Universidad de los Andes
High Energy laboratory

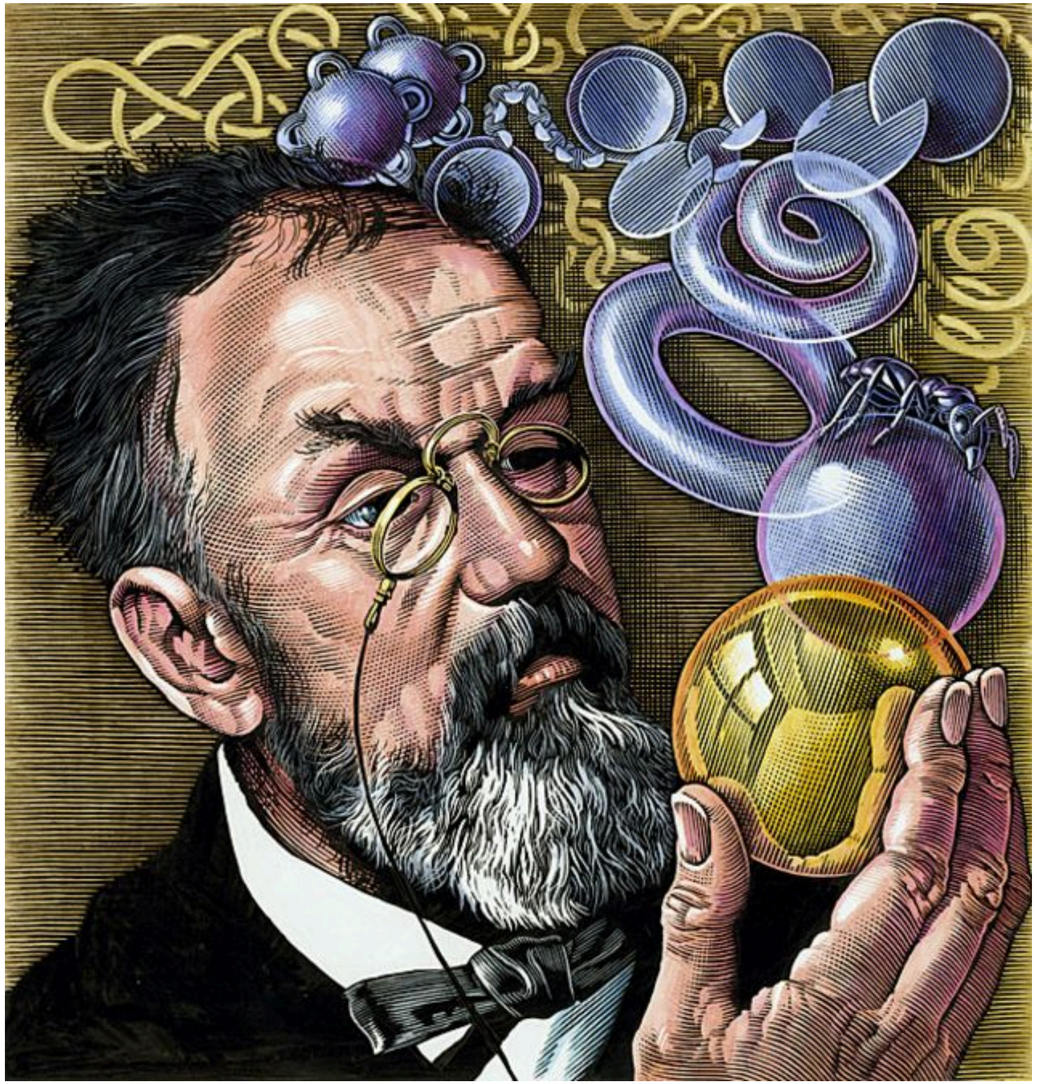
# In-line X-ray Phase Contrast Imaging and Applications

## Where are we and where will we go?

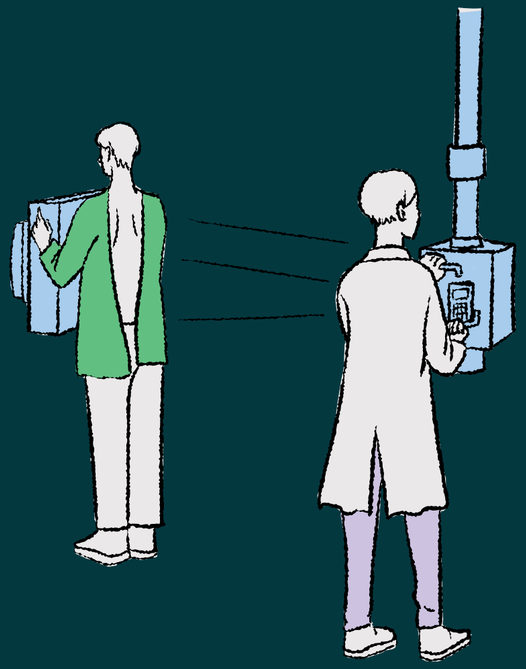
Steven Cely Iza

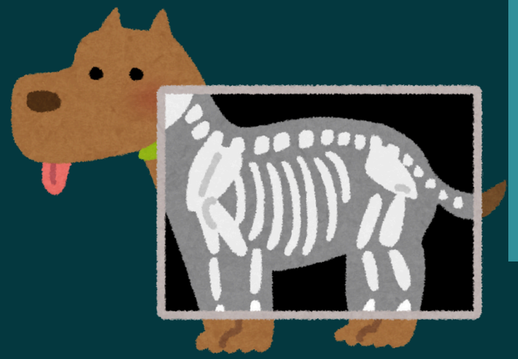
“X-rays provided the medical field with a window into the secrets of the body, turning the fight against many diseases into a battlefield with unparalleled knowledge”

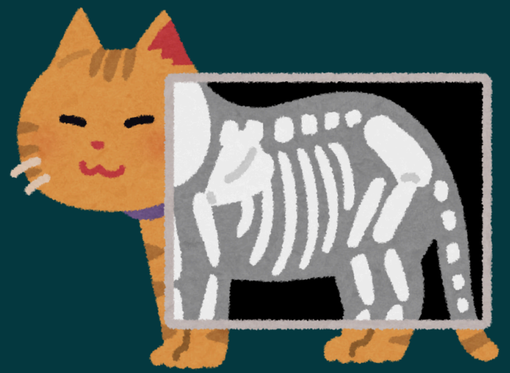
ANONYMOUS



# X-ray Imaging

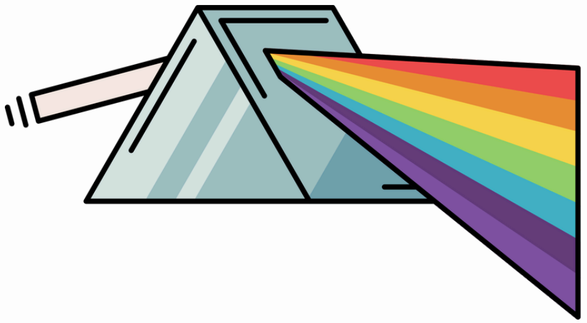






# X-ray Imaging

X-ray propagation through matter



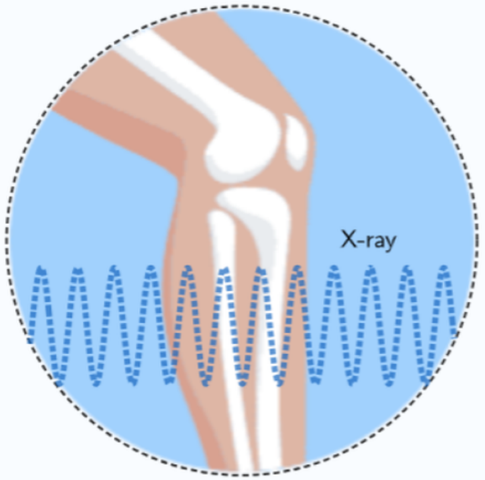
**Maxwell's wave equations**

Air

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\varphi(x,y,z,t) = 0 \qquad \textbf{(1)}$$

