Quantum Thermal Machines

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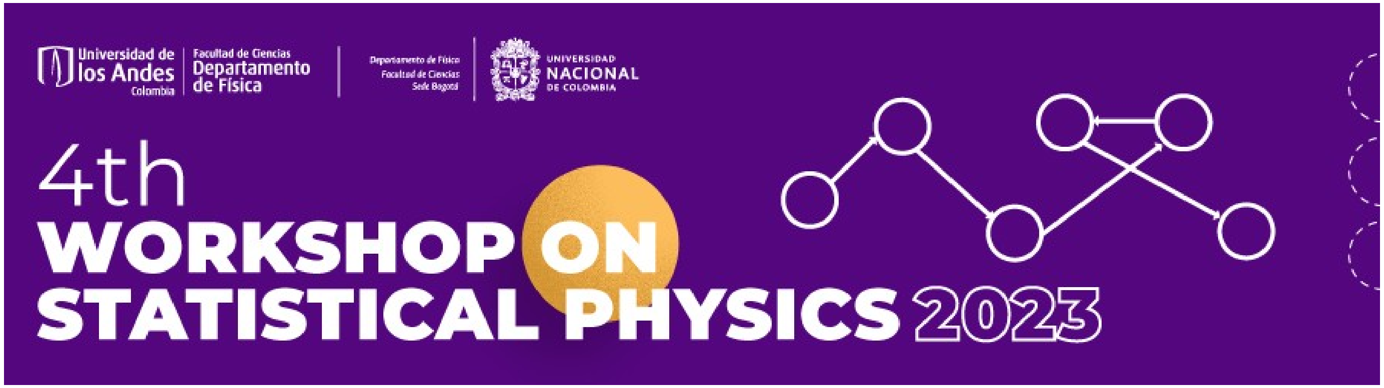
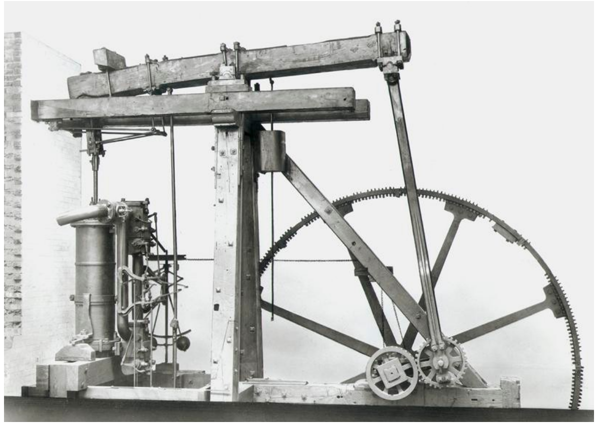
MINI-COURSE:

Quantum thermodynamics: fluctuations and thermal machines

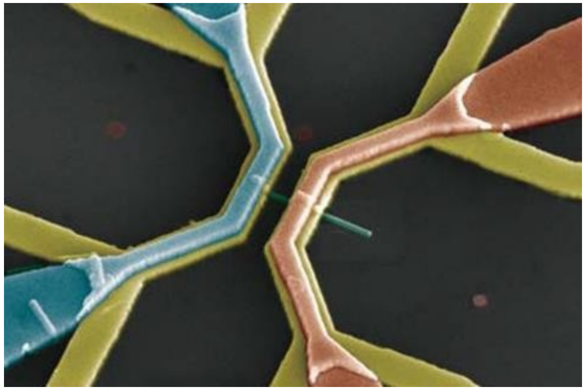


Macroscopic (classical) heat engines



(Watt's steam engine, 1769)

Microscopic (quantum) heat engines



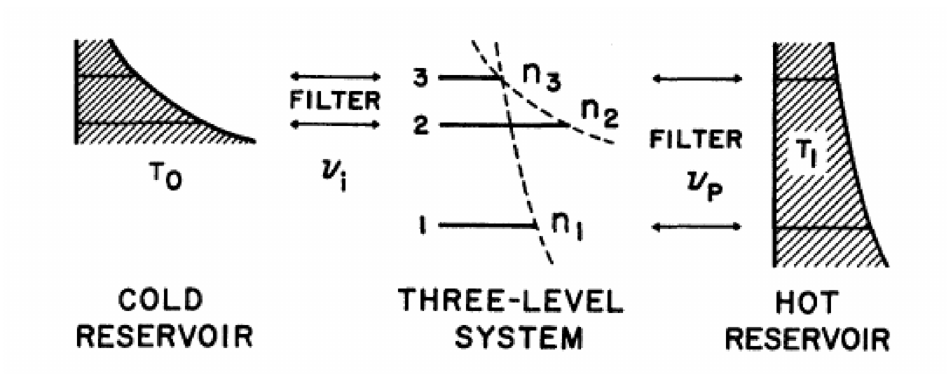
(Quantum-dot engine, 2018
H. Linke group, Sweden)

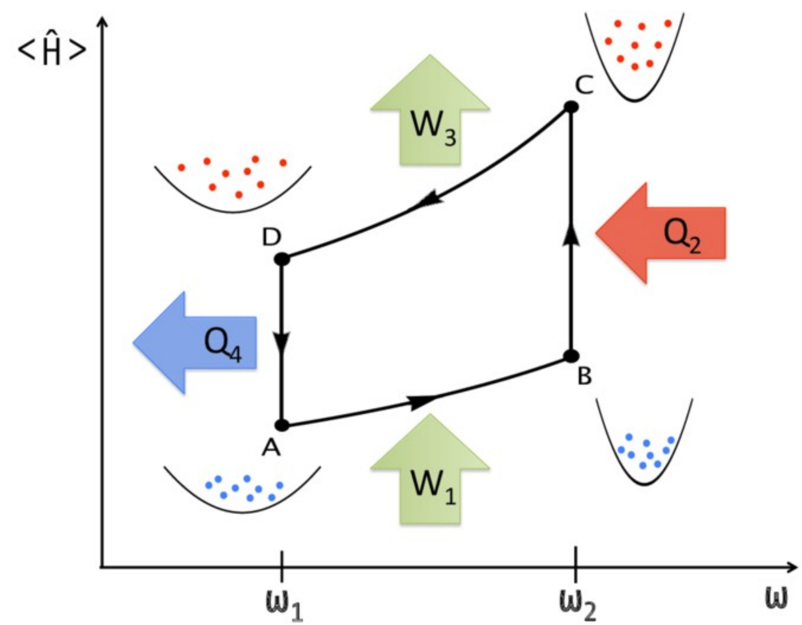
+ Large number of degrees of freedom
+ Fluctuations become negligible
+ Classical thermodynamics

+ Small systems (nanoscale)
+ Fluctuations are important
+ Stochastic and quantum thermodynamics
+ Quantum-mechanical enhancements

+ **Pioneering works in 60s**:

Three-Level Masers as Heat Engines Scovil and Schulz-Dubois, *Phys. Rev. Lett.* ***2*** *(1959)*.





+ **Cyclic vs. continuous operation engines**

Quantum Carnot, Otto and Stirling cycles Quan, Liu, Sun, Nori, *Phys. Rev. E* ***76*** *(2007)*.

Steady-state heat engines: lasers, masers, solar cells, thermoelectric devices ...

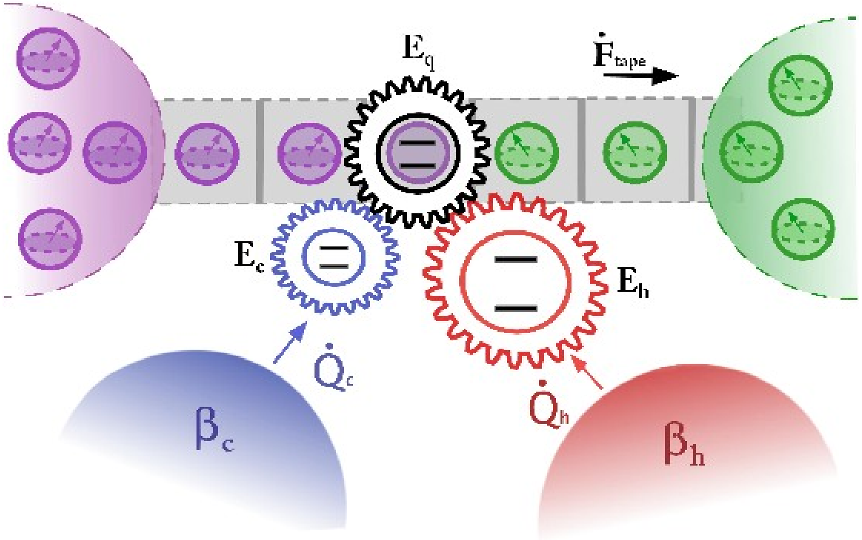
Kosloff & Levy, *Annu. Rev. Phys. Chem.* ***65*** *(2014)*.

Why developing and studying models of quantum heat engines?

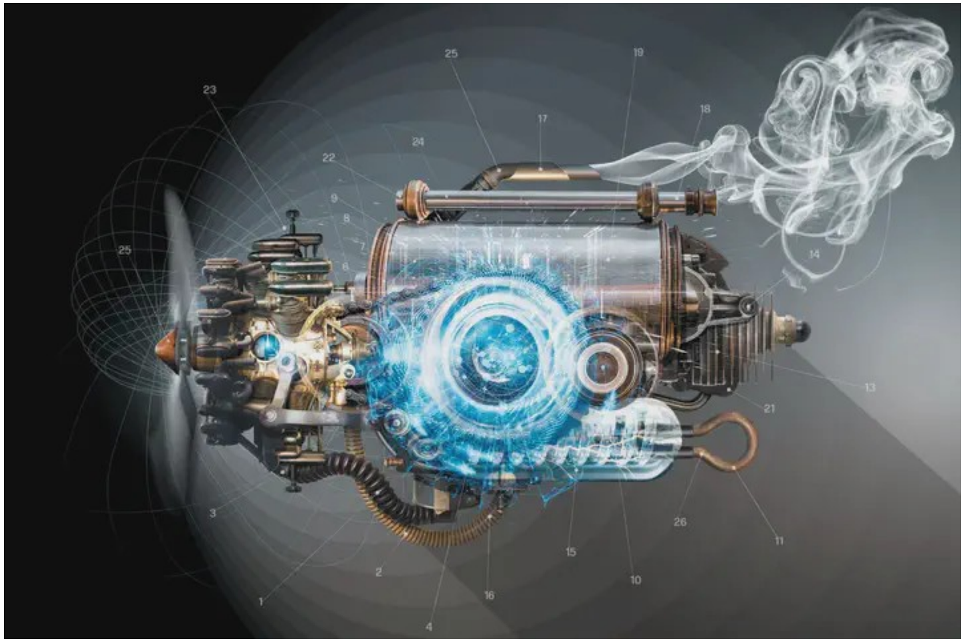
+ Simple setups to explore and test fundamental issues in quantum thermodynamics

+ Is there any miniaturization limit for heat engines and refrigerators ?

+ Is there any systematic quantum-thermodynamic advantage in the performance with respect to classical engines?



