Thermodynamics of Gambling Demons

Theory and Experiment

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IFISC

AGENCIA ESTATAL DE INVESTIGACIÓN

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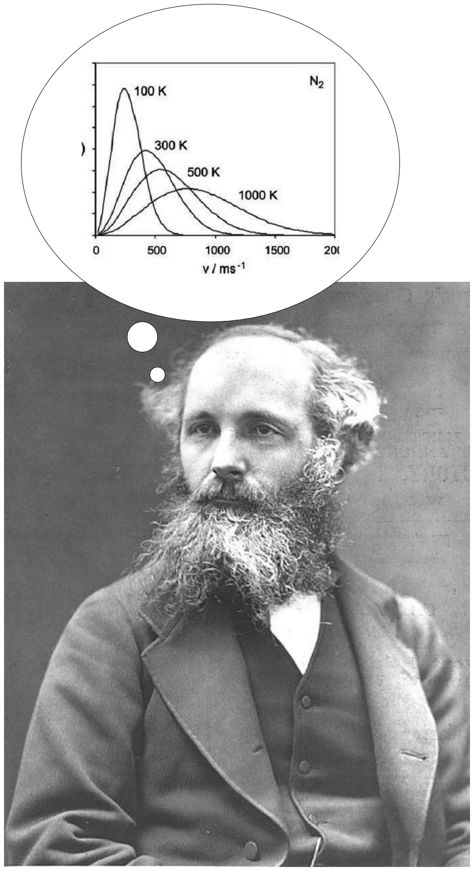
Universidad de los Andes Colombia | Facultad de Ciencias Departamento de Física | Universidad Nacional de Colombia

4th WORKSHOP ON STATISTICAL PHYSICS 2023

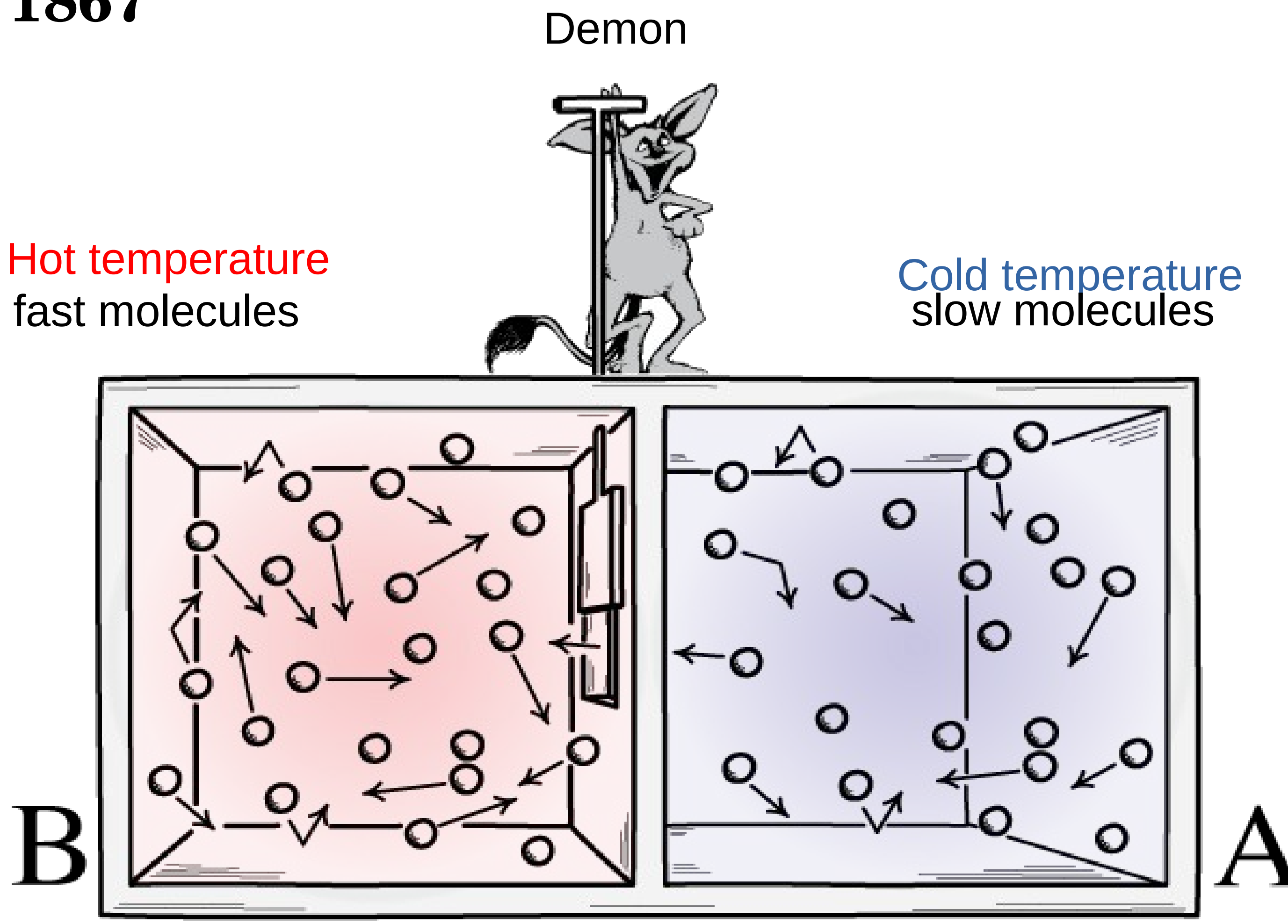
Outline of this talk:

- **Introduction**
- **Idea of gambling demons**
- **Martingale theory**
- **Theoretical results**
- **Experiment: single-electron box**

Maxwell's demon 1867



James C. Maxwell
(1831 – 1879)