Lineal Medium)

$$\left[\varepsilon(\boldsymbol{r})\mu_o\frac{\partial^2}{\partial t^2} - \nabla^2\right]\varphi(x,y,z,t) = 0 \qquad \textbf{(2)}$$



# X-ray Imaging

X-ray propagation through matter

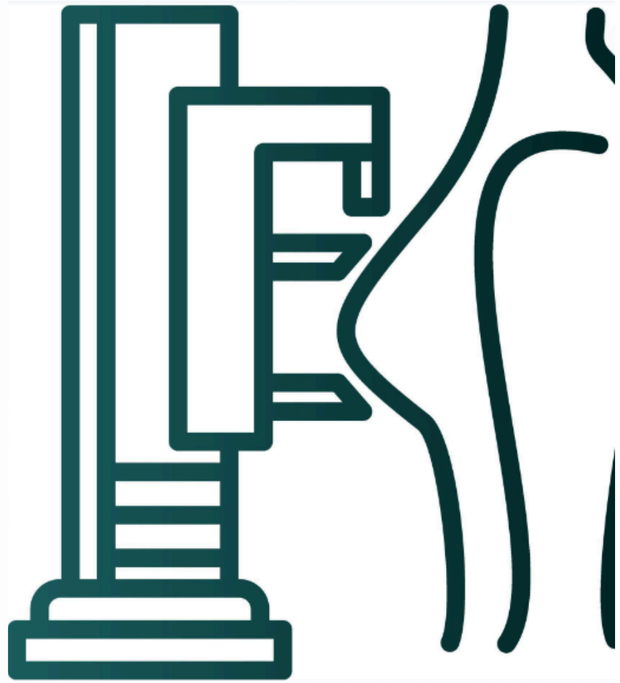
**Helmholtz equation**

Air

$$[k^2 + \nabla^2]\psi(x, y, z) = 0 \qquad (3)$$

Lineal Medium)

$$[n(\boldsymbol{r})^2 k^2 + \nabla^2]\psi(x, y, z) = 0 \qquad (4)$$



# X-ray Imaging

## X-ray propagation through matter

**Solution (Lineal Medium): Neccesary Assumptions**

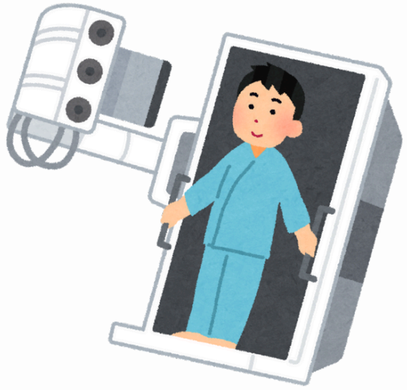
Paraxial aproximation

$$\psi(x,y,z) = \rho(x,y,z)e^{ikz} \tag{5}$$

$$\left[2i\frac{\partial}{\partial z} + \nabla_t{}^2 + k^2(n^2-1)\right]\rho(x,y,z) = 0$$

Projection approximation

$$\left[2i\frac{\partial}{\partial z} + k^2(n^2-1)\right]\rho(x,y,z) = 0 \tag{6}$$



# X-ray Imaging

## X-ray propagation through matter

**Solution (Lineal Medium)**

$$\rho(x,y,z) = \rho(x,y,0)e^{\frac{1}{2}ik\int_0^z (n(\boldsymbol{r})^2-1)dz} \quad (7)$$

Refractive index: Complex number

$$n(\boldsymbol{r}) = 1 - \delta(\boldsymbol{r}) + i\beta(\boldsymbol{r}) \quad (8)$$

Spatial and time solution

$$\varphi(x,y,z,t) = \rho(x,y,0)e^{-k\int_0^z \beta(\boldsymbol{r})dz}e^{i\left[-k\int_0^z \delta(\boldsymbol{r})dz + kz - \omega t\right]} \quad (9)$$

# X-ray Imaging

X-ray propagation through matter

**Solution (Lineal Medium)**

Beer-Lambert equation

$$\alpha(x,y,z,t) = \rho(x,y,0)e^{-k\int_0^z \beta(\boldsymbol{r})dz}$$

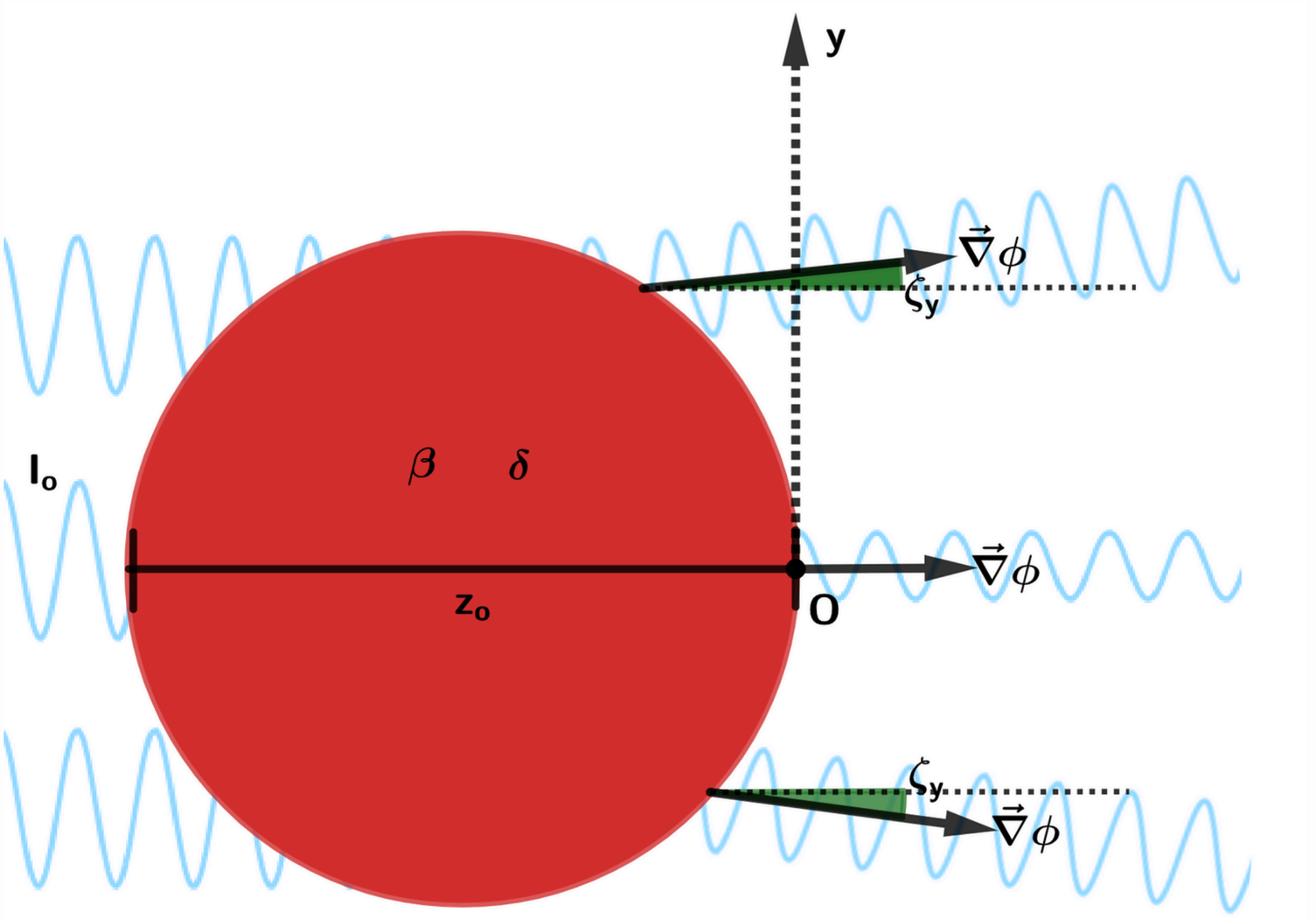
$$I(x,y,z,t) = |\rho(x,y,0)|^2 e^{-2k\int_0^z \beta(\boldsymbol{r})dz}$$