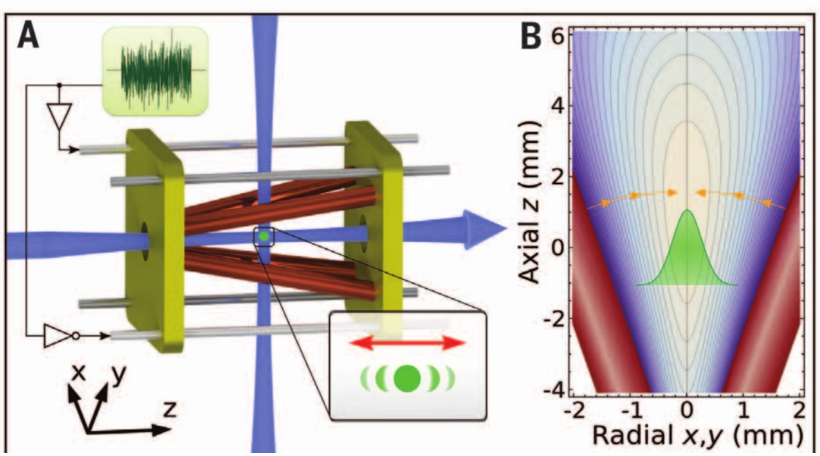
K. Hammam *et al.* NJP **23** 043024 (2021)



QTD Maryland 2024 conference webpage picture

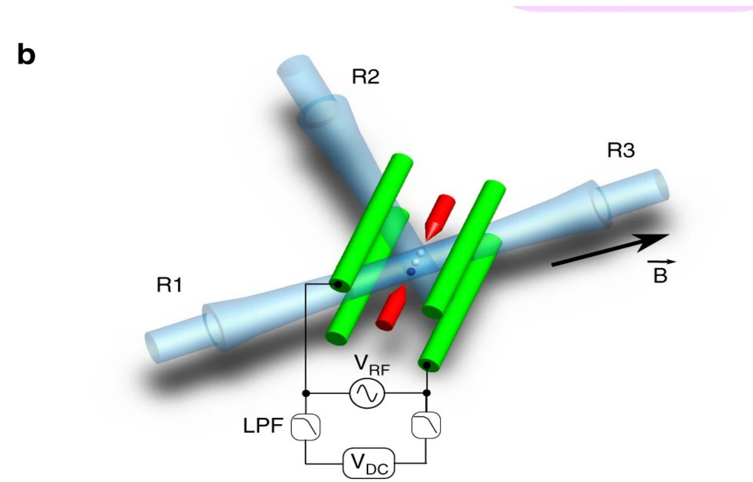
First prototypes

Single-atom quantum engine



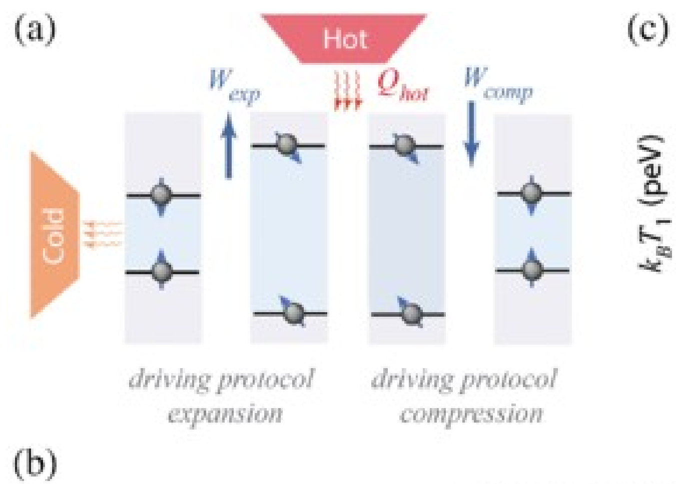
Roßnagel, *et al.* Science (2016)

Quantum absorption refrigerator



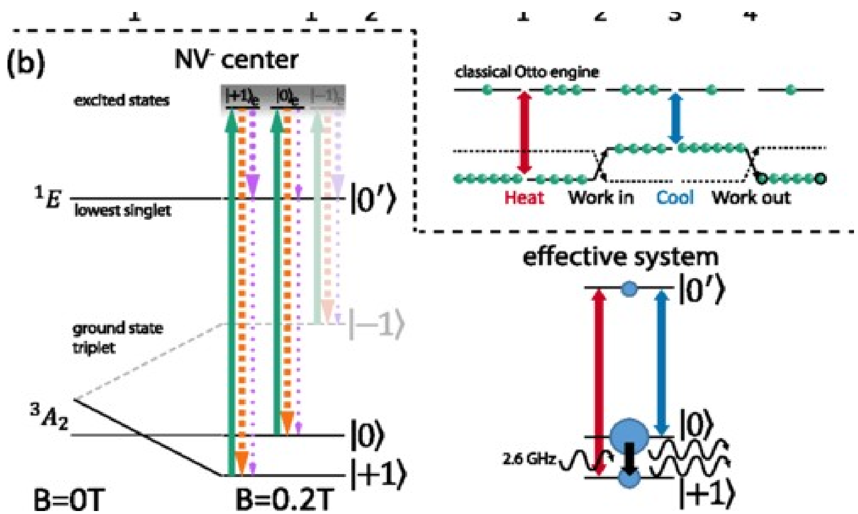
Maslennikov, *et al.* Nat Commun. (2019)

Spin Otto cycle with NMR techniques



Peterson, *et al.* Phys. Rev. Lett **123** (2019)

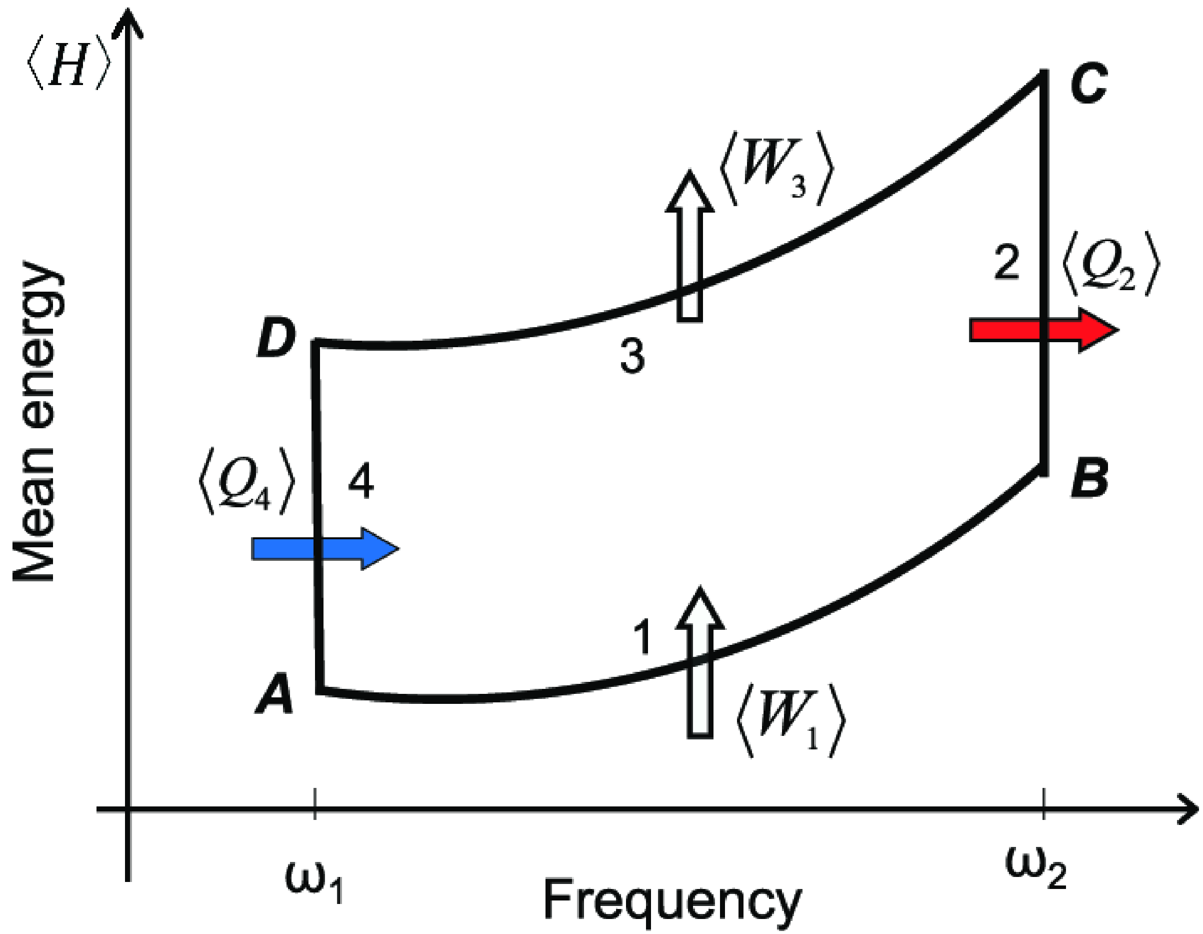
Continuous three-level engine with NV centers



Klatzow, *et al.* Phys. Rev. Lett **122** (2019)

Quantum Otto cycle

+ Quantum system (working substance) is driven by an external agent and selectively put in contact with two thermal reservoirs at different temperatures in a cyclic way



O. Abbah, M. Paternostro, E. Lutz PRR **2** (2020)

Example: Harmonic oscillator

$$H = \hbar\omega a^\dagger a \qquad \omega_1 \leftrightarrow w_2$$

Four strokes:

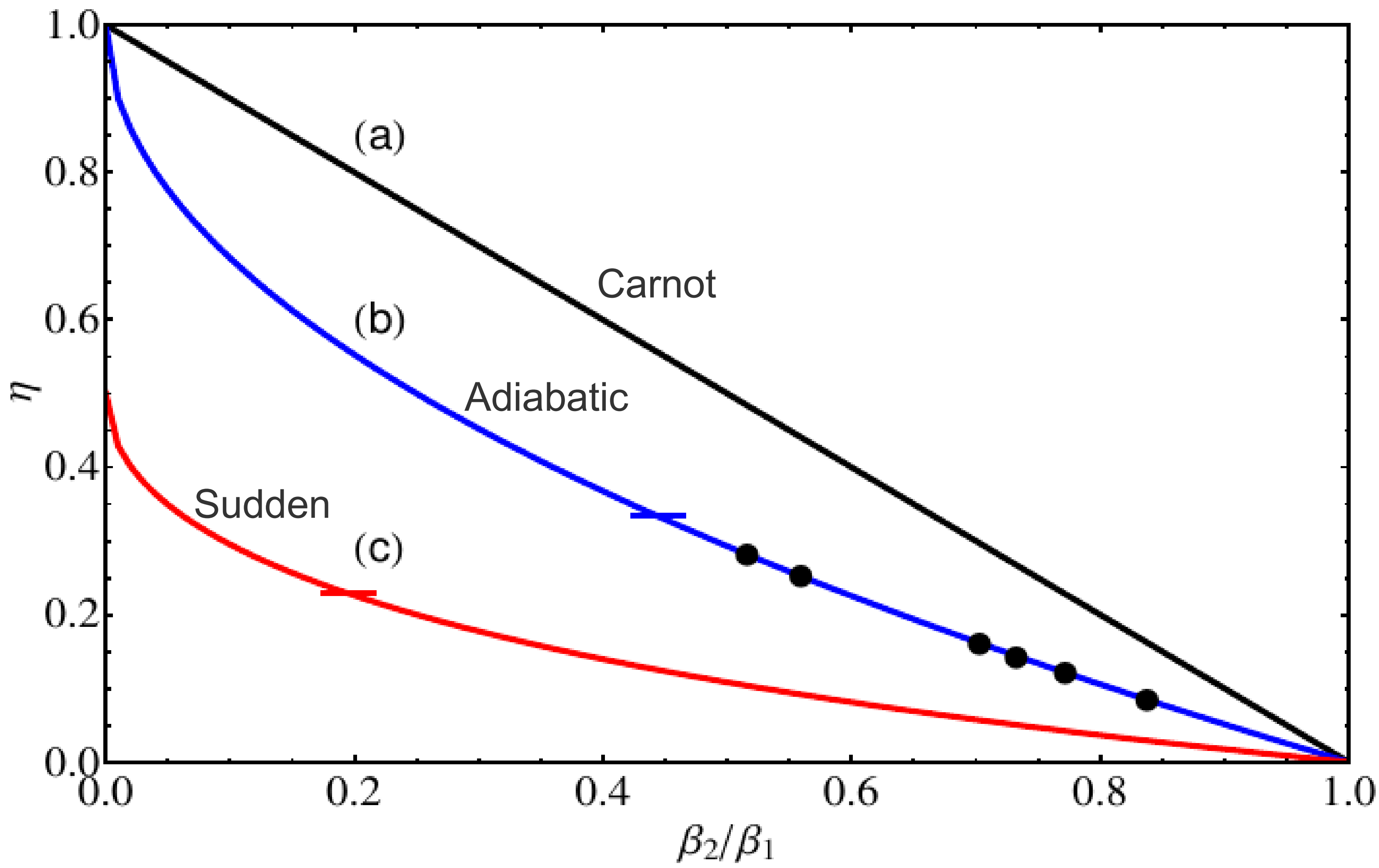
1: A $\rightarrow$ B Adiabatic compression

2: B $\rightarrow$ C Hot isochore

3: C $\rightarrow$ D Adiabatic expansion

4: D $\rightarrow$ A Cold isochore

Efficiency at maximum power:

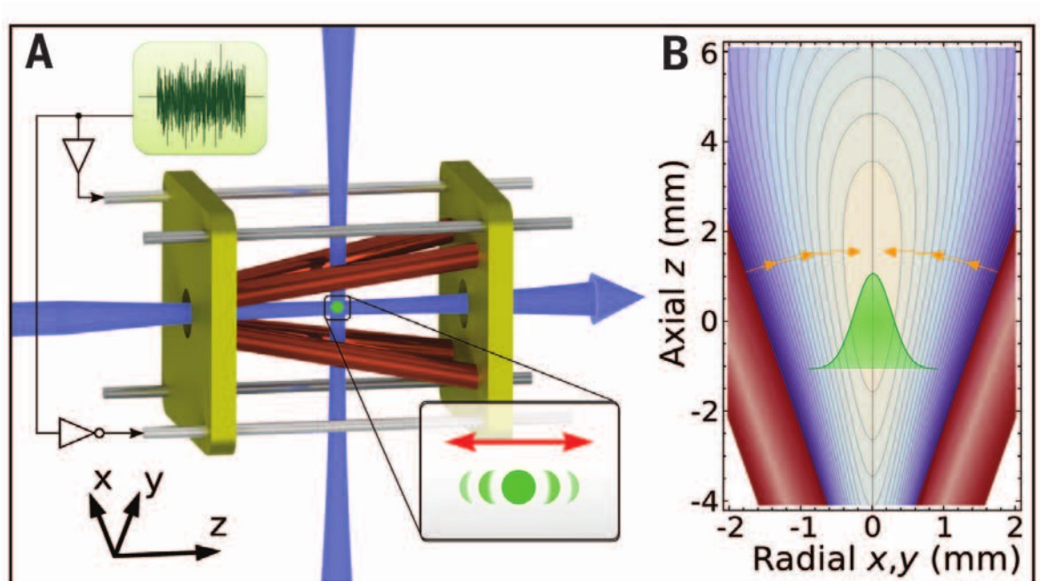


Typical car gasoline-fueled Otto engine ~ 0.2

Single-atom quantum engine experiment:

J. Roßnagel, *et al.* Science **352** 325-329(2016)

Single Calcium ion in a linear Paul trap (with tapered geometry) → harmonic potential



Axial z position determines frequency of radial potential

$$\omega_{x,y} = \frac{\omega_{0x,0y}}{(1 + z\tan\theta/r_0)^2}.$$

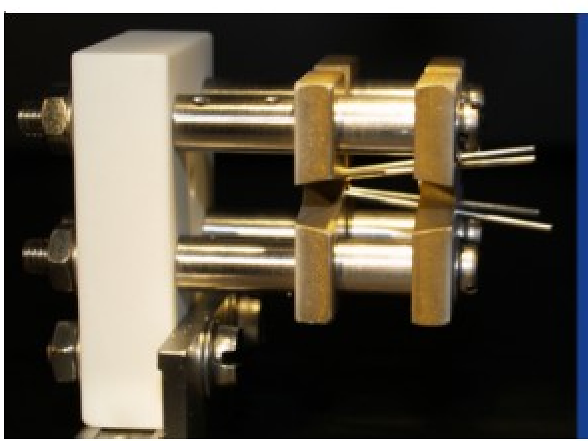
Hot bath → electric field noise

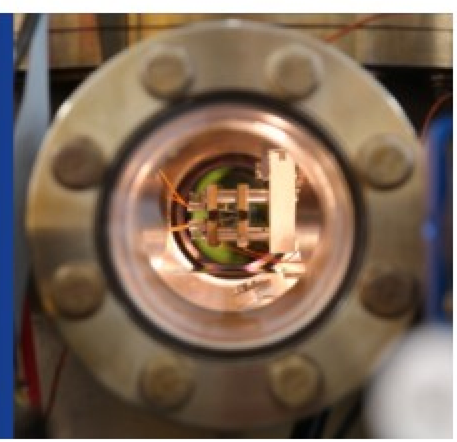
Cold bath → laser cooling

Force in the z direction:

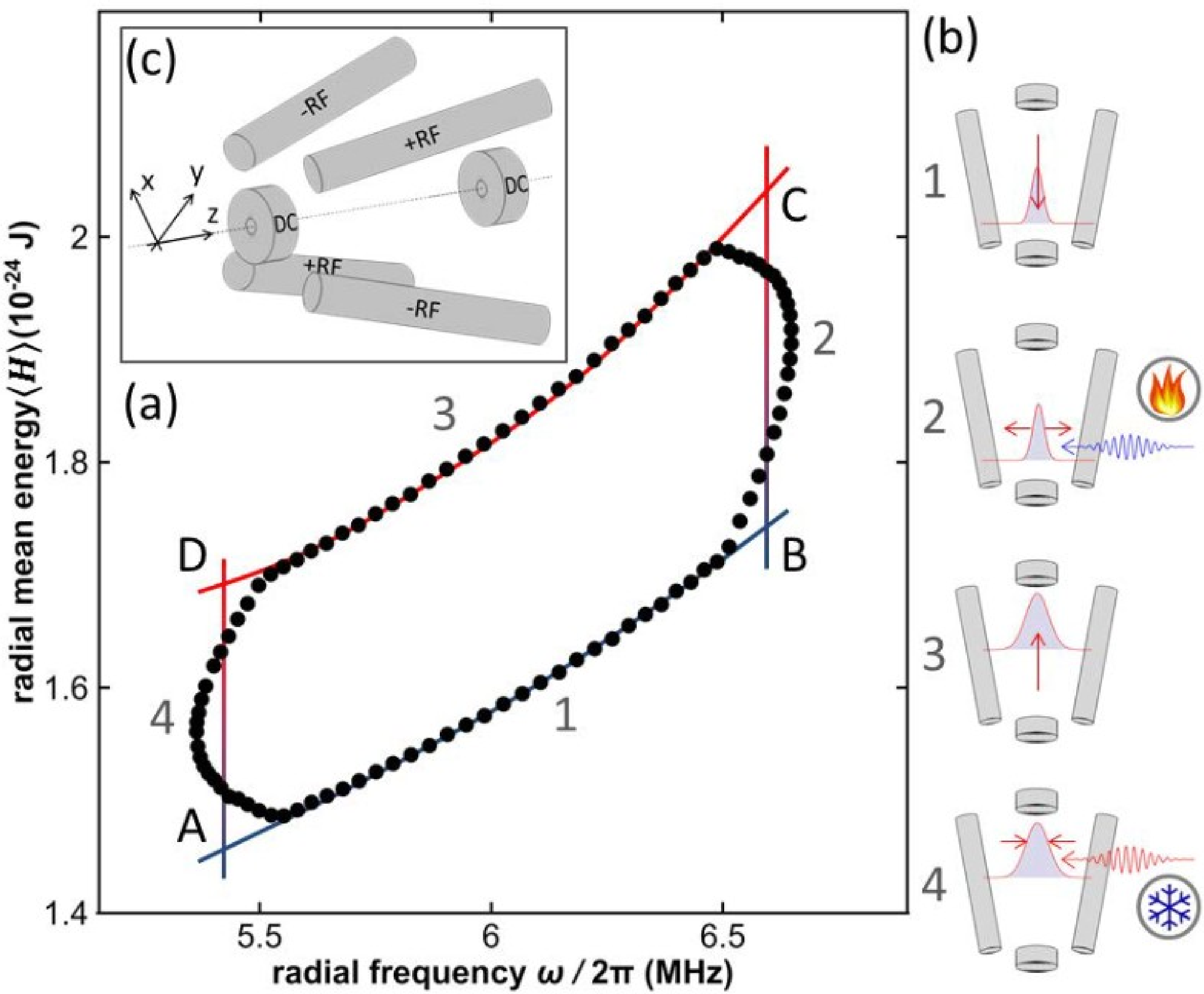
$$F_z(T)$$

depends on the temperature





Modulate the radial frequency to perform an Otto cycle:



(adiabatic) compression

Hot isochore

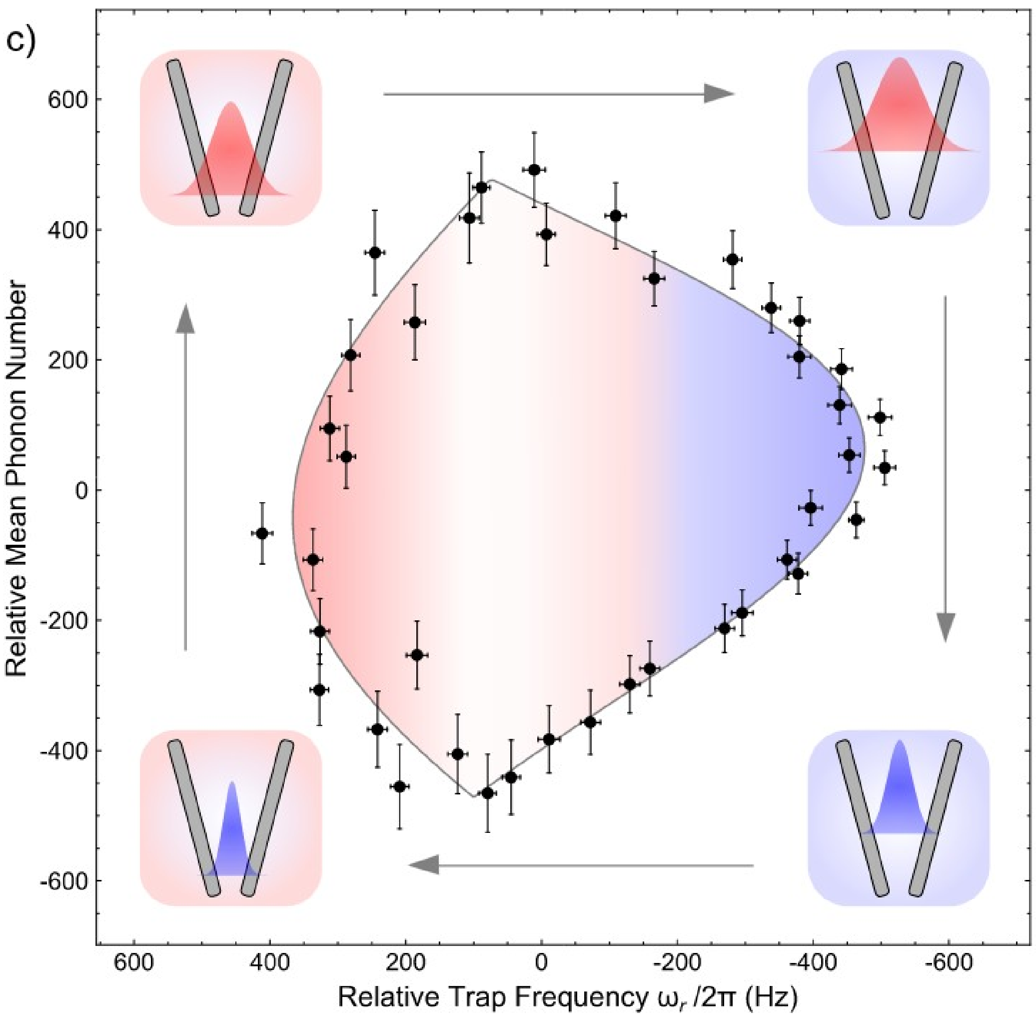
(adiabatic) expansion

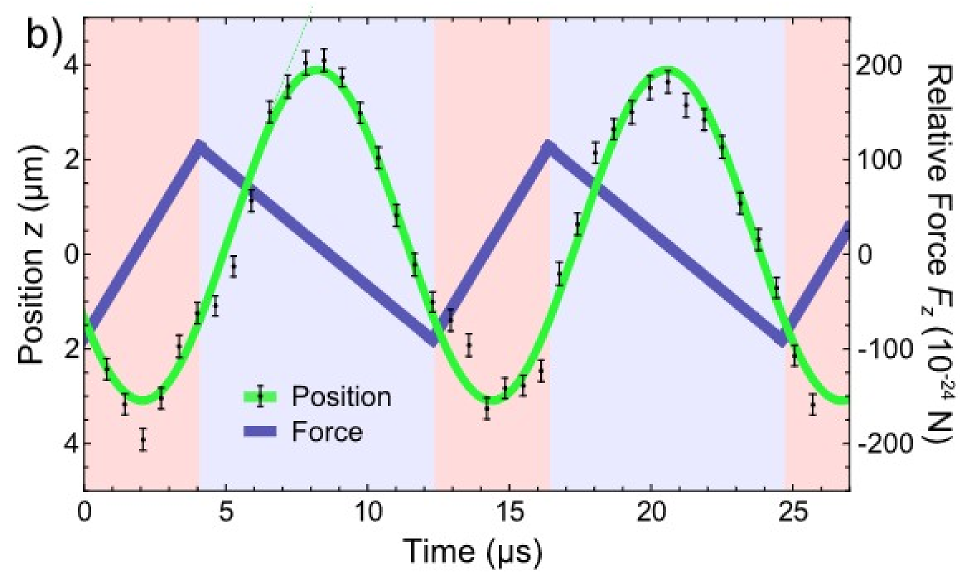
Cold isochore

O. Abah, *et al.* PRL **109** 203006(2012)

Single-atom heat engine

Actual implementation of the cycle:





Keep laser cooling (cold bath)

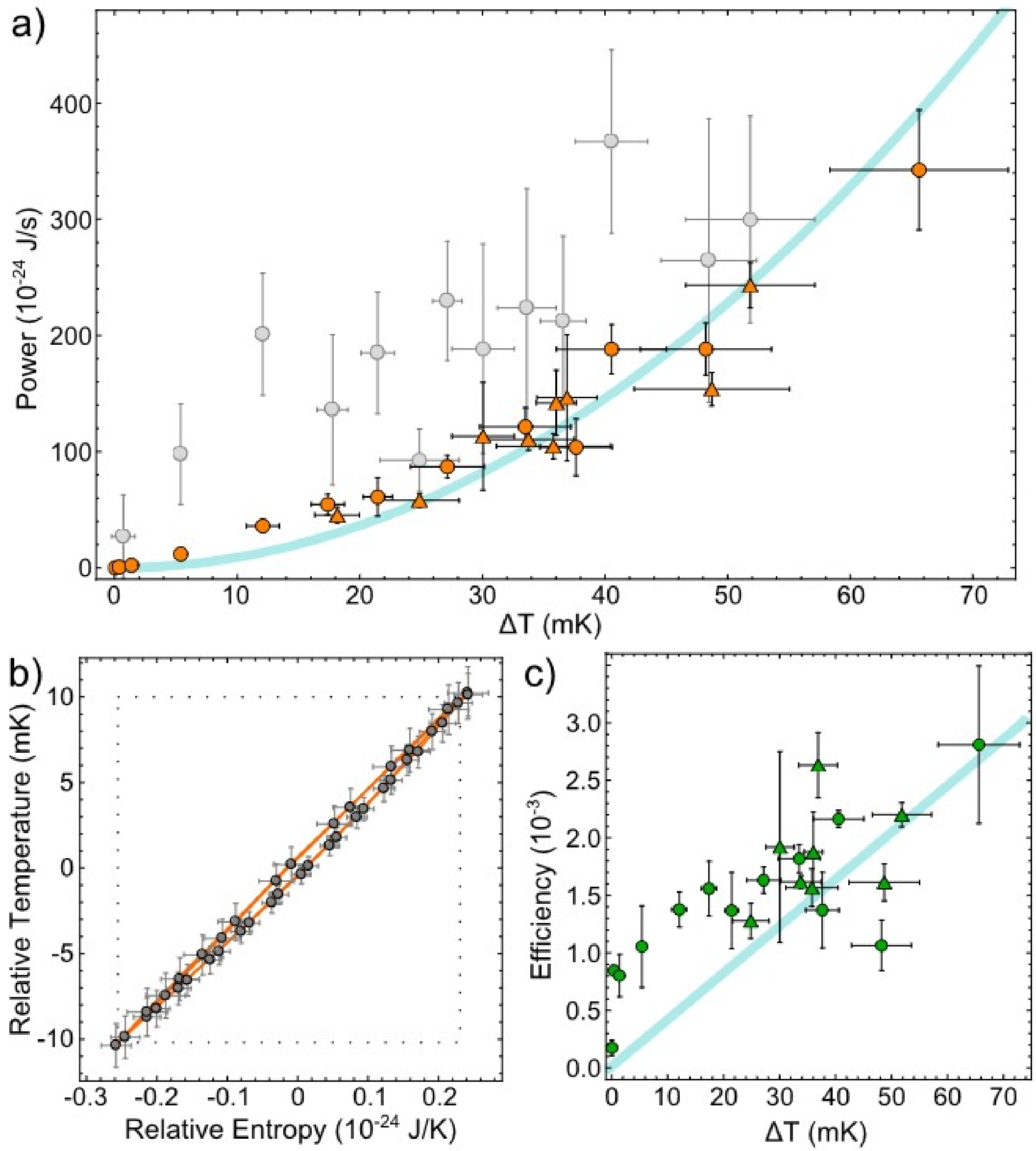
Switch on/off extra electric noise (hot bath)

Work accumulated in axial movement

Working temperature $\sim 568mK$

$$\Delta T \sim 20mK$$

Performance of the engine:



Power $\sim 10^{-21} J/s$

approx $k_B T$ per second

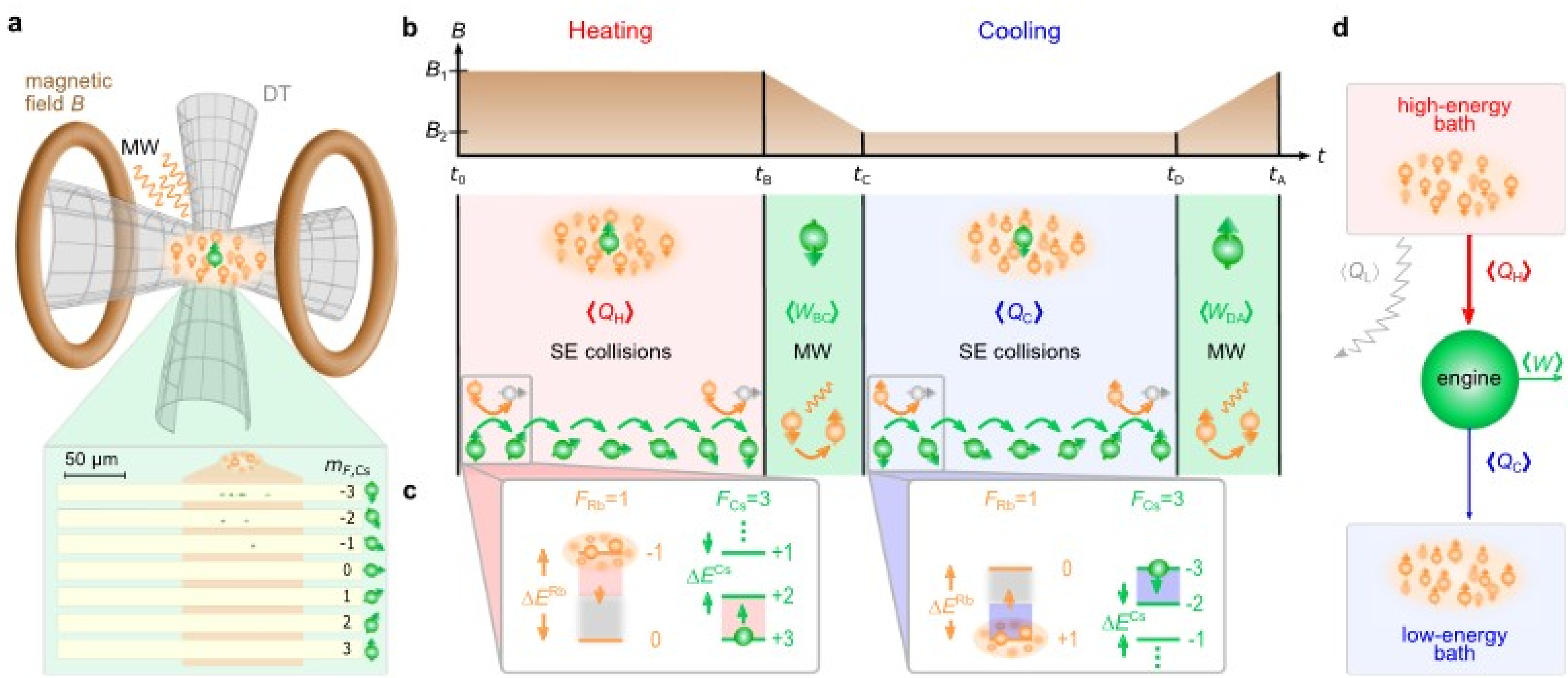
(at room temperature)

Efficiency $\sim 0.28\%$

$\eta_{\text{Carnot}} \sim 3,4\%$

(Watts engine 1783 ~ 5–7%)

Otto cycle driven by atomic collisions: Q. Bouton, *et al.* Nat. Commun. **12,** 2063 (2021)



Cesium impurities in ultracold Rubidium cloud in optical trap ⟶ More realistic bath!