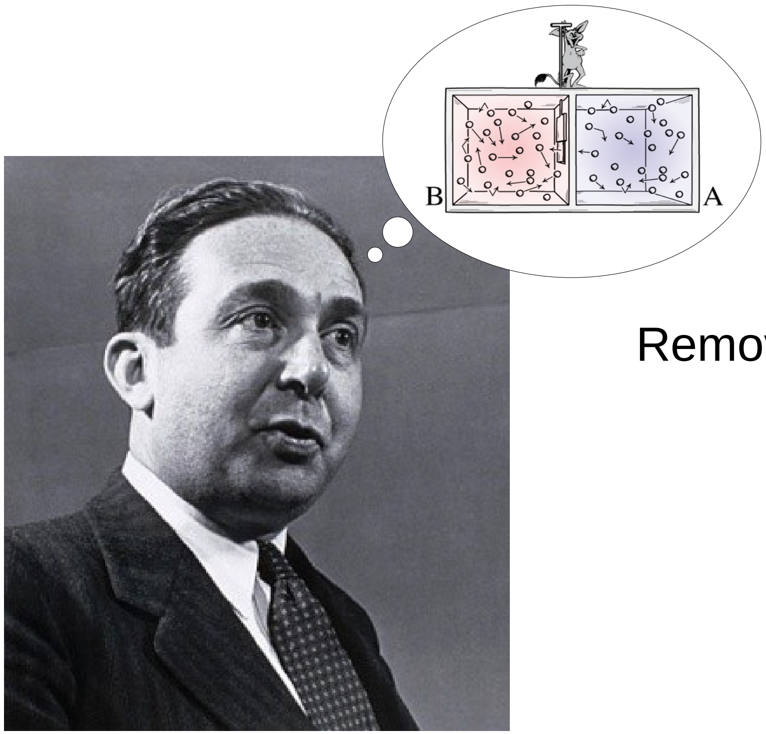


Spontaneous heat flow →

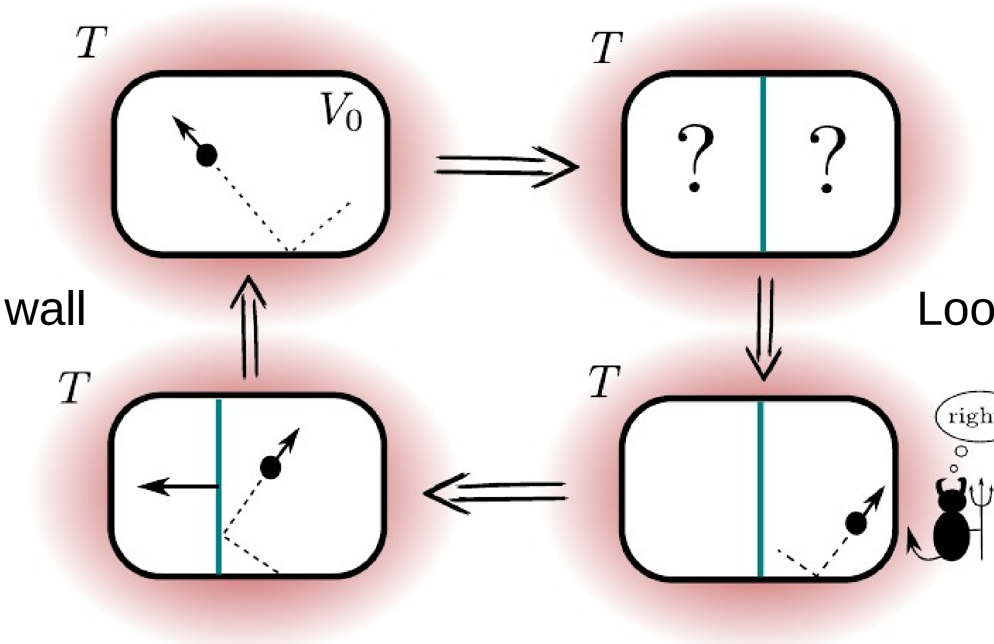
Paradoxical effect ←

Drawing: Jonh D. Norton

Szilard’s engine 1929



Leó Szilard (1898 - 1964)

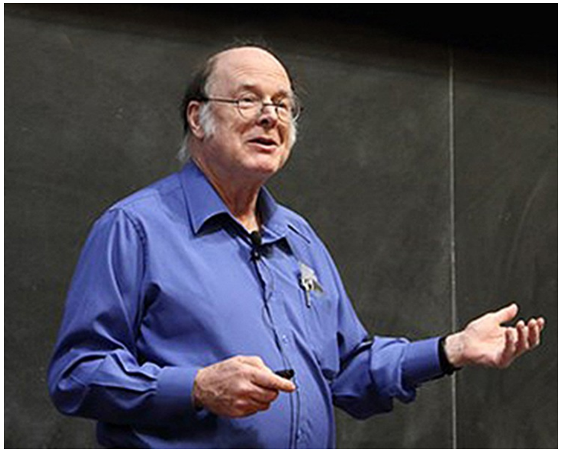


Optimal work extraction per cycle: $$W_{\mathrm{ext}} = k_B T \log\left(\frac{V_0}{V_0/2}\right) = k_B T \log 2$$

Szilard’s conclusion: thermodynamic cost of measurement?

Informational Exorcism

Only apparent “violations” of second law. Take into account informational costs!



Charles H. Bennett

Landauer’s Erasure 1961

$$W \geq k_B T \log 2$$

Erasing information (1 bit of) has a minimum work cost even if doing it reversibly

NO FREE LUNCH!

Information is physical!

Rolf W. Landauer (1927 - 1999)

Thermodynamics of feedback control: $$\Delta S - \frac{Q}{T} \geq -\mathcal{I}$$ information acquired /stored

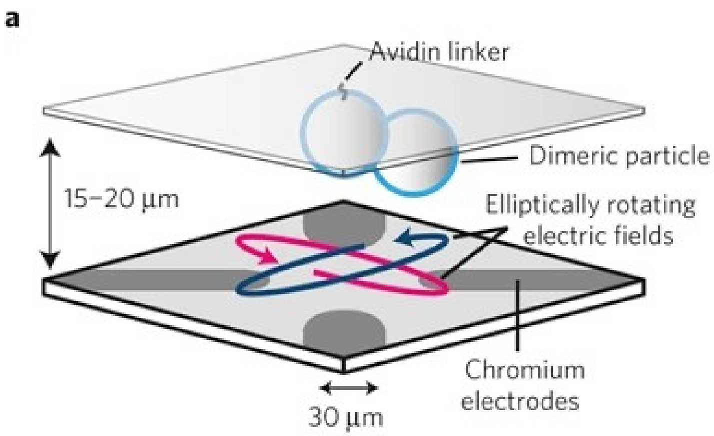
Recall Jonh Bechoefer and Carlos Alvarez talks !

Theory Review: JMR Parrondo, JM Horowitz, and T Sagawa, Nat. Phys. (2015).

Experiments

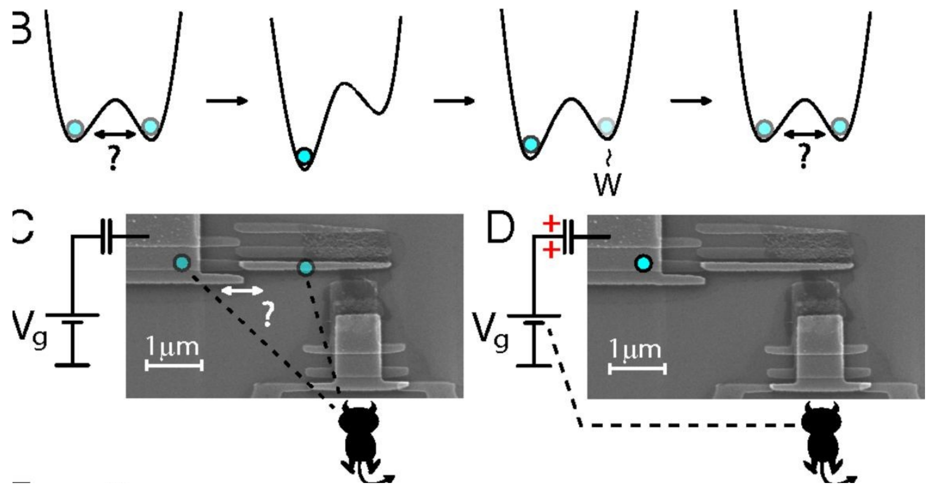
Colloidal particles in optical traps

S Toyabe *et al.* Nat. Phys. **6** (2010)



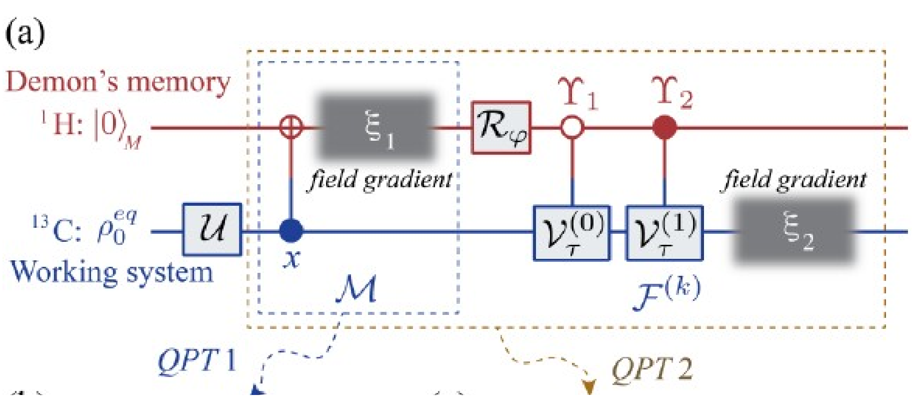
Electronic devices

JV Koski *et al.* PNAS **111** (2014)



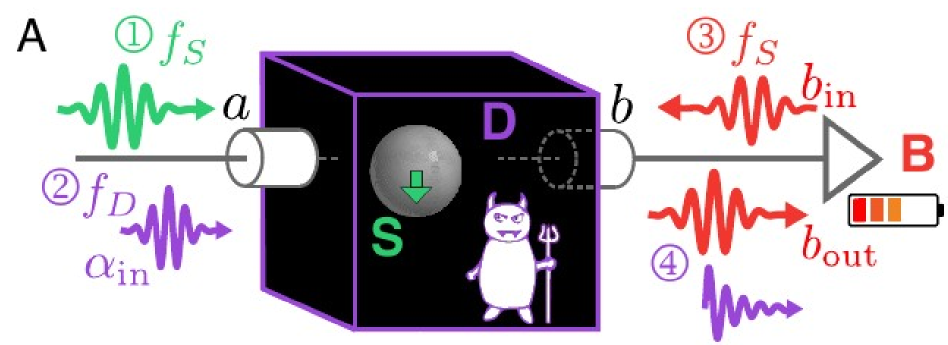
Nuclear spins and NMR spectrometry

PA Camati *et al.* PRL **117** (2016)