Phase shift

$$\Delta\phi(x,y,z,t) = -k\int_0^z \delta(\boldsymbol{r})dz$$

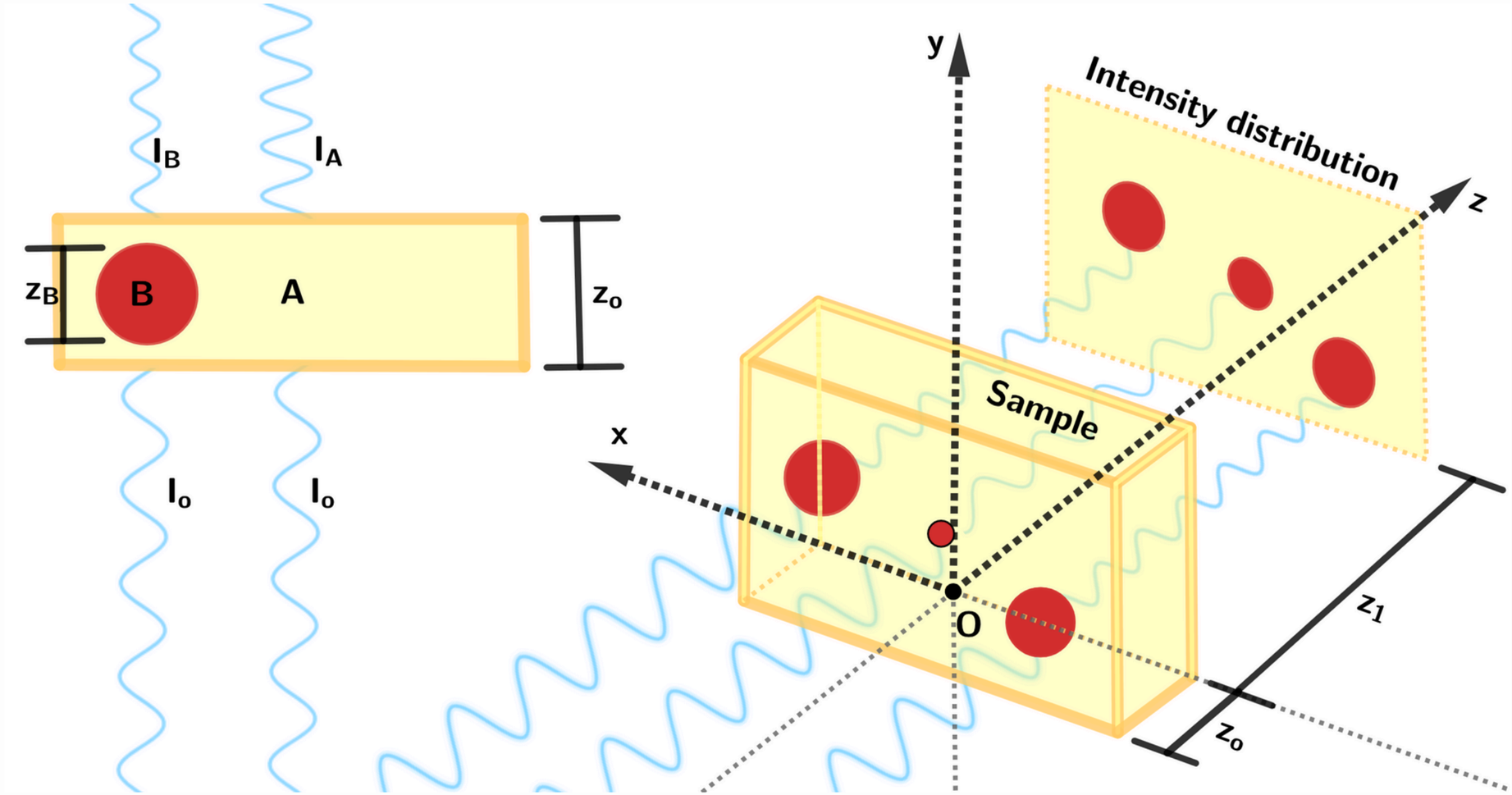
# X-ray Imaging

## Beer-Lambert Equation-Phase Shift



# Absorption Imaging

## Beer-Lambert Equation

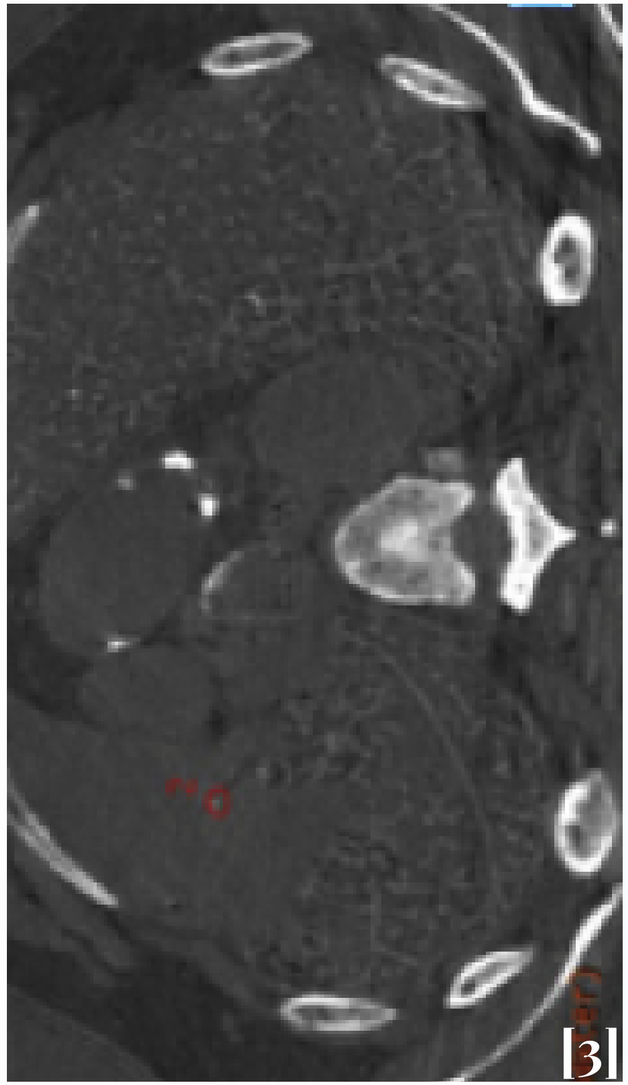


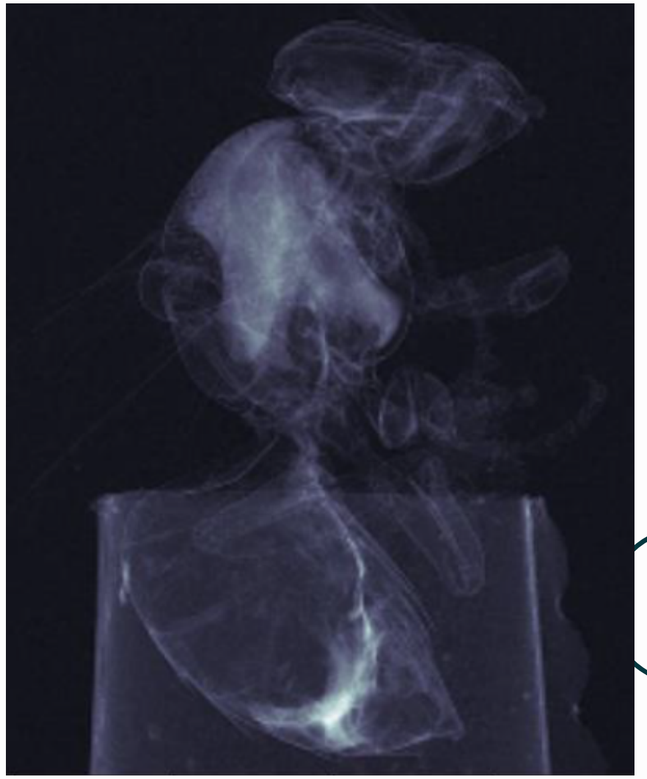
# Absorption Imaging

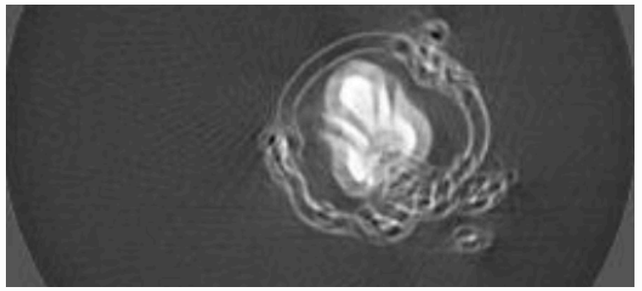
## Applications


[1]