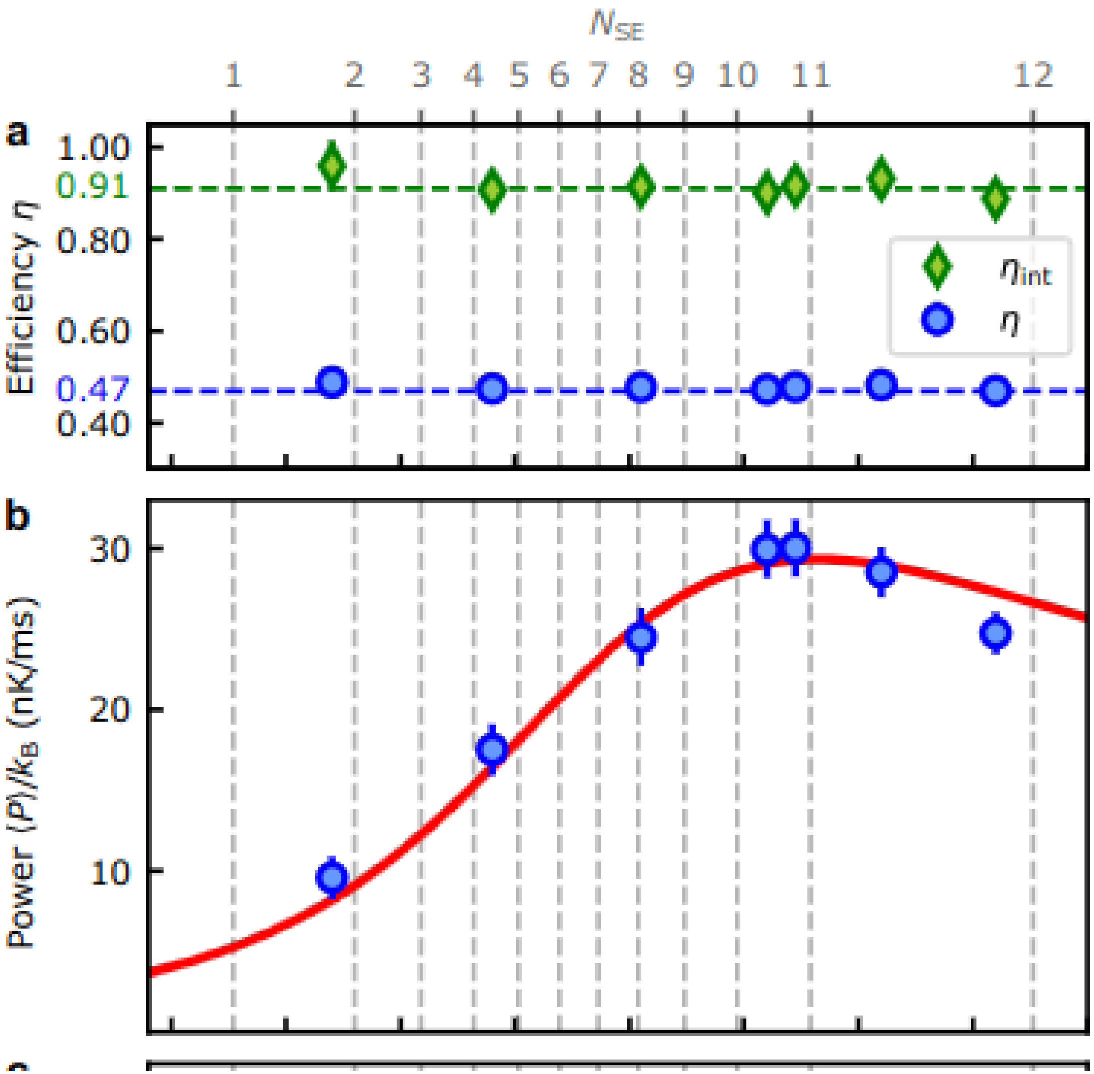
Magnetic field to implement the working substance (Cs impurity) adiabatic driving

$$E_n = n\lambda B \qquad n = 0, 1, ..., 7 \qquad B_1 \rightarrow B_2$$

Microwaves to tune the temperature hot/cold of the Rb spins bath

More recent experiments

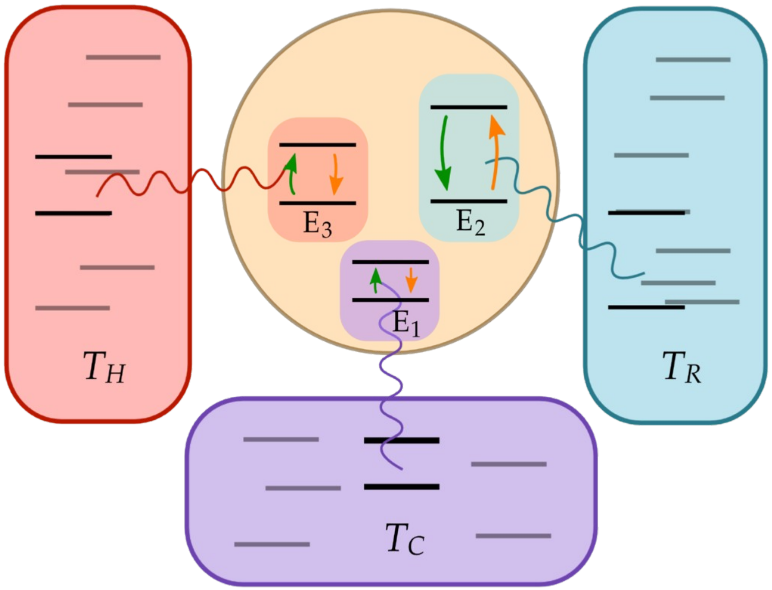
Otto cycle driven by atomic collisions: Q. Bouton, *et al.* Nat. Commun. **12,** 2063 (2021)



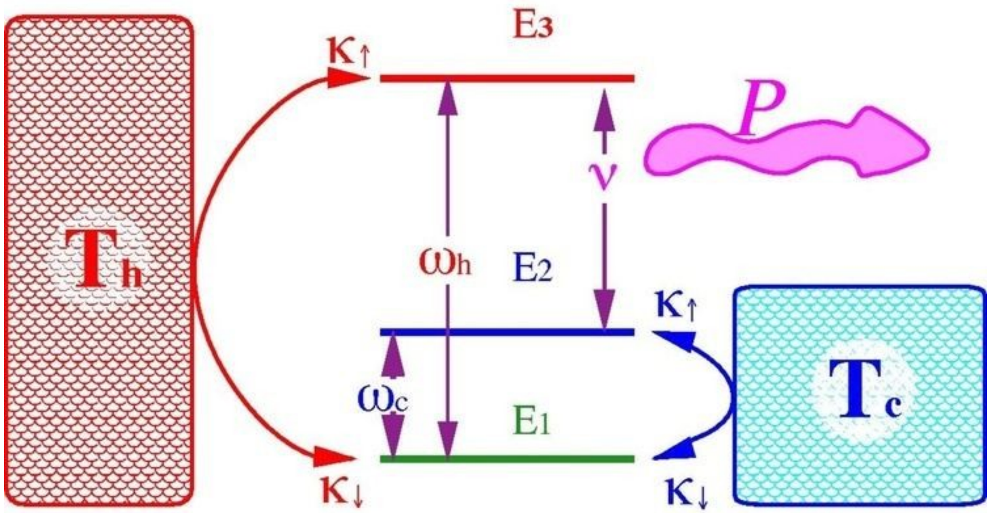
Much better efficiency!

Much less power than the single-atom engine

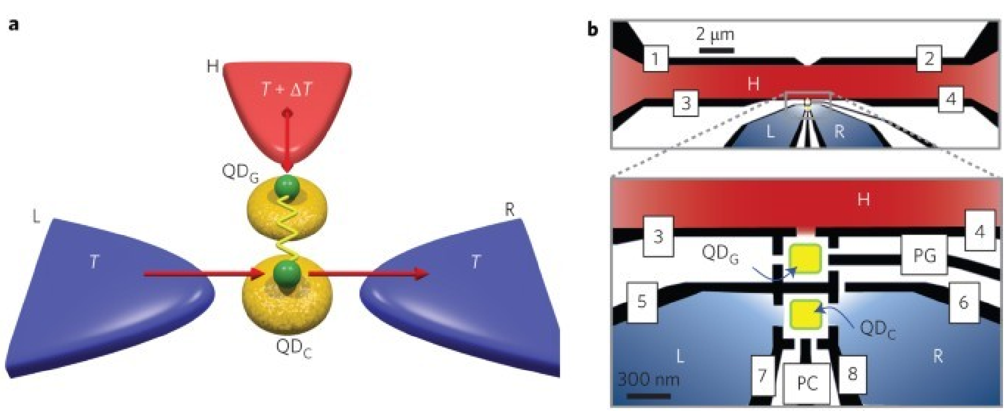
+ Work in non-equilibrium steady-state conditions



Small absorption refrigerators
Linden et al. PRL (2010)