Circuit QED setups

N Cottet *et al.* PNAS **114** (2017)



Experiments Review: S Ciliberto, E. Lutz, Physics Today (2015)

Gambling Demons

New version of Maxwell's demon based on stopping strategies:

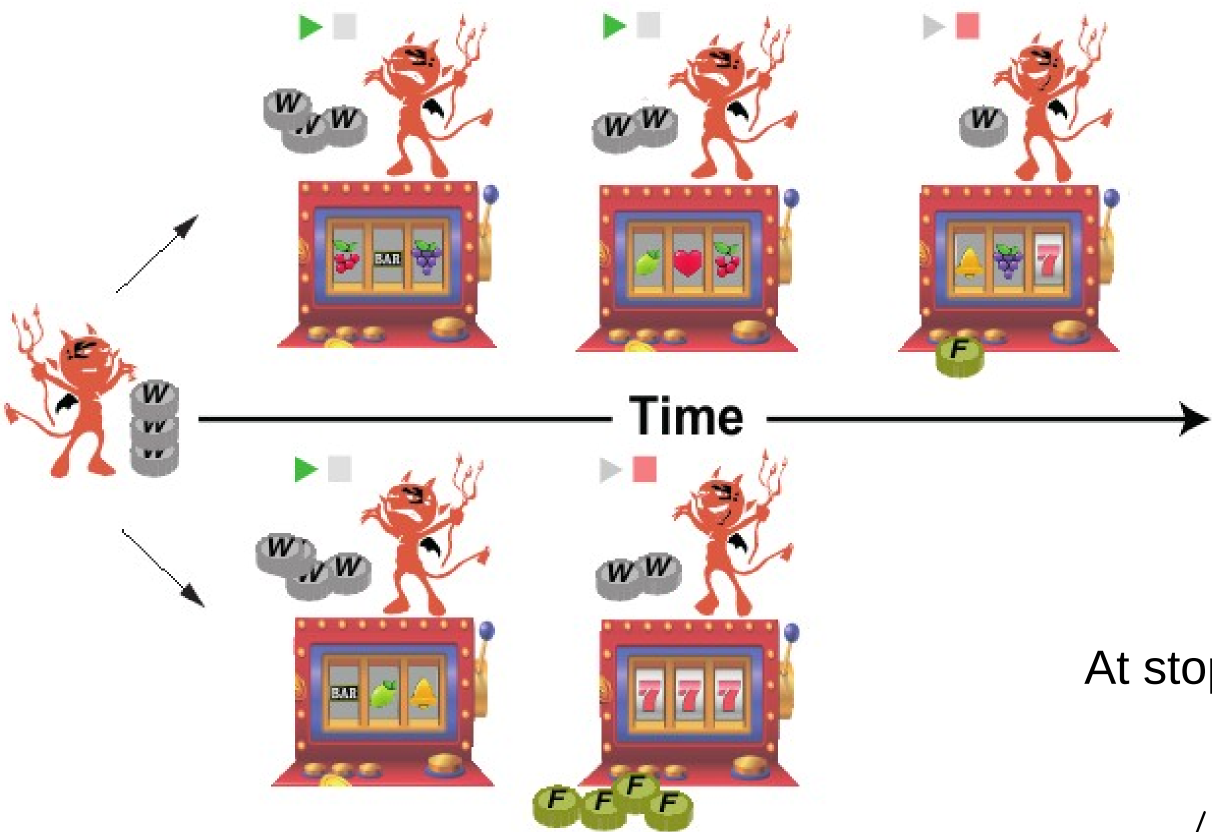
(I) gather information about microscopic dynamics → ok

(II) feedback control (trapdoor / piston) → **Reduce to minimal expression!** only stop (or not) the dynamics

Players in the casino or agents in the market can decide to play or not a game, but cannot change the rules of the game (feedback).

Gambling demon idea

Analogous to a gambler in a slot machine:



τ

$\mathcal{T} \leq \tau$

At final time: τ

$$\langle W \rangle > \langle \Delta F \rangle$$

At stopping times: $\mathcal{T}$

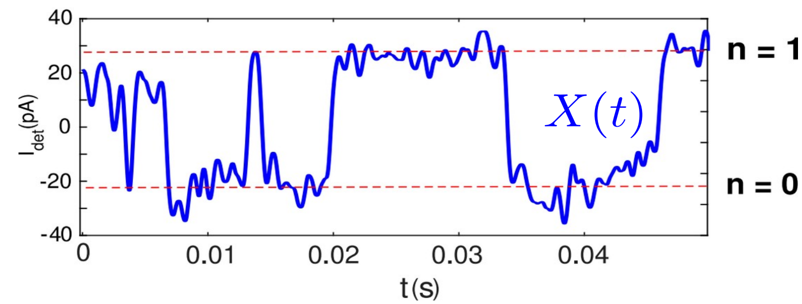
$$\langle W \rangle_{\mathcal{T}} \overset{?}{<} \langle \Delta F \rangle_{\mathcal{T}}$$

Average over trajectories of different length !

Second law at the microscale: Fluctuation relations

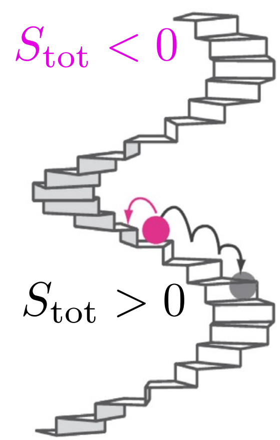
System degree of freedom: $X(t)$

follows stochastic paths $\mathbf{X}_\tau = \{X(t)\}_{t=0}^{\tau}$

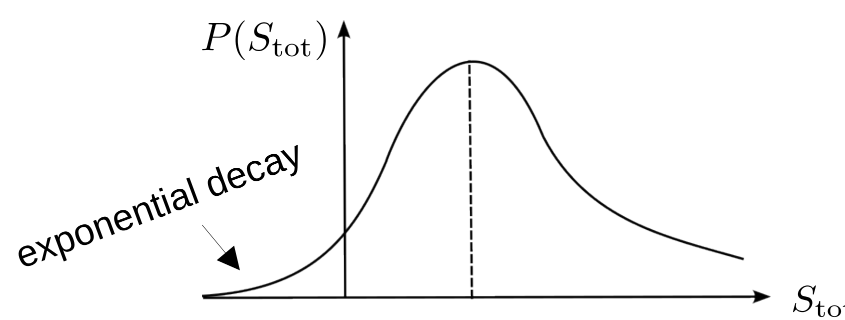


Stochastic entropy production: $$S_{\rm tot}(\tau) = \log\left(\frac{P(\mathbf{X}_\tau)}{\tilde{P}(\tilde{\mathbf{X}}_\tau)}\right) = \Delta S - \sum_k \frac{Q_k}{T_k}$$

system reservoirs

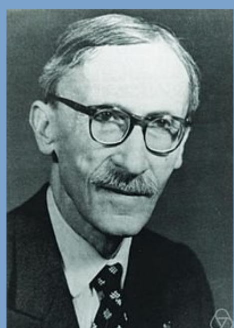


$$\langle e^{-S_{\rm tot}}\rangle = 1 \qquad \langle S_{\rm tot}\rangle \geq 0$$



U. Seifert, Rep. Prog. Phys. (2012) M. Esposito *et al*. Rev. Mod. Phys. (2009)

Martingale theory:

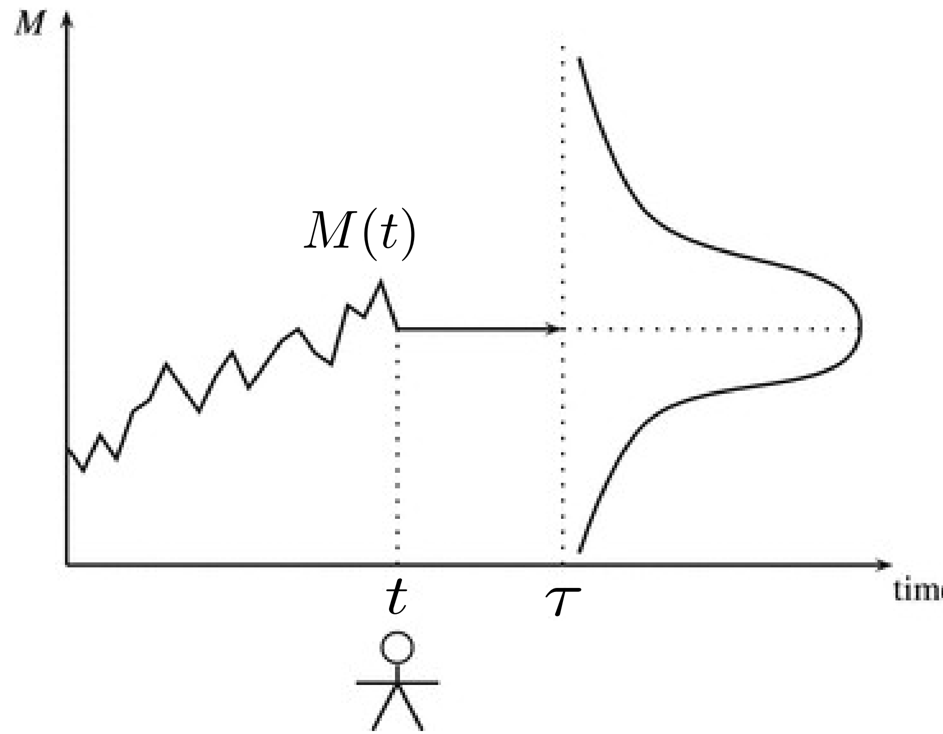


Martingale process

Introduced in prob. theory by Paul Lévy in 1934

$$\langle M(\tau)|\mathbf{X}_t\rangle = M(t)$$

$$\text{for} \quad 0 \leq t \leq \tau$$



Martingales verify many interesting properties:

- Doob's optional stopping theorem: $\langle M(t)\rangle_{\mathcal{T}} = \langle M(0)\rangle$ at stochastic times $\mathcal{T}$
- Doob's maximal inequality: $\Pr[M_{\max}(\tau) \geq m] \leq \langle M(\tau)\rangle / m$ maximum in an interval

Entropy production is an exponential martingale:

$$\langle e^{-S_{\rm tot}(\tau)} | \mathbf{X}_t \rangle = e^{-S_{\rm tot}(t)}$$

Nonequilibrium stationary processes

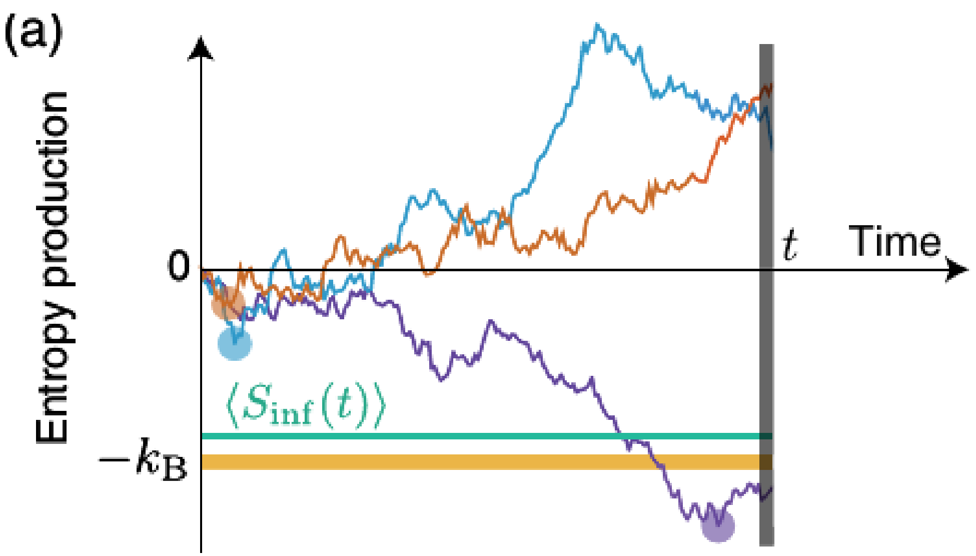
Stronger than fluctuation theorem! if $t=0$ $\langle e^{-S_{\rm tot}(\tau)} | X_0 \rangle = e^{-S_{\rm tot}(0)} = 1$

+ Stopping-times fluctuation relation

$$\langle e^{-S_{\rm tot}(\mathcal{T})} \rangle = 1 \qquad \langle S_{\rm tot}(\mathcal{T}) \rangle \geq 0$$

+ Statistics of EP finite-time minimum

$$\Pr\left(S_{\rm min}(t) \leq -\xi\right) \leq e^{-\xi}$$



I. Neri, É. Roldán, F. Julicher PRX **7** (2017), R. Chétrite *et al.* EPL **124** (2019), I. Neri *et al.* JSM (2019) ...

Recent review: É.Roldán *et al. arXiv:2210.09983 (2022)*

Generalized stopping-times fluctuation relations

Extension of martingale theory for **driven systems** (with fixed protocol):

$$\langle e^{-S_{\rm tot}(\tau)} | \mathbf{X}_t \rangle \neq e^{-S_{\rm tot}(t)} \quad \text{but} \quad \langle e^{-S_{\rm tot}(\tau)-\delta(\tau)} | \mathbf{X}_t \rangle = e^{-S_{\rm tot}(t)-\delta(t)}$$

with the extra term "stochastic asymmetry":

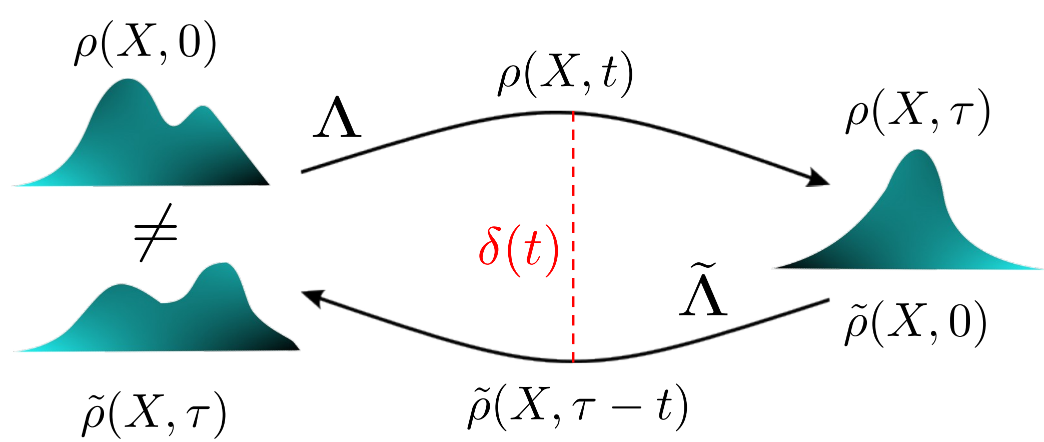
$$\delta(t) = \log\left(\frac{\rho[X(t),t]}{\tilde{\rho}[X(t),\tau-t]}\right)$$

asymmetry of the prob. density under time-reversal

$$\langle e^{-\beta[W-\Delta F]-\delta} \rangle_{\mathcal{T}} = 1$$

Entropy production Extra term

$$\langle W \rangle_{\mathcal{T}} - \langle \Delta F \rangle_{\mathcal{T}} \geq -k_B T \langle \delta \rangle_{\mathcal{T}}$$