[2]

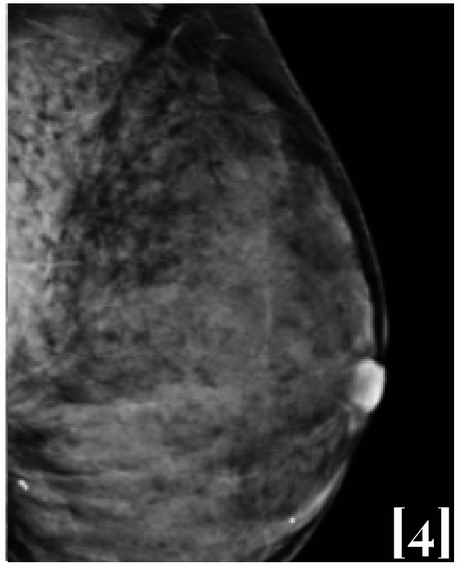

[3]

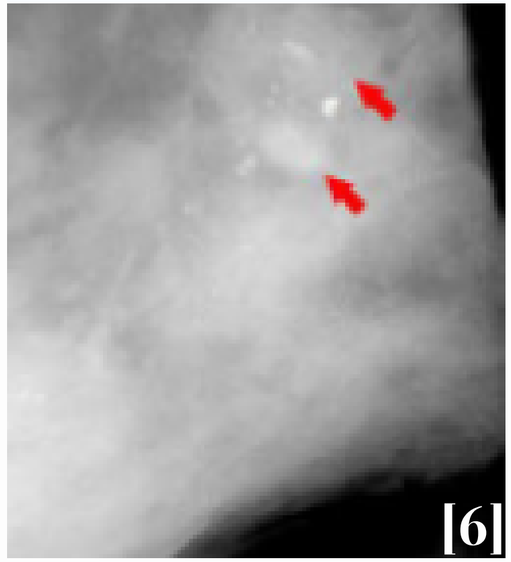


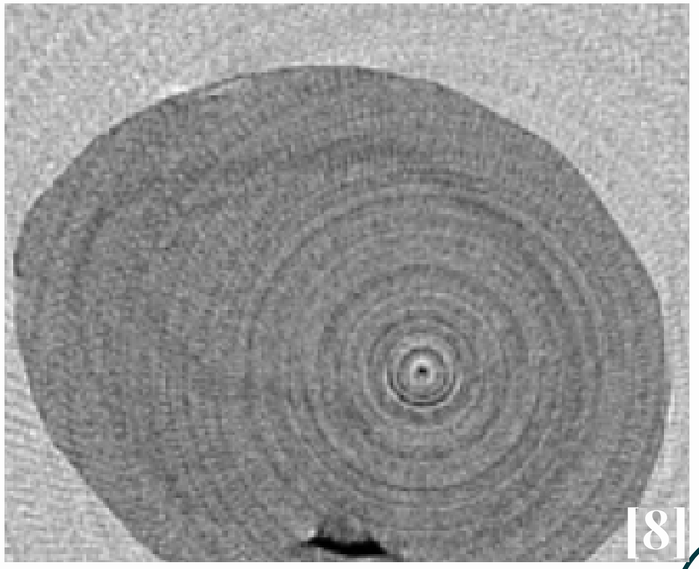


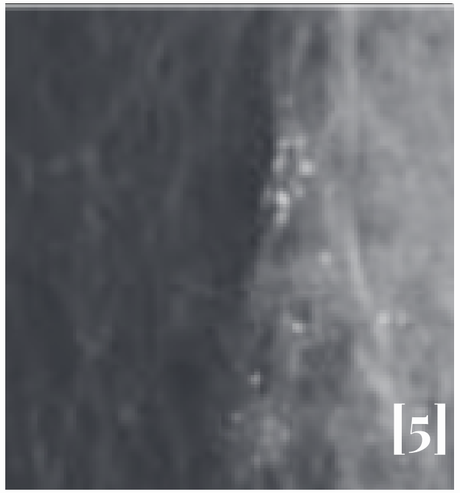
# Absorption Imaging

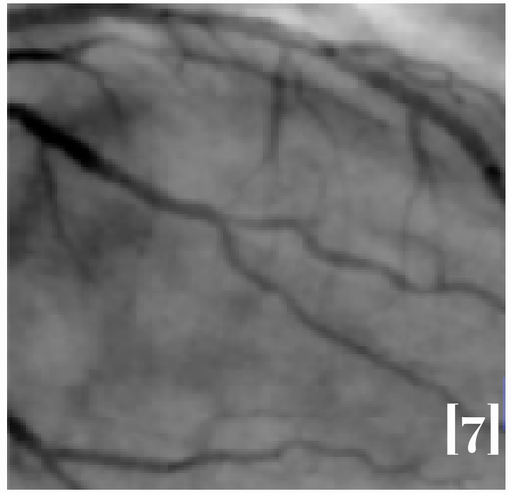
## Limitations

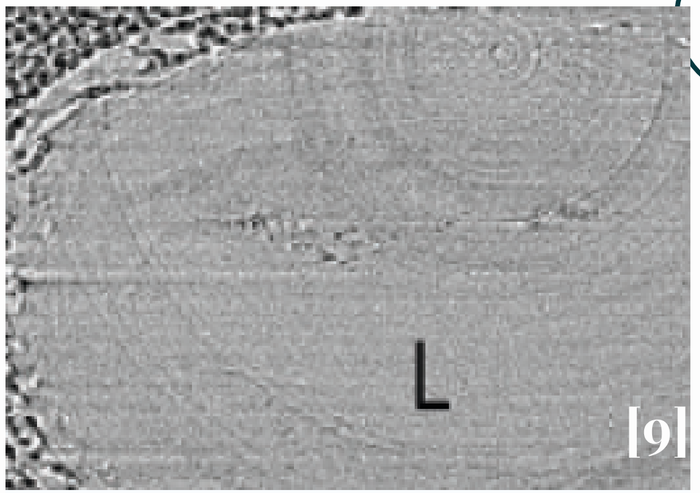

[4]


[6]


[8]


[5]


[7]



[9]