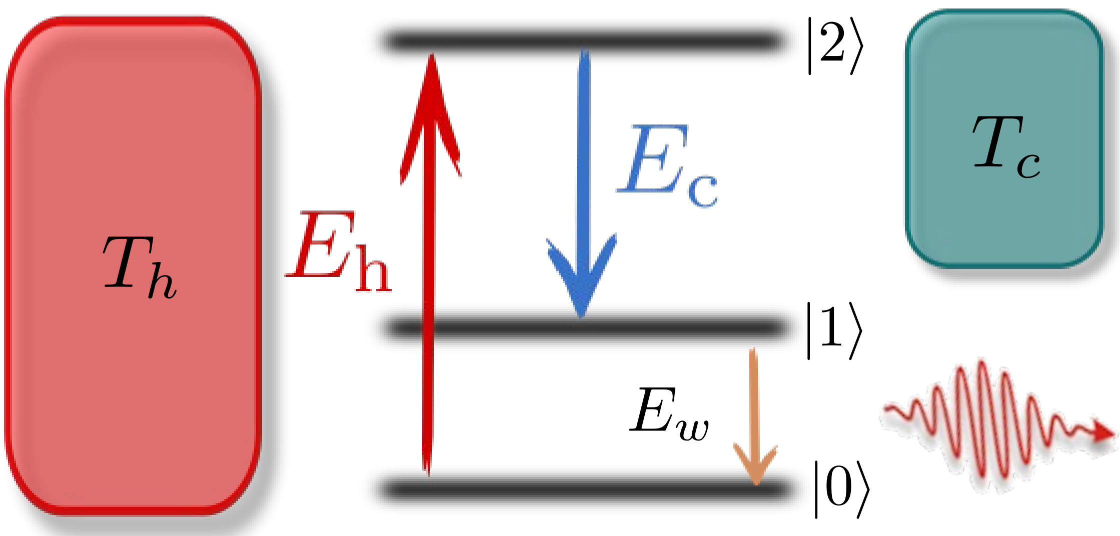


Three-level laser / amplifier
Kosloff et al, *Annu. Rev. Phys. Chem. (2014)*



Quantum-dot energy harvesters
Thierschmann et al. Nat. Nano (2015), Sanchez et al. PRB (2011)

Three-level laser / amplifier



Engine Hamiltonian:

$$H = E_h|2\rangle\langle 2| + E_w|1\rangle\langle 1|$$

$$E_h = E_w + E_c$$

External driving field (weak):

$$V = \epsilon|1\rangle\langle 0|e^{-iE_w t} + \text{h.c.}$$

Lindblad master equation (rotating frame):

$$\dot{\rho} = -i[V_R, \rho] + \sum_{i=h,c} \mathcal{D}_{\downarrow}^{(i)}[\rho] + \mathcal{D}_{\uparrow}^{(i)}[\rho]$$

driving dissipation

hot

$$L_{\downarrow}^{(h)} = \sqrt{\gamma_{\downarrow}^{(h)}}|0\rangle\langle 2|$$

$$L_{\uparrow}^{(h)} = \sqrt{\gamma_{\uparrow}^{(h)}}|2\rangle\langle 0|$$

cold

$$L_{\downarrow}^{(c)} = \sqrt{\gamma_{\downarrow}^{(c)}}|1\rangle\langle 2|$$

$$L_{\uparrow}^{(c)} = \sqrt{\gamma_{\uparrow}^{(c)}}|2\rangle\langle 1|$$

Dissipators: $\mathcal{D}_{\uparrow\downarrow}^{(i)}[\rho] = L_{\uparrow\downarrow}^{(i)}\rho L_{\uparrow\downarrow}^{(i)\dagger} - \frac{1}{2}\{L_{\uparrow\downarrow}^{(i)\dagger}L_{\uparrow\downarrow}^{(i)}, \rho\}$

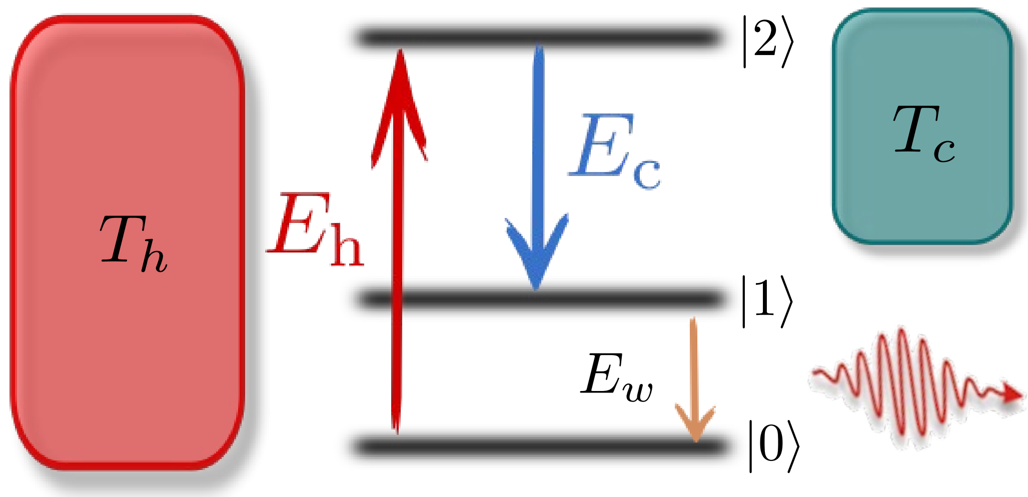
Local detailed balance: $\gamma_{\downarrow}^{(i)} = \gamma_{\uparrow}^{(i)} e^{\beta_i E_i}$

Steady-state:

$$\dot{\rho} \equiv 0 \quad \rightarrow \pi$$

Solving as a matrix:

$$\dot{\vec{\pi}} = W\vec{\pi} \equiv 0$$



Heat currents (into machine): $\dot{Q}_i = \mathrm{Tr}[H(\mathcal{D}_{\downarrow}^{(i)}[\pi] + \mathcal{D}_{\uparrow}^{(i)}[\pi])]$ for $i = h, c$

Hot: $\dot{Q}_h = E_h(\gamma_{\uparrow}^{(h)}\pi_0 - \gamma_{\downarrow}^{(h)}\pi_2)$ Cold: $\dot{Q}_c = E_c(\gamma_{\uparrow}^{(c)}\pi_1 - \gamma_{\downarrow}^{(c)}\pi_2)$

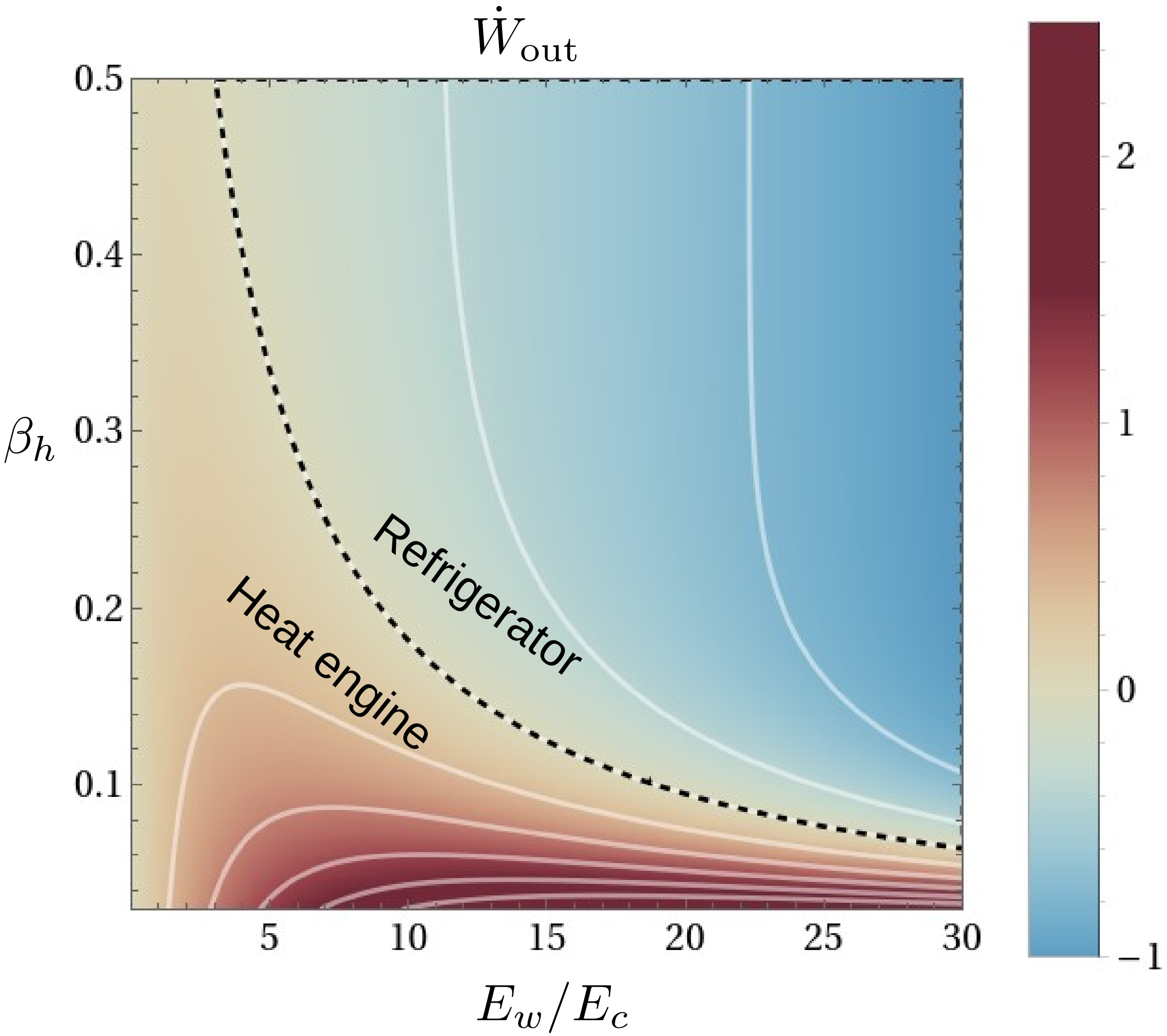
Driving (input) power: $\dot{W} = \mathrm{Tr}[(\dot{V})_R \; \rho_{ss}] = \epsilon E_w i(c - c^*)$

First law: $\dot{E} = \dot{Q}_c + \dot{Q}_h + \dot{W} = 0$ **Second law:** $\dot{S}_{\mathrm{tot}} = -\beta_c \dot{Q}_c - \beta_h \dot{Q}_h \geq 0$

Efficiency: $\eta = \frac{\dot{W}_{\text{out}}}{\dot{Q}_h} = 1 - \frac{E_c}{E_h}$

As before: $\eta \leq \eta_C$

with output work: $\dot{W}_{\text{out}} = -\dot{W}$

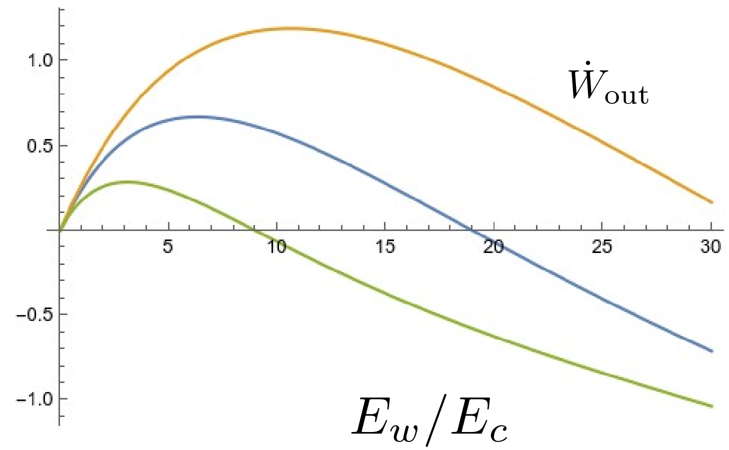


Heat engine regime:

$$\dot{W}_{\text{out}} \geq 0 \Leftrightarrow \beta_c E_c \geq \beta_h E_h$$

Refrigerator regime:

$$\dot{Q}_c \geq 0 \Leftrightarrow \beta_c E_c \leq \beta_h E_h$$



Many models of engines are **based on quantum effects** (e.g. tunneling) or even show an **intrinsic quantum dynamics**, leading e.g. to entanglement in multipartite systems, but...