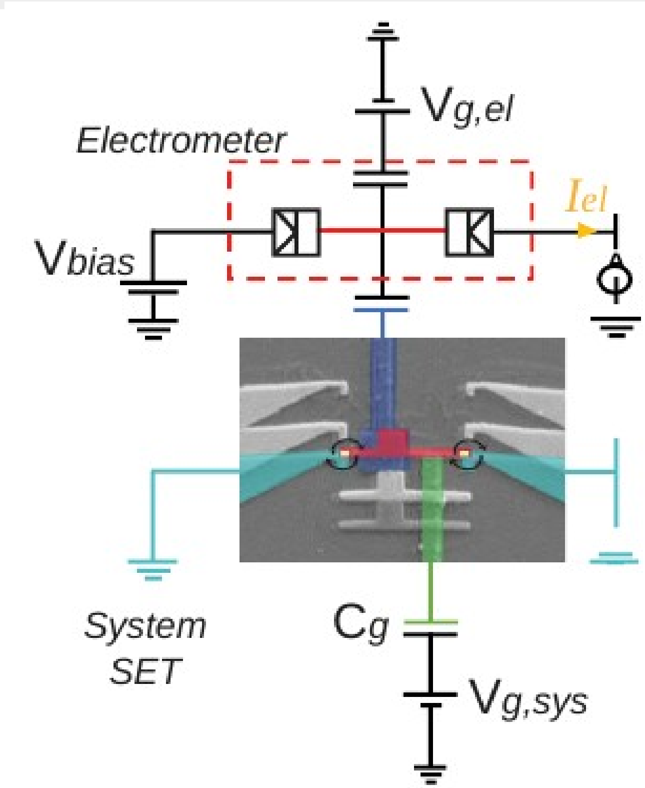
Single Electron Box (SEB): [PICO group (Helsinki)]

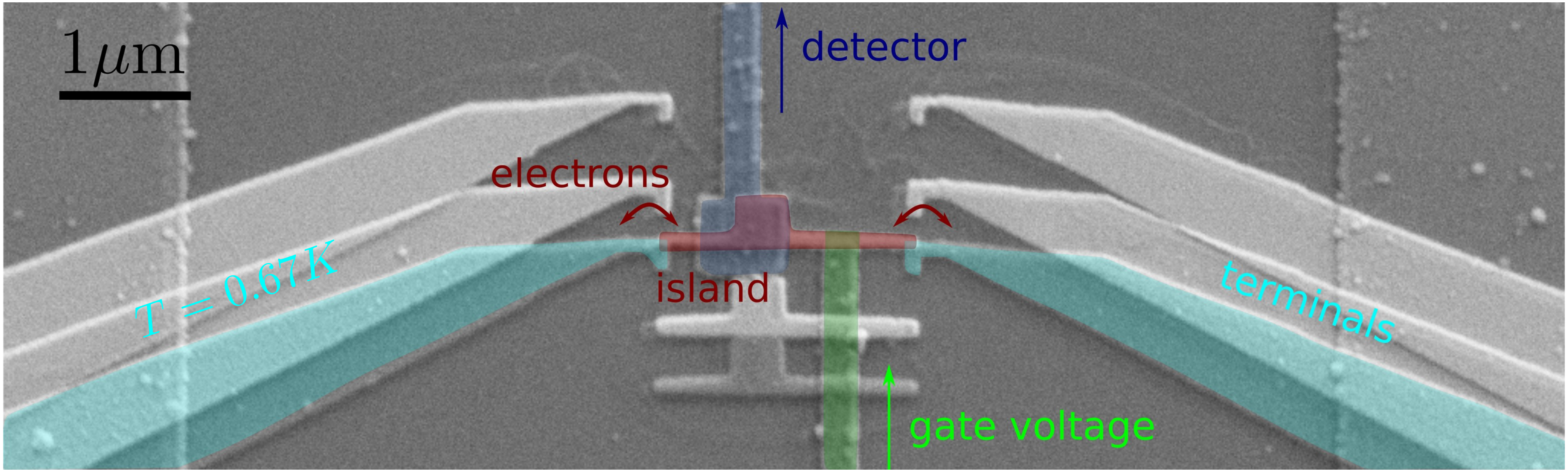
System: Cu island

Thermal reservoir: Al superconducting leads

Driving protocol: gate voltage following a linear ramp

Detector: SET monitors tunneling events



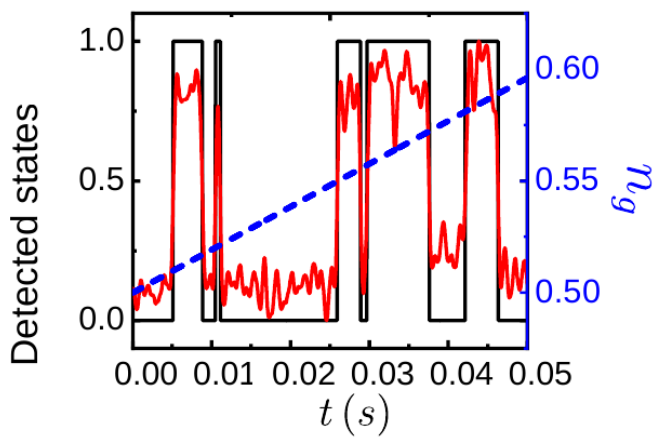


scanning electron micrograph

Experiment

Driven two-level system:

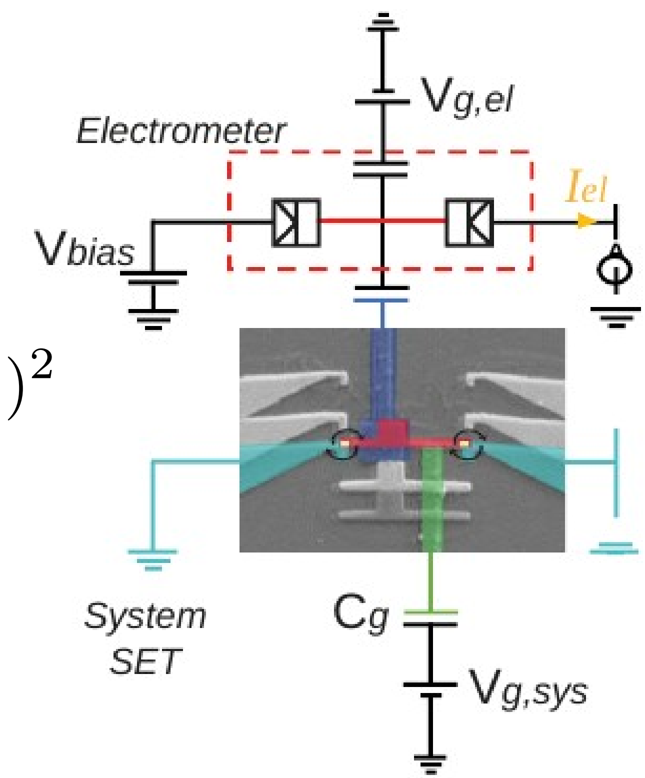
n=0 / n=1 (excess) electron in the box

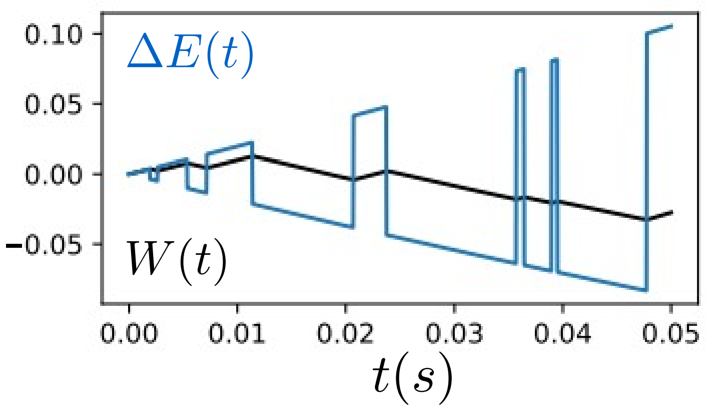


$$H(n) = E_c(n - n_g)^2$$

charge state

offset charge





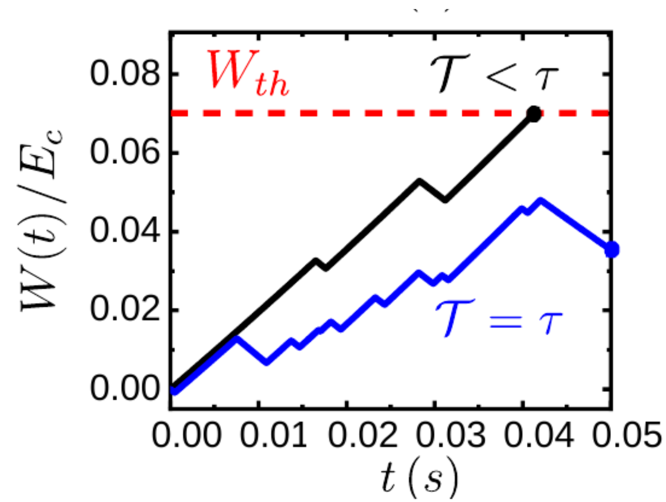
$$E_c = 109\mu\text{eV} \simeq 1.9 k_B T$$