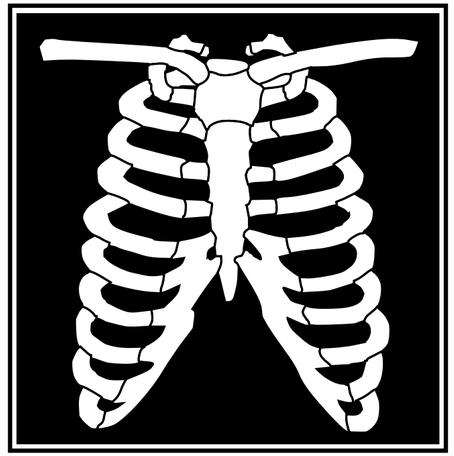
# Absorption Imaging

## Limitation

**Dependence with energy**

$$I(x,y,z,t) = |\rho(x,y,0)|^2 e^{-2k\int_0^z \beta(\boldsymbol{r})dz}$$

$$\beta(\boldsymbol{r},E) \sim \frac{Z^4}{E^{4.5}} \qquad \textbf{(10)}$$

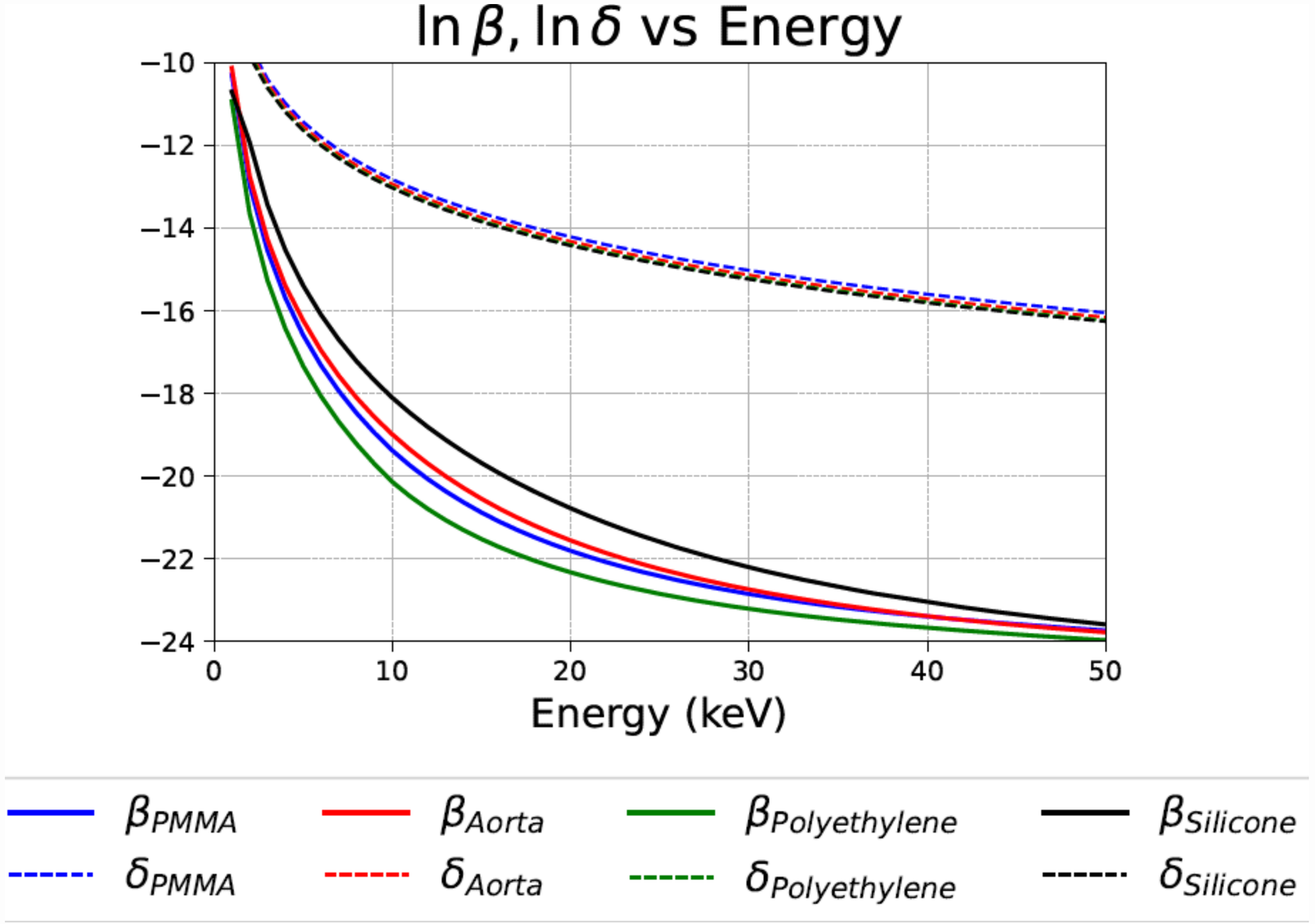


$$\Delta\phi(x,y,z,t) = -k\int_0^z \delta(\boldsymbol{r})dz$$

$$\delta(\boldsymbol{r},E) \sim \frac{1}{E^2} \qquad \textbf{(11)}$$