Quantum-thermodynamic advantage?

+ Define and compare to classical analogs, introduce extra dephasing ...

R. Uzdin, et al. PRX (2015), L. Correa, et al. PRE (2019), ...

It can be ambiguous depending how one defines the "analog"...

+ Breaking of classical nonequilibrium inequalities such as TUR as a witness

$\rightarrow$ ***model independent***

Agarwalla and Segal PRB (2018), Ptaszyński PRB (2018), Kalaee et al. PRE (2021)
G. Manzano and R. Lopez, PRR (in press), arXiv:2302.09414

Thermodynamic and Kinetic Uncertainty Relations

Classical Markovian processes in nonequilibrium steady states

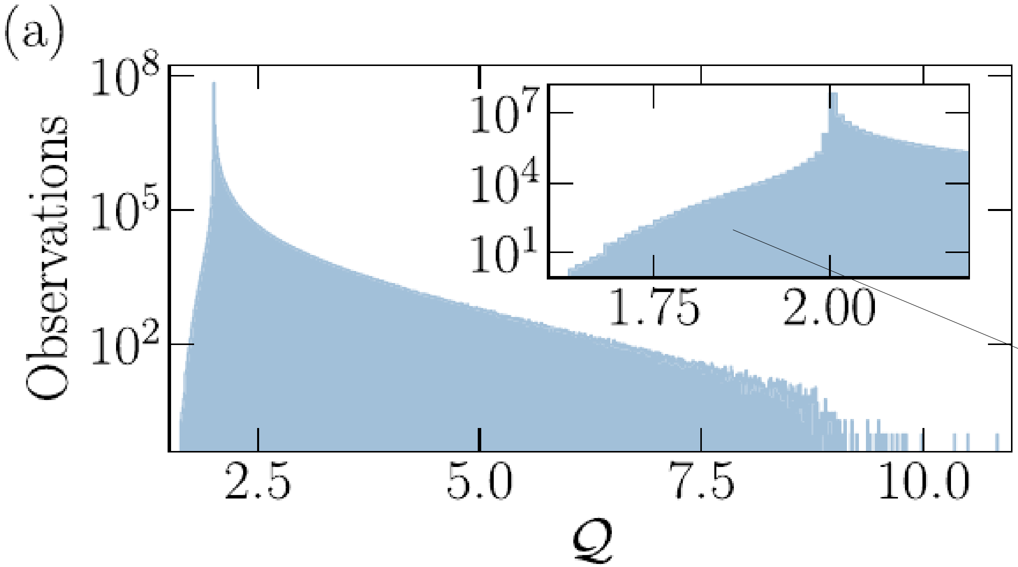
TUR

$$\frac{\mathrm{Var}[J(t)]}{\langle J(t)\rangle^2} \geq \frac{2k_B}{\dot{S}_{\mathrm{tot}}}$$

Entropy production (EP) rate / dissipation

Arbitrary current

Barato and Seifert, PRL (2015), Gingrich *et al.* PRL (2016), ...



Kalaee *et al.*, PRE **104** (2021)

Or in other words, in classical engines:

$$\mathcal{Q} \equiv \dot{S}_{\mathrm{tot}} \frac{\mathrm{Var}[\dot{W}]}{\dot{W}^2} \geq 2$$

VIOLATION!

Quantum-thermodynamic advantage!

Continuous three-level engine

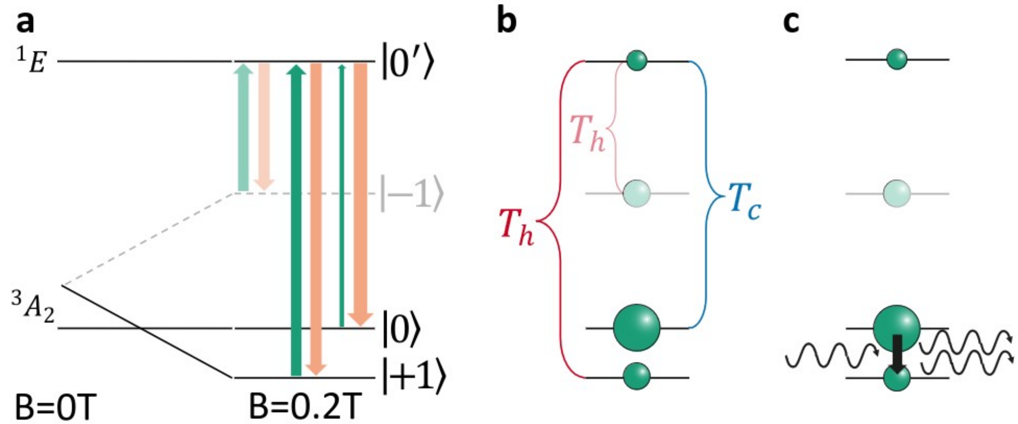
NV thee-level quantum heat engine: Klatzow, *et al.* Phys. Rev. Lett, **122** (2019)

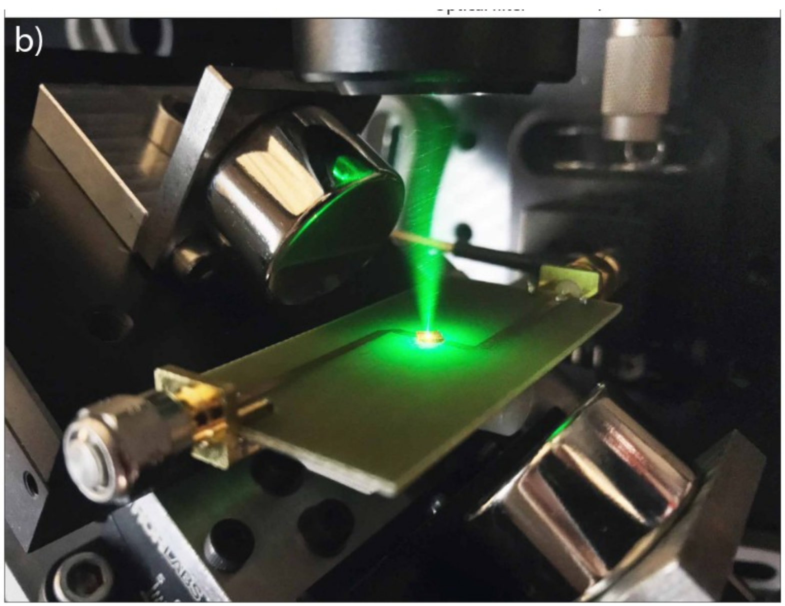
Nitrogen-vacancy centers in diamond

Effective 3-level system

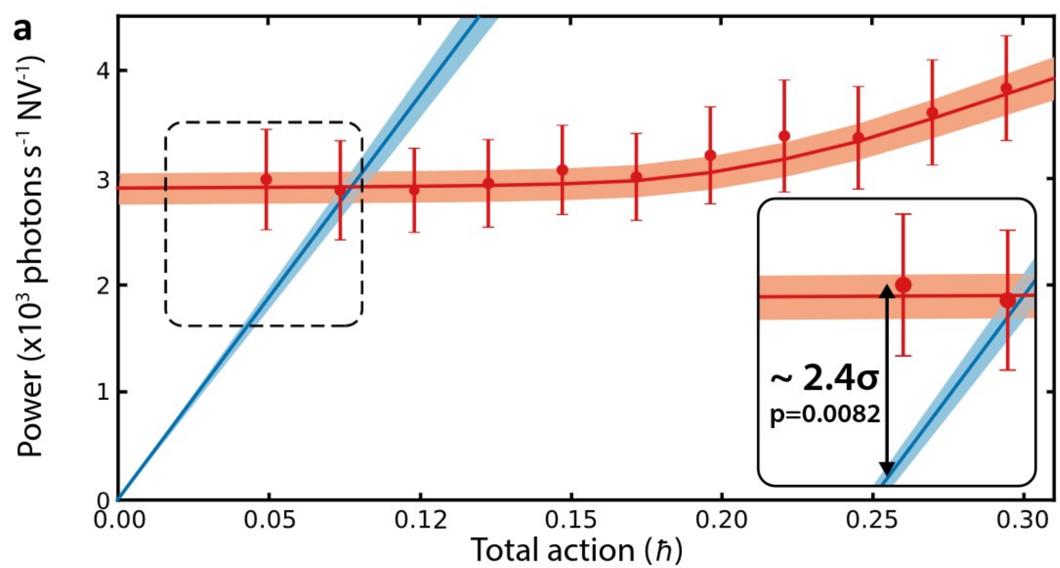
Effective baths using optical pumping and decay by different mechanisms

Driving with microwave field (work)