More details (setup): J.P. Pekola, Nat. Phys. **11** (2015) O. Maillet *et al*. PRL **122** (2019)

Strategy: *Finite-horizon work first-passage times*

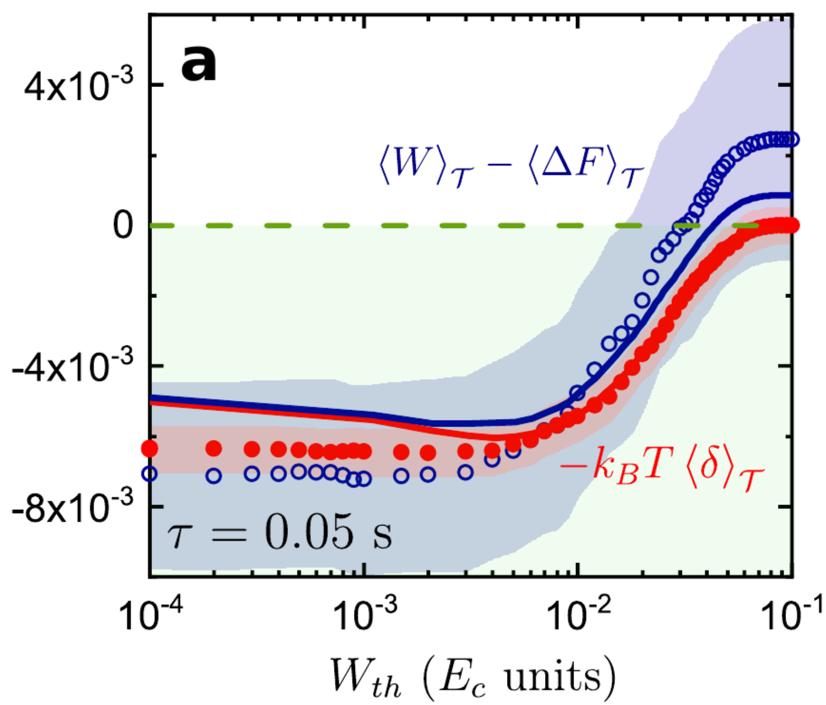
Stop trajectories when the work reaches a threshold

If threshold is not reached, we continue until the end.

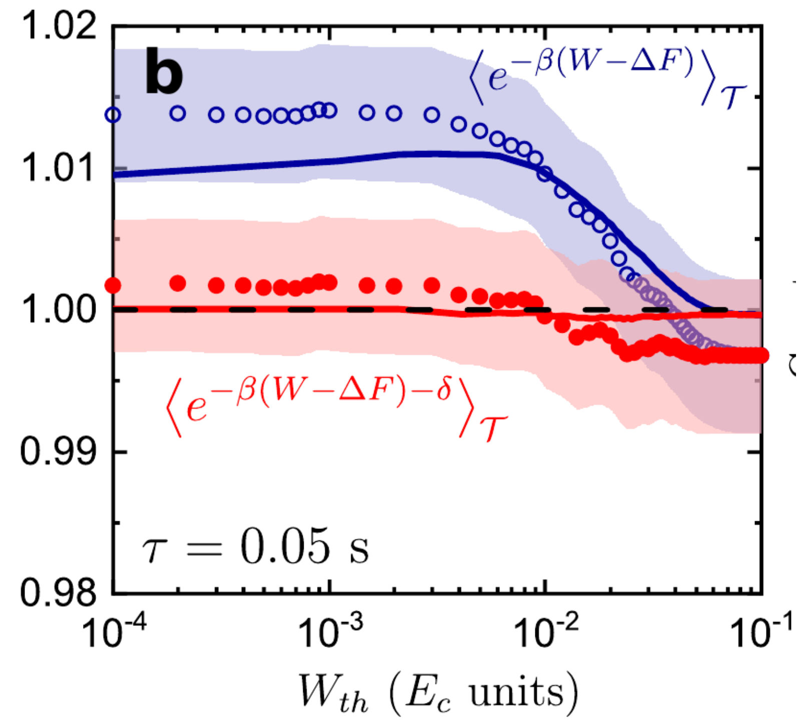


Experimental results:

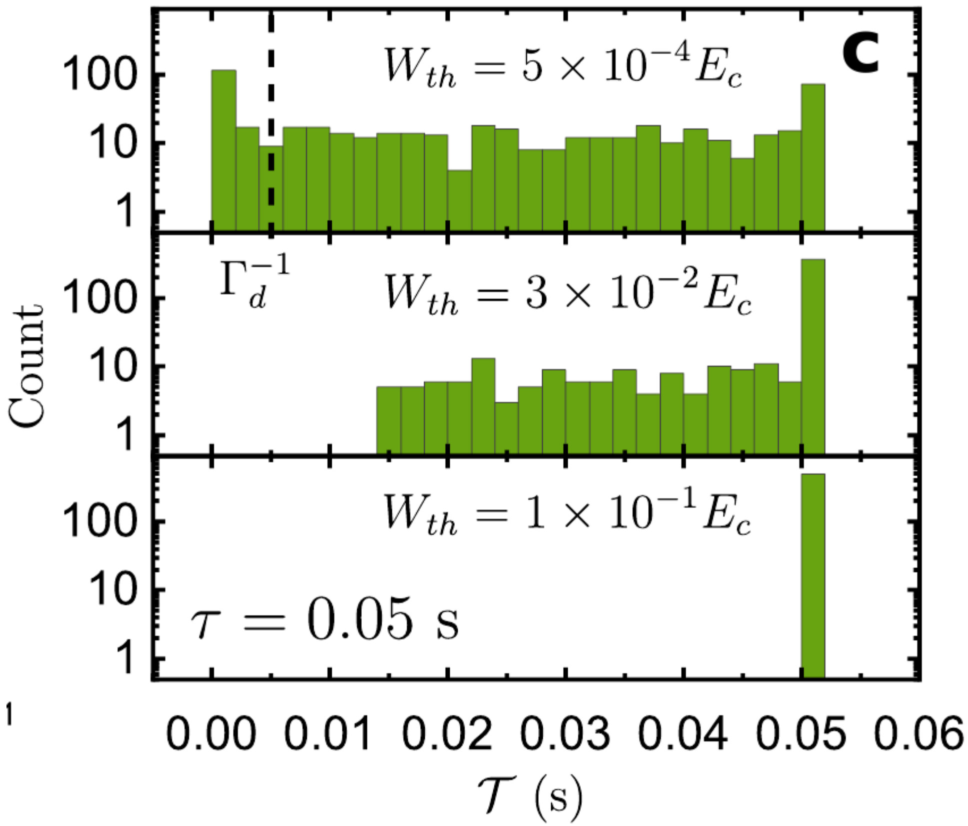
S-L inequality



fluctuation theorem



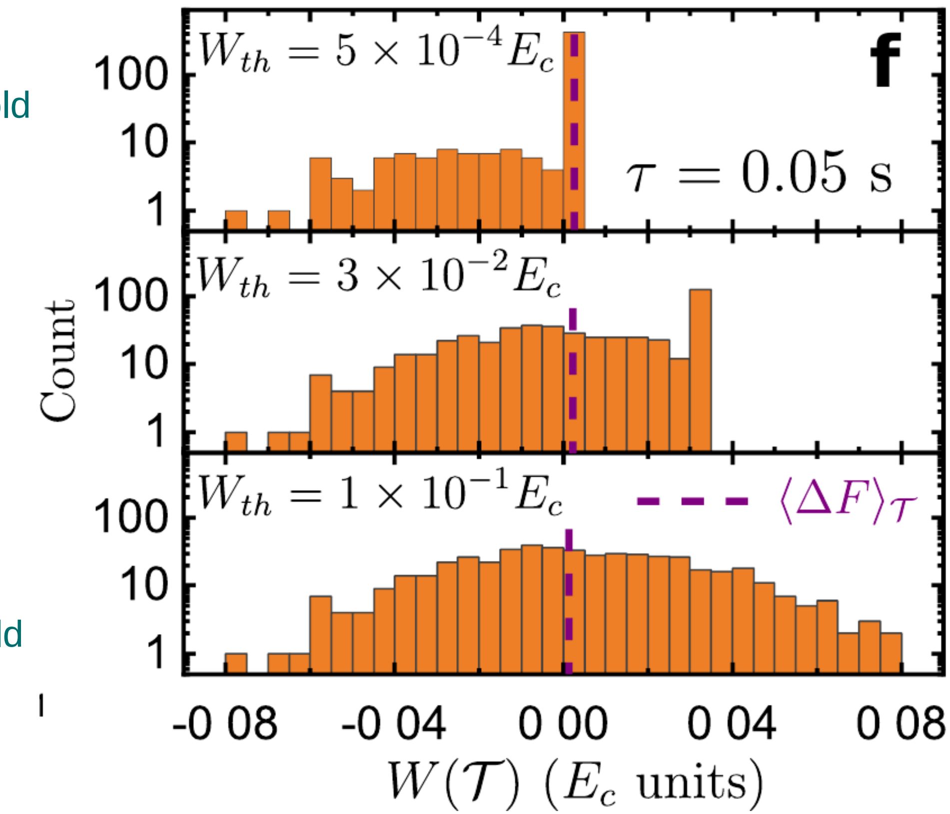
Stopping times histogram



small threshold

big threshold

Work prob. distribution



We can extract work by stopping at small thresholds (on average) !

At large thresholds almost all trajectories stop at the final time: $\mathcal{T} \simeq \tau$

Summarizing

Remarks:

$$\langle W\rangle_{\mathcal{T}} - \langle \Delta F\rangle_{\mathcal{T}} \;\geq\; -k_B T \langle \delta \rangle_{\mathcal{T}}$$

Successful gambling when we have:

1. Time-reversal asymmetry — $\langle \delta \rangle_{\mathcal{T}} > 0$
2. A good strategy / Information — monitor the system to decide when to stop

Outlook: compare with information inequalities ?

$$\langle W\rangle - \langle \Delta F\rangle \geq -k_B T \mathcal{I}$$

nature physics
LETTERS
https://doi.org/10.1038/s41567-019-0481-0

Large work extraction and the Landauer limit in a continuous Maxwell demon

M. Ribezzi-Crivellari[1,2] and F. Ritort[1,3*]

In the continuous limit:

$$\mathcal{I} \longrightarrow \infty$$

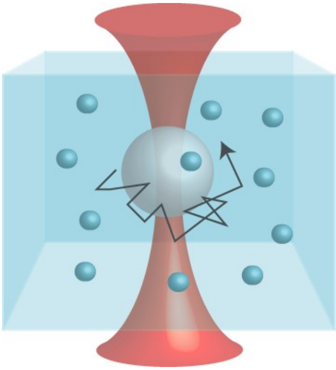
Looser bounds

Other recent experiment:

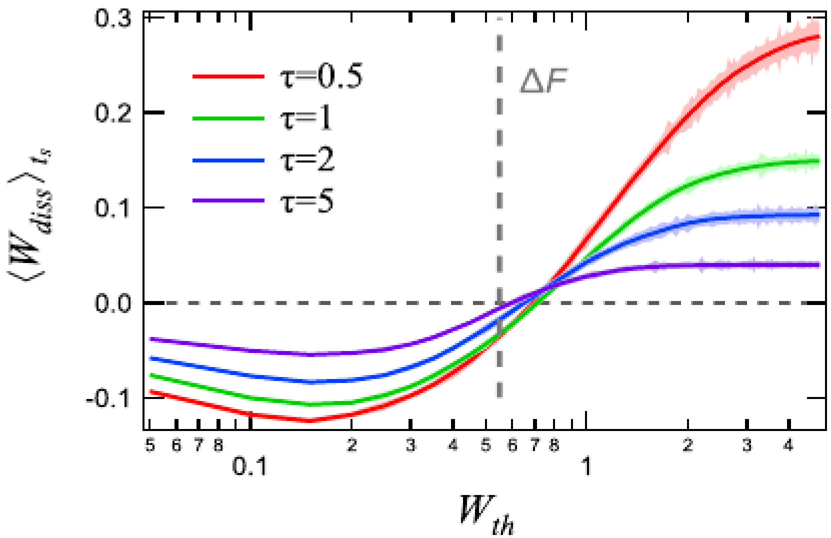
J.A.C. Albay, Y. Jun and P-Y Lai, PRR **5**, 023115 (2023)

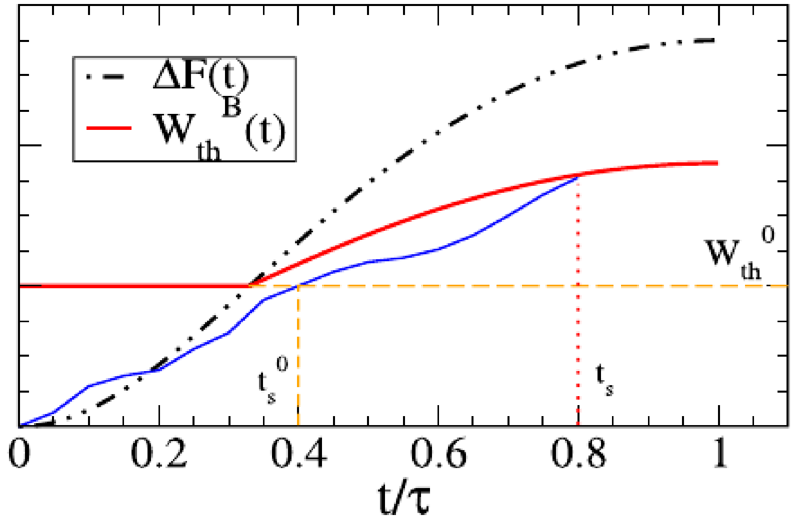
Overdamped Brownian particle under harmonic trapping (optical tweezers)

$$V(x,\lambda) = \frac{1}{2}\lambda(t)x^2 \qquad \lambda(t) = \lambda_0[2 - \cos\left(\frac{\pi t}{\tau}\right)]$$


$T = 300K$

Testing improvements modifying the stopping thresholds:





Main conclusions

- We introduced a "gambling demon" that stops a driven nonequilibrium thermodynamic process at stochastic times.

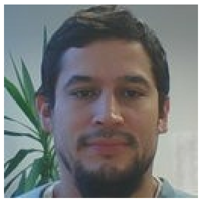
  + Since it does not require feedback control, its applicability should be easier.

- We derived universal fluctuation relations for work and entropy production at stopping times and second-law-like inequalities.

  + Valid in more general situations e.g. arbitrary strategies, several baths, etc.

- We tested our results experimentally in a single electron box, where we have found a simple and good strategy allowing work extraction.

  + Optimal strategies?