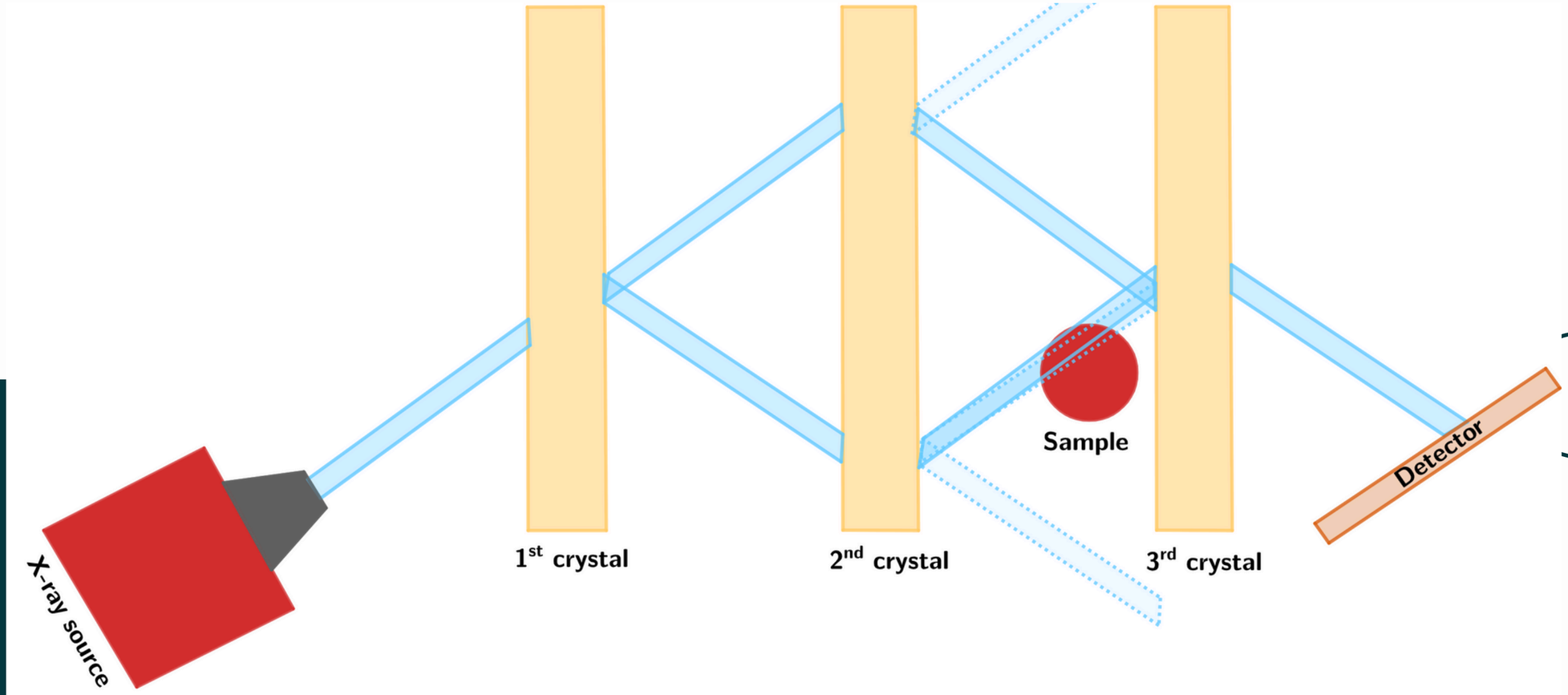
# X-ray Imaging

Absorption and dispersion of materials



# Phase Contrast Imaging

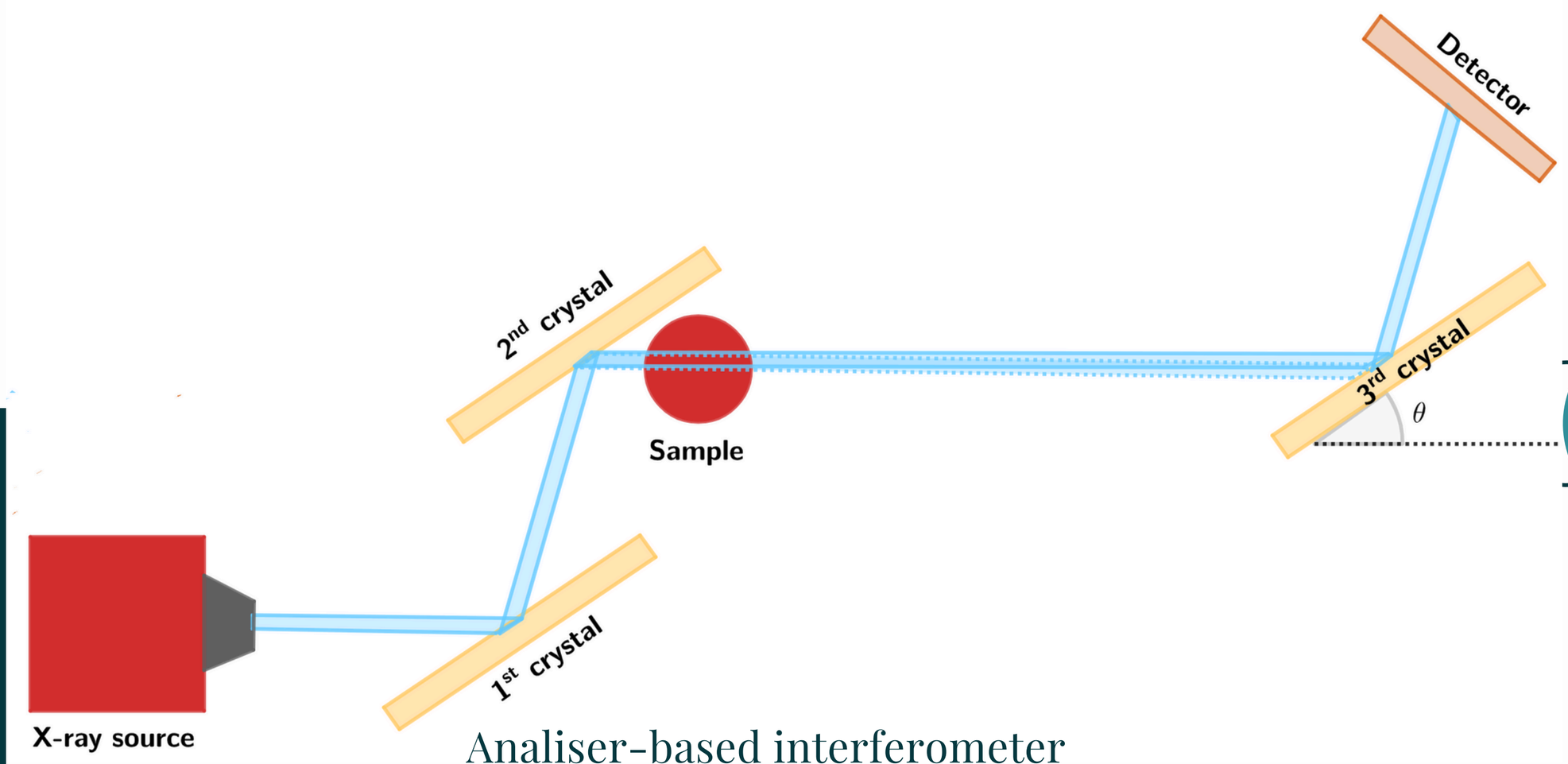
Beer-Lambert Equation-Phase shift




Bonse-Hart interferometer

# Phase Contrast Imaging

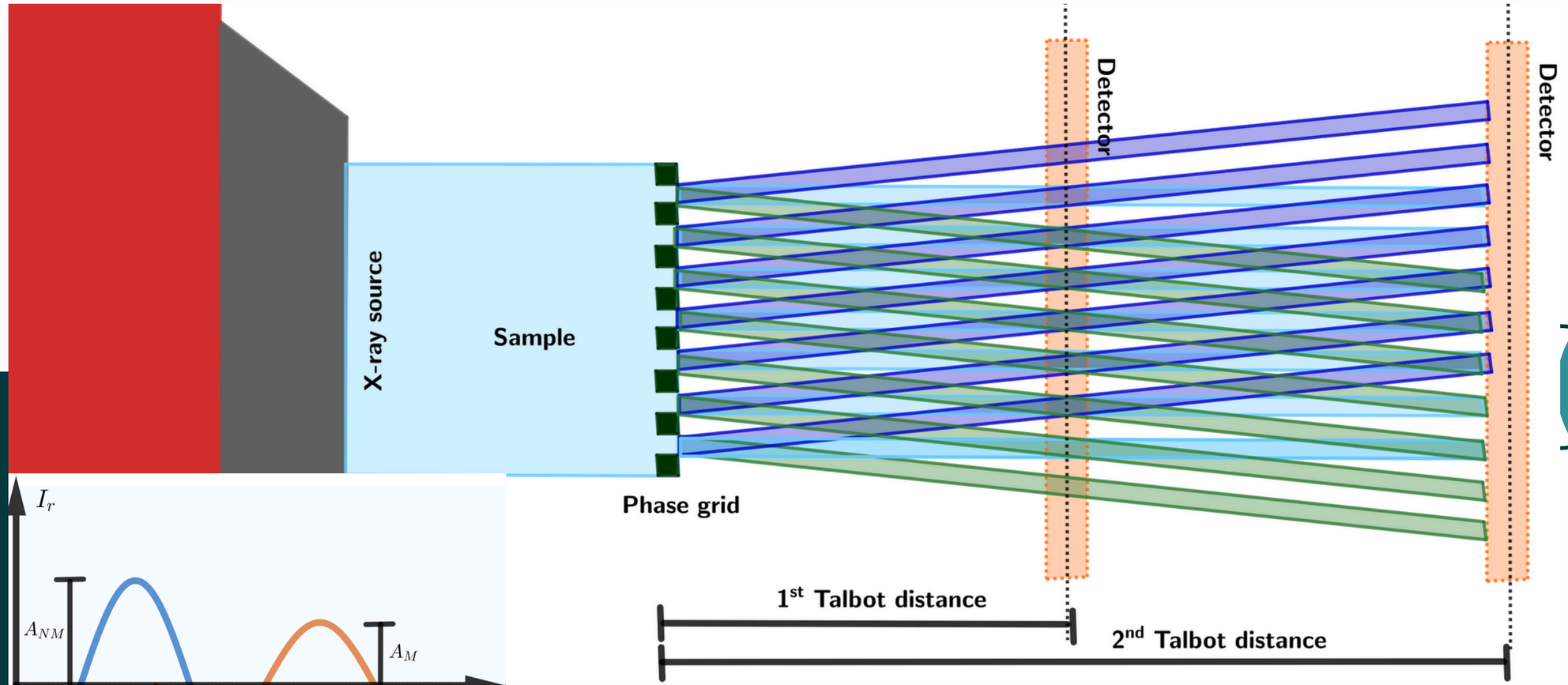
## Beer-Lambert Equation-Phase shift




Analiser-based interferometer

# Phase Contrast Imaging

## Beer-Lambert Equation-Phase shift



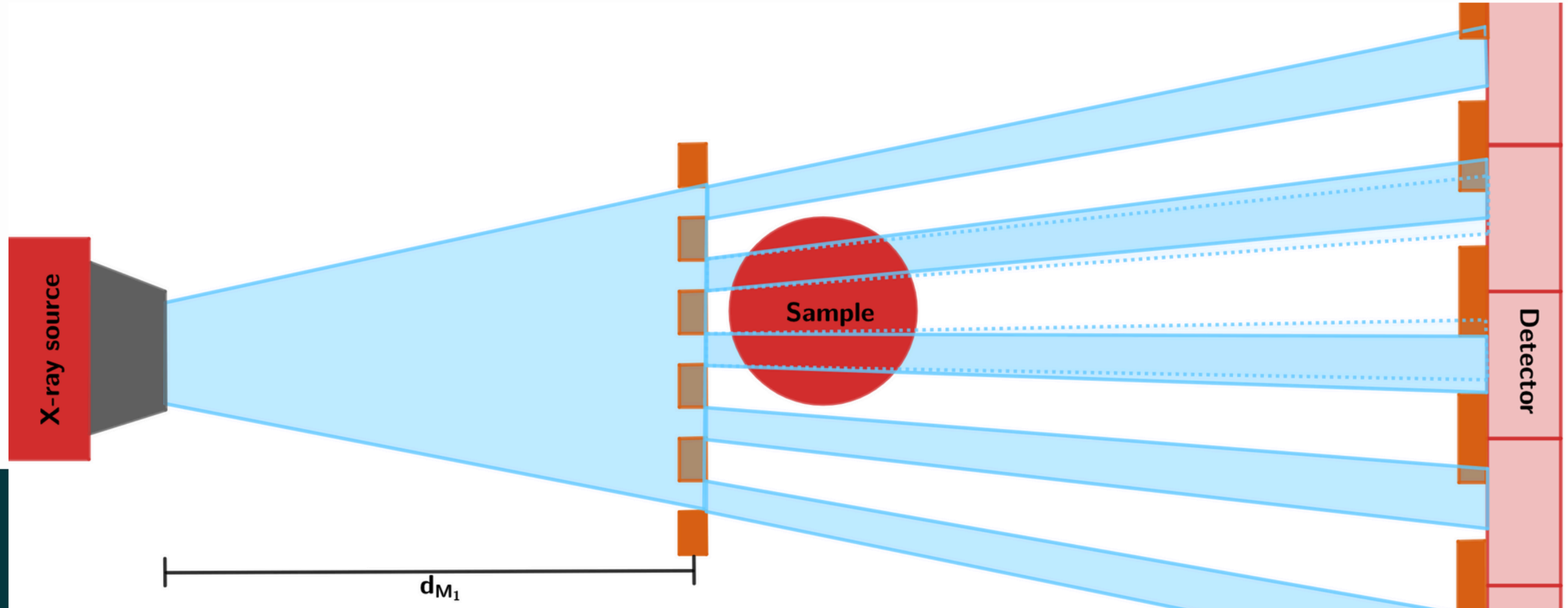