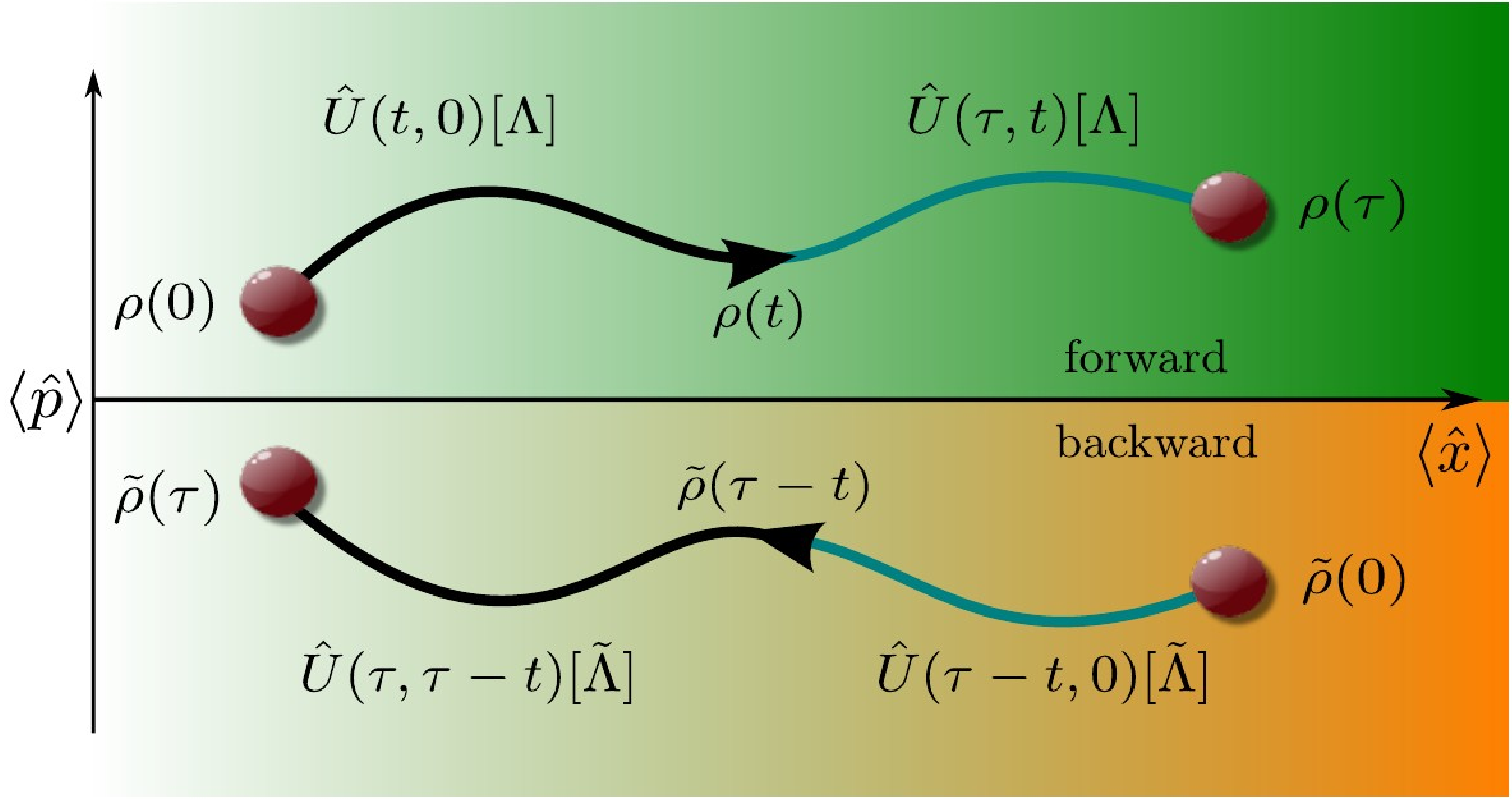
Focus is to demonstrate a quantum signature



THANK YOU

for your attention

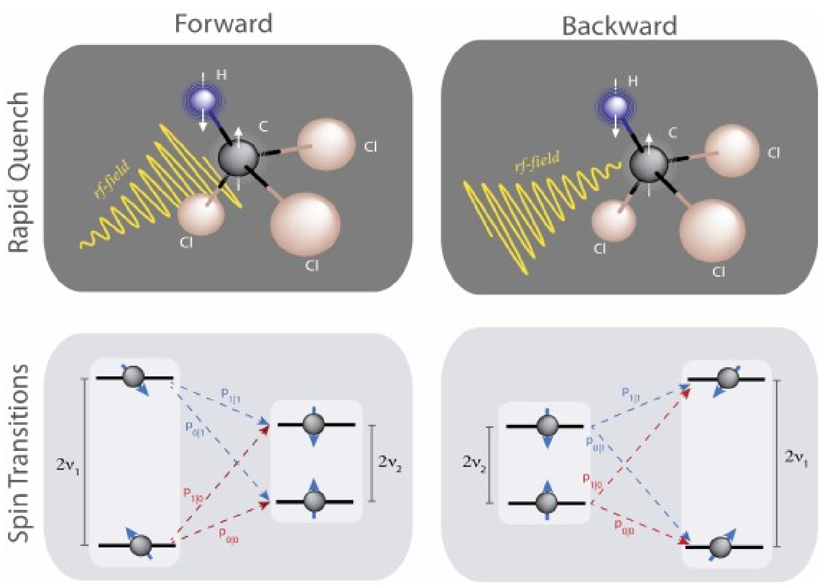
Micro-reversibility relation for non-autonomous systems:

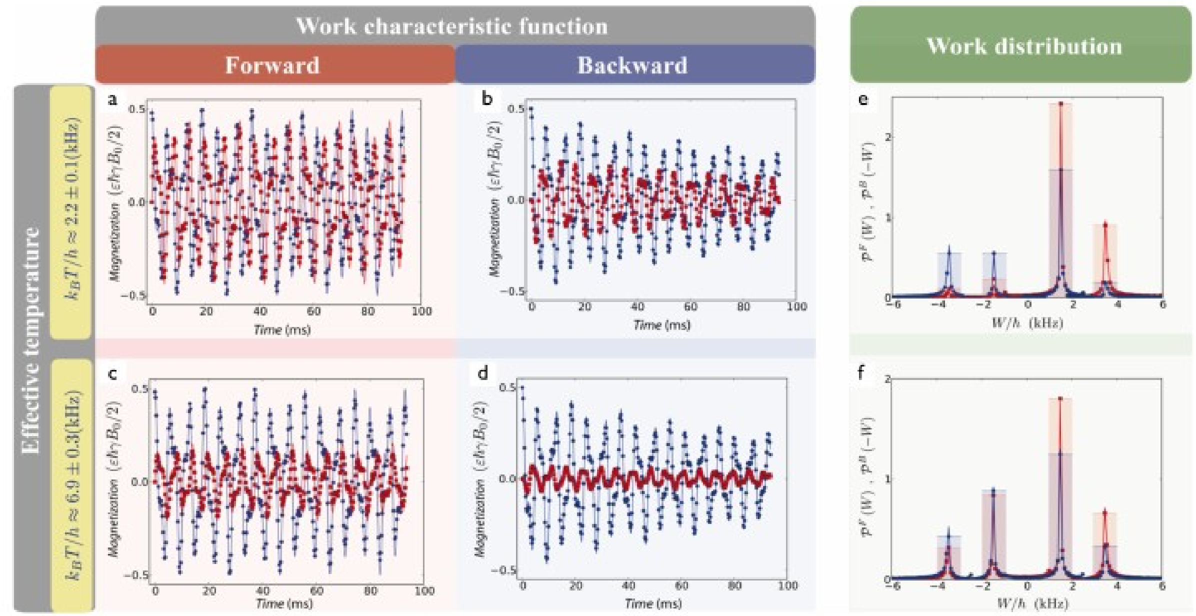


$$U^\dagger(\tau,t)[\Lambda] = \Theta^\dagger U(\tau-t,0)[\tilde{\Lambda}]\Theta$$

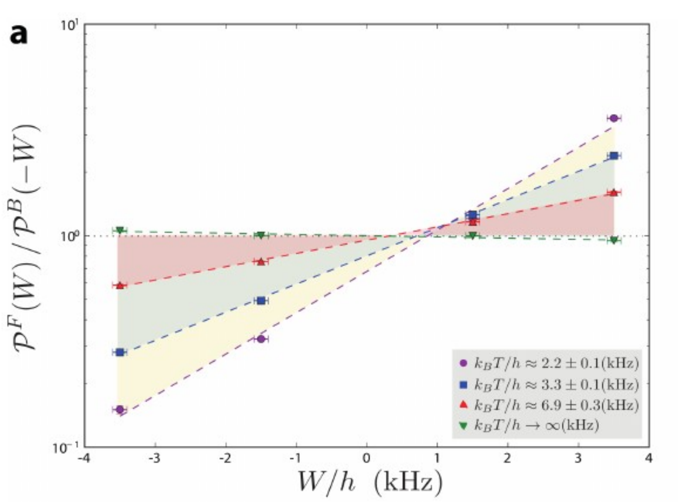
Liquid-state NMR plarform

T.B. Batalhao, *et al.* Phys. Rev. Lett, **113** (2014)



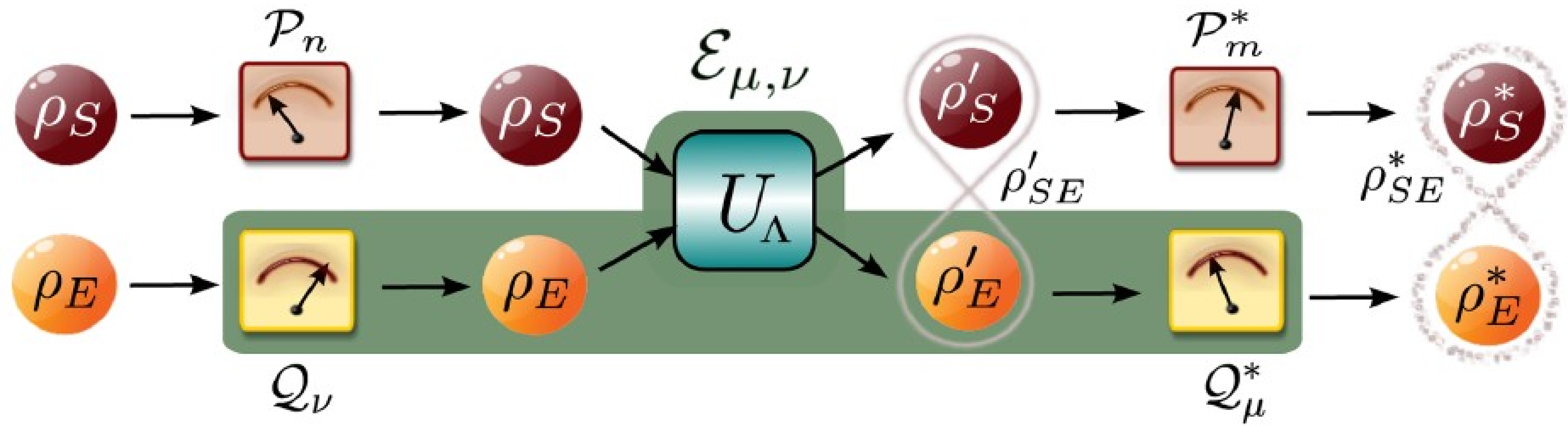


Testing Crooks fluctuation theorem (logarithm scale) and Jarzynski equality:



k_BT/h (kHz)	2.2 ± 0.1	3.3 ± 0.1	6.9 ± 0.3
$\log \langle e^{-\beta W} \rangle$	0.38 ± 0.03	0.20 ± 0.02	0.06 ± 0.01
$\beta \Delta F$	0.39 ± 0.06	0.22 ± 0.02	0.05 ± 0.01
Theory	0.43 ± 0.03	0.22 ± 0.01	0.053 ± 0.003

Fluctuation theorems for arbitrary environments setup: G. Manzano, *et al.* PRX, **8** (2018)



Initial state: $\rho_S = \sum_n p_n \mathcal{P}_n, \qquad \rho_E = \sum_\nu q_\nu \mathcal{Q}_\nu,$

Global evolution: $U_\Lambda \equiv \mathcal{T}_+ \exp\left(-\frac{i}{\hbar}\int_0^\tau dt\, H(\lambda_t)\right),$

Evolved state: $\rho'_{SE} = U_\Lambda(\rho_S \otimes \rho_E)U_\Lambda^\dagger,$

Post-measurement state $\rho^*_{SE} = \sum (\mathcal{P}^*_m \otimes \mathcal{Q}^*_\mu)\rho'_{SE}(\mathcal{P}^*_m \otimes \mathcal{Q}^*_\mu)$