Aalto University

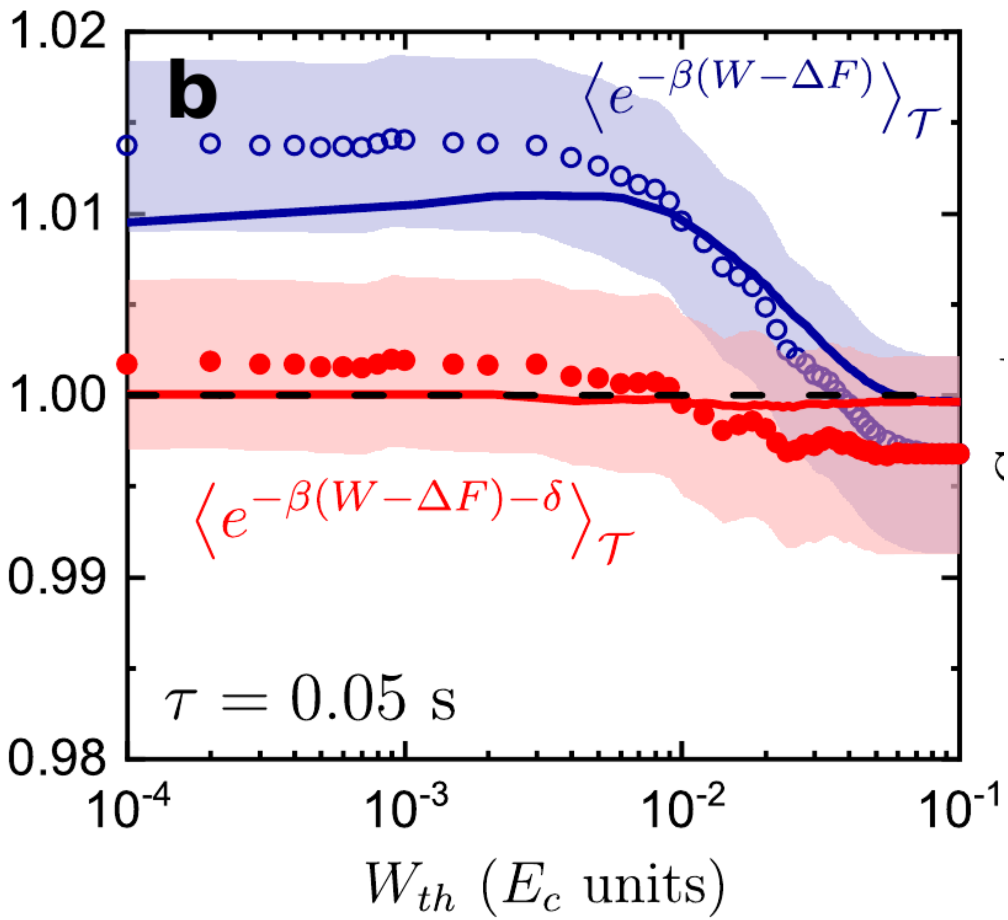
THANK YOU

for your attention

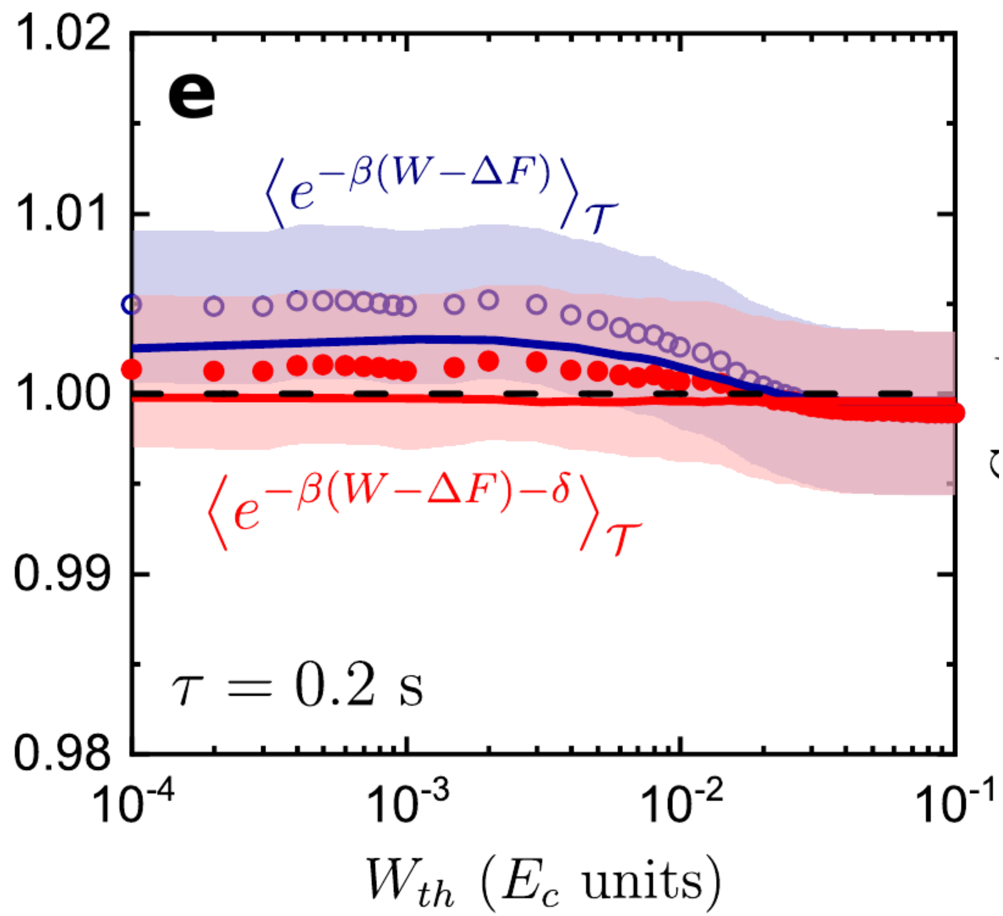

ICTP

For more information: Phys. Rev. Lett. **126**, 080603 (2021)

Experimental results: Fluctuation theorem at stopping times



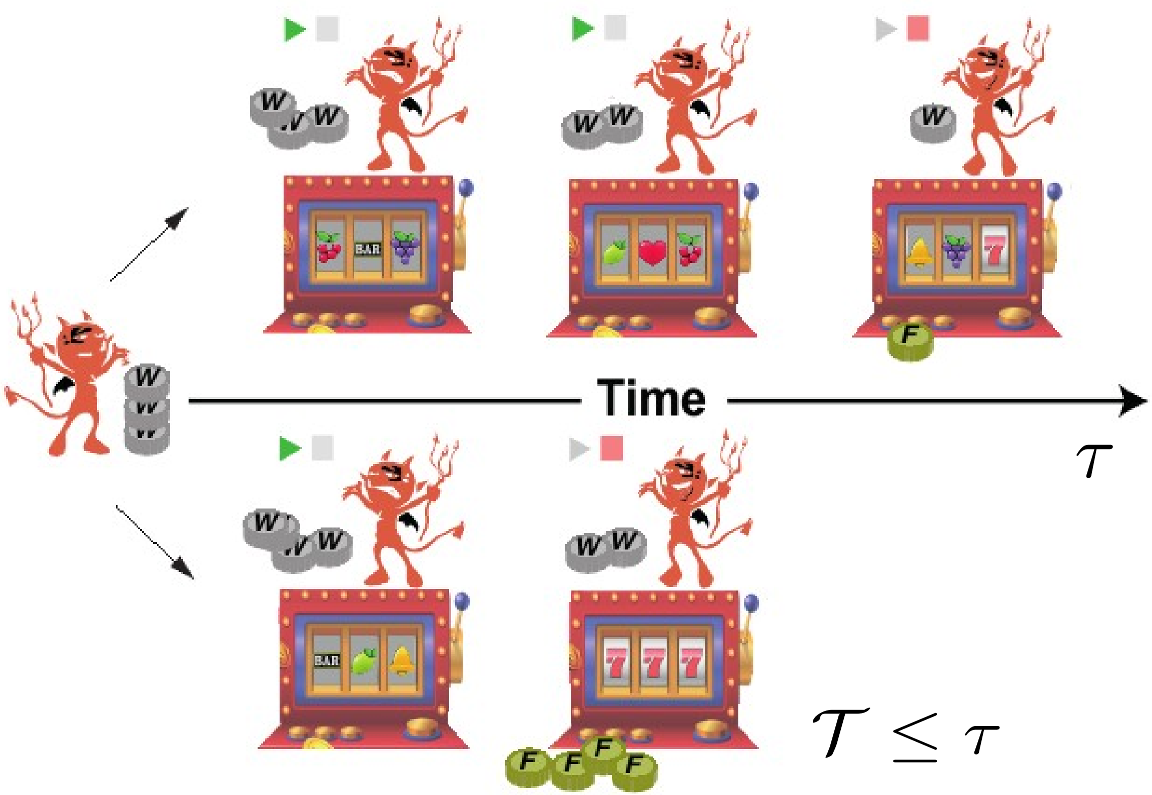
Faster protocol (larger “violations”)



Slower protocol (smaller “violations”)

- **Gambling is possible !!**

$$\langle W \rangle_{\mathcal{T}} - \langle \Delta F \rangle_{\mathcal{T}} \geq -k_B T \langle \delta \rangle_{\mathcal{T}} \quad \text{if} \quad \langle \delta \rangle_{\mathcal{T}} > 0$$ (time-reversal asymmetry)



at stopping times: $\mathcal{T}$

$$\langle W \rangle_{\mathcal{T}} < \langle \Delta F \rangle_{\mathcal{T}}$$

Is possible !

(but it is not guaranteed)