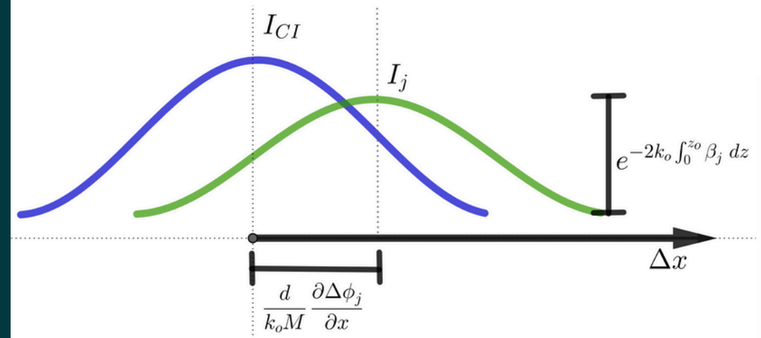


Grating interferometer

# Phase Contrast Imaging

## Beer-Lambert Equation-Phase shift

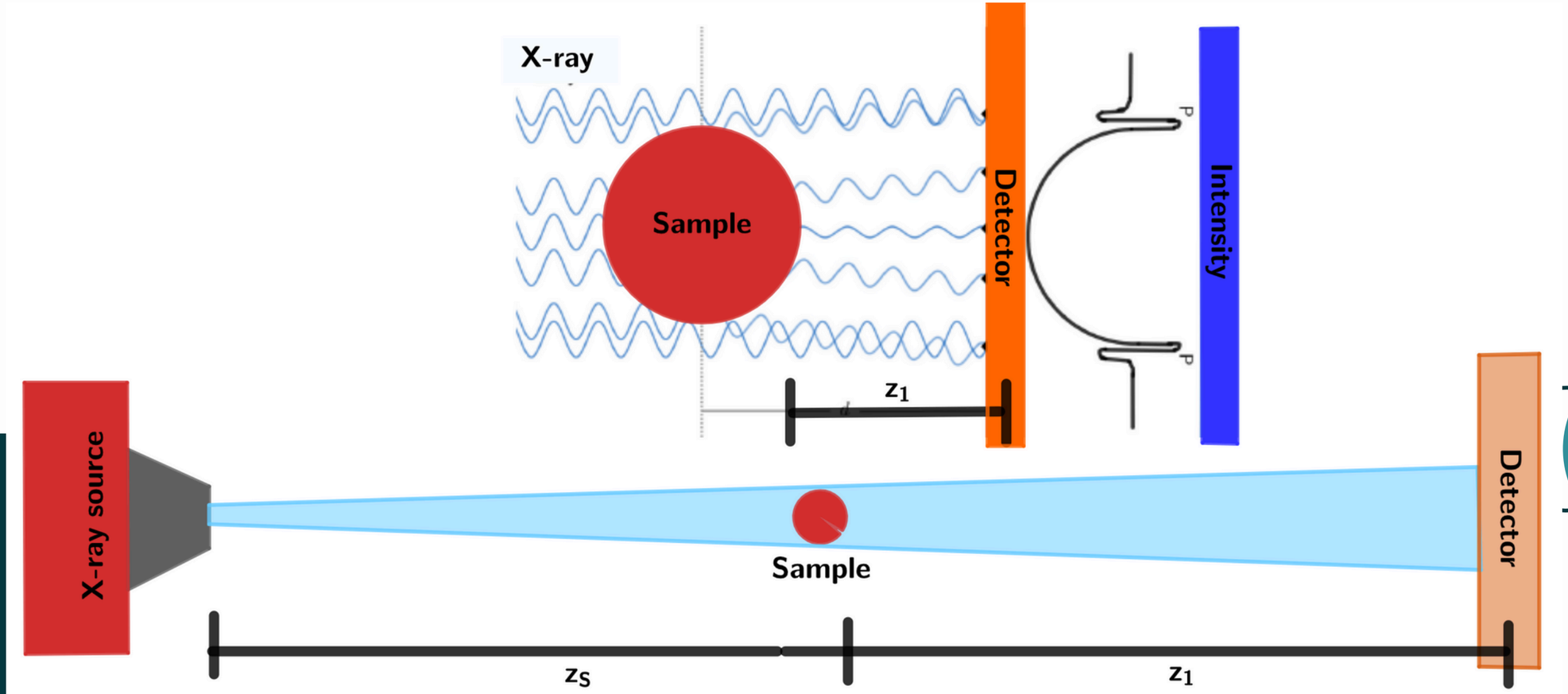



Edge Illumination

# Phase Contrast Imaging

Beer-Lambert Equation-Phase shift




Inline (Free-propagation)

# Phase Contrast Imaging

## Inline (Free-Propagation)

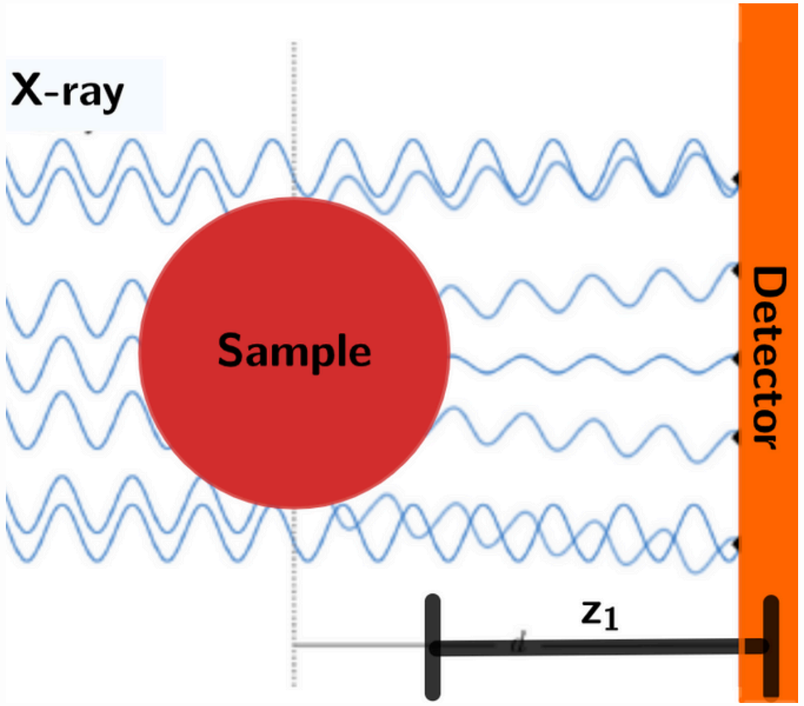
**Solution (Air): Neccesary Assumption**

