q) = \frac{1}{q^2} \left\{ \mathcal{G}(q) \left( q^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \xi \frac{q^\mu q^\nu}{q^2} \right\}$  the complete structure of the fermion-boson interaction vertex  $\Gamma_\nu(k, p)$  so that

$$S_F(p) = \frac{\mathcal{F}(p)}{p - \mathcal{M}(p)}$$

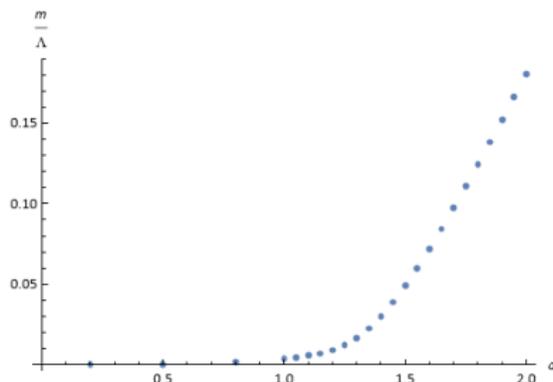
The special case where the bare propagator is  $\mathcal{M}(p) = m_0$  y  $\mathcal{F}(p) = 1$ , we deduce one coupled fermion equations on tracing with the unit matrix  $p$

$$\frac{\mathcal{M}(p)}{\mathcal{F}(p)} = m_0 + \frac{\alpha}{4\pi} (3 + \xi) \int_0^{k^2} dk^2 \frac{\mathcal{F}(k) \mathcal{M}(k)}{k^2 + \mathcal{M}(k)^2} \left[ \theta_+ \frac{k^2}{p^2} + \theta_- \right]$$

# Results

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We initiate a change of variable  $k^2 = \Lambda^2 x$ , so  $dk^2 = \Lambda^2 dx$ . We will use the program *Wolfram Mathematica: Modern Technical computing* considering the conditions  $\xi = 0$  and a  $\alpha \gg m_0$  and  $m_0 \rightarrow 0$  doing a quadrature Gaussian weights and taking different values for  $0.5 < \alpha < 2$ .



$$\mathcal{M}(p) = m_0 + \frac{\alpha}{4\pi} (3) \int_0^{\Lambda^2 x} dx \frac{\mathcal{M}(k)}{x + \left(\frac{\mathcal{M}(k)}{\Lambda}\right)^2} \left[ \theta_+ \frac{k^2}{p^2} + \theta_- \right]$$

# Summary

# Summary

- In a non-perturbative phenomena such as confinement and dynamic chiral symmetry breaking are addressed naturally in this context.
- In studies of such phenomena, it is necessary a truncation of this tower of equations, which can be done by making a sustainable guess about a or more of the Green functions relevant to the study.
- The non-perturbative nature of ESD leads us to study this type of phenomena in plausible scenarios, which which in many theories represents a formidable mathematical complexity. However, in the rainbow approximation, valuable information about the nature of the particles can be obtained. fundamental, especially on the origin of their masses.