Paraxial approximation

$$\left[2i\frac{\partial}{\partial z} + \nabla_t{}^2 - k^2\right]\rho(x,y,z) = 0 \quad (12)$$

$$2ik\frac{\partial[\rho^*\rho]}{\partial z} + \rho^*\nabla_t{}^2\rho - \rho\nabla_t{}^2\rho^* = 0 \quad (13)$$

$$\rho(x,y,z) = \rho(x,y,0)e^{-ik\int_0^z(\delta(r)-i\beta(r))dz} \quad (14)$$



# Phase Contrast Imaging

## Inline (Free-Propagation)

**Transport-Intensity equation**

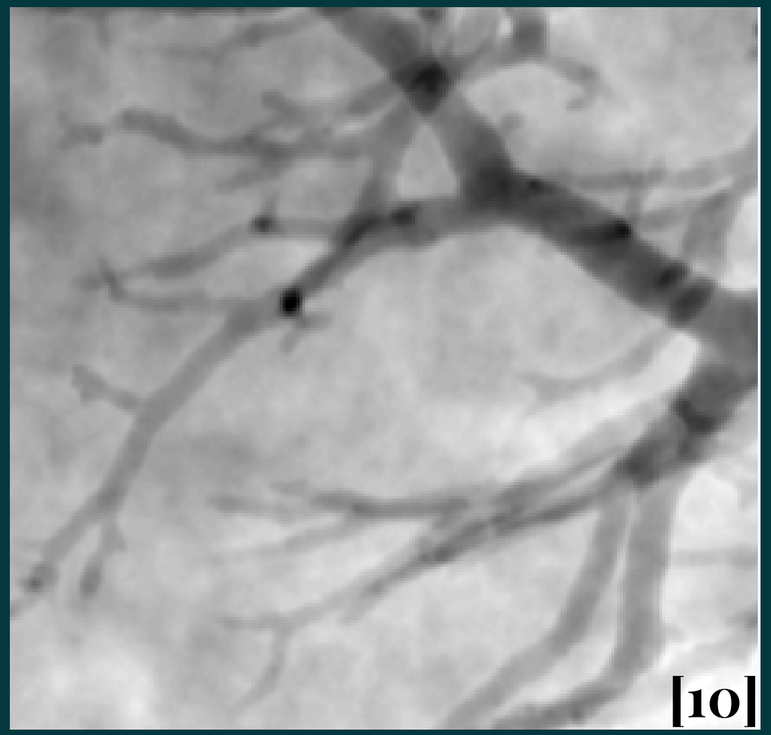
General

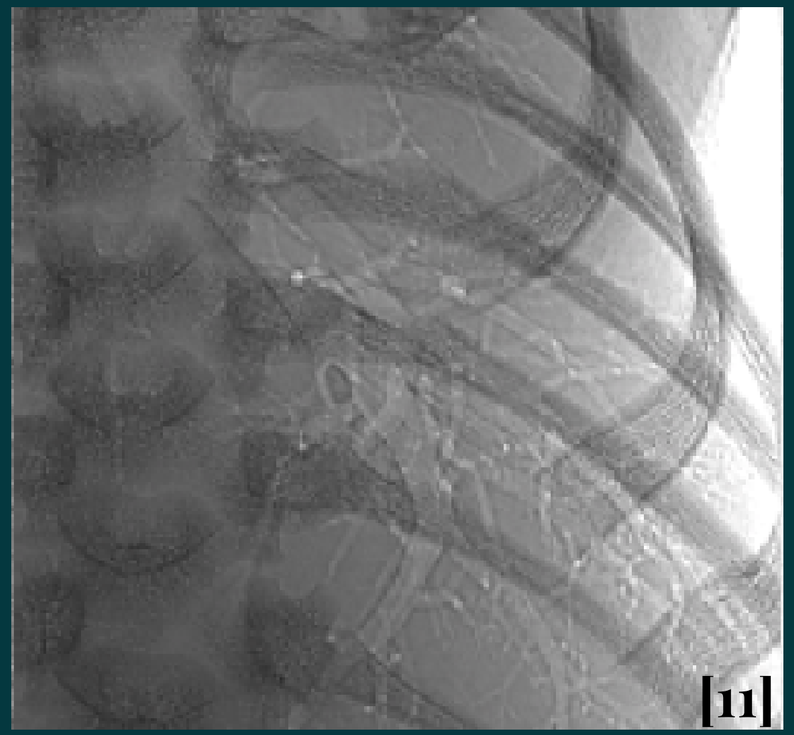
$$k\frac{\partial I}{\partial z} + \vec{\nabla}_t I \cdot \vec{\nabla}_t \Delta\phi + I\nabla_t{}^2\Delta\phi = 0 \qquad (15)$$

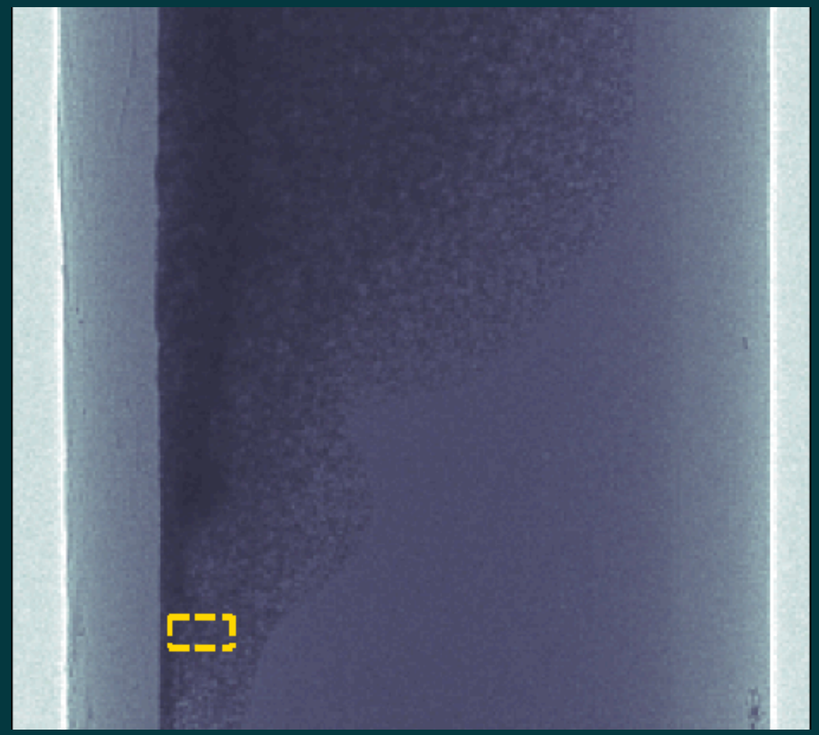
$$I_{z_1} = I_{z_o}\left[1 - \frac{z_1}{k}\nabla_t{}^2\Delta\phi\right] \qquad (16)$$

# Phase contrast Imaging: Inline

Applications


[10]


[11]



# Phase Contrast Imaging

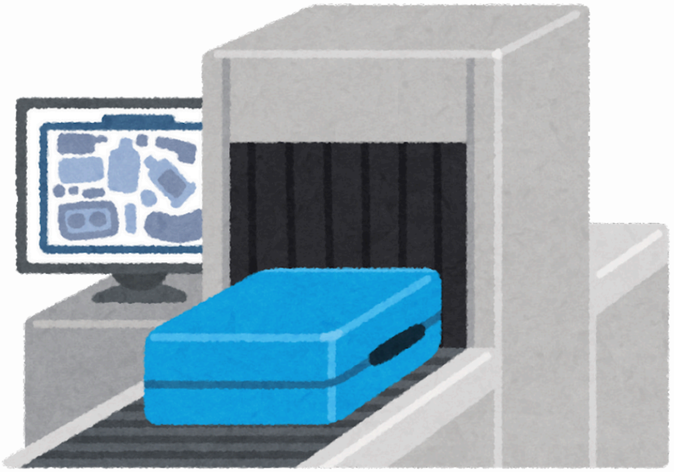
## Inline (Free-Propagation)

**Phase retrieval: Paganin approach**

Assumption $\gamma = \frac{\delta(\boldsymbol{r})}{\beta(\boldsymbol{r})}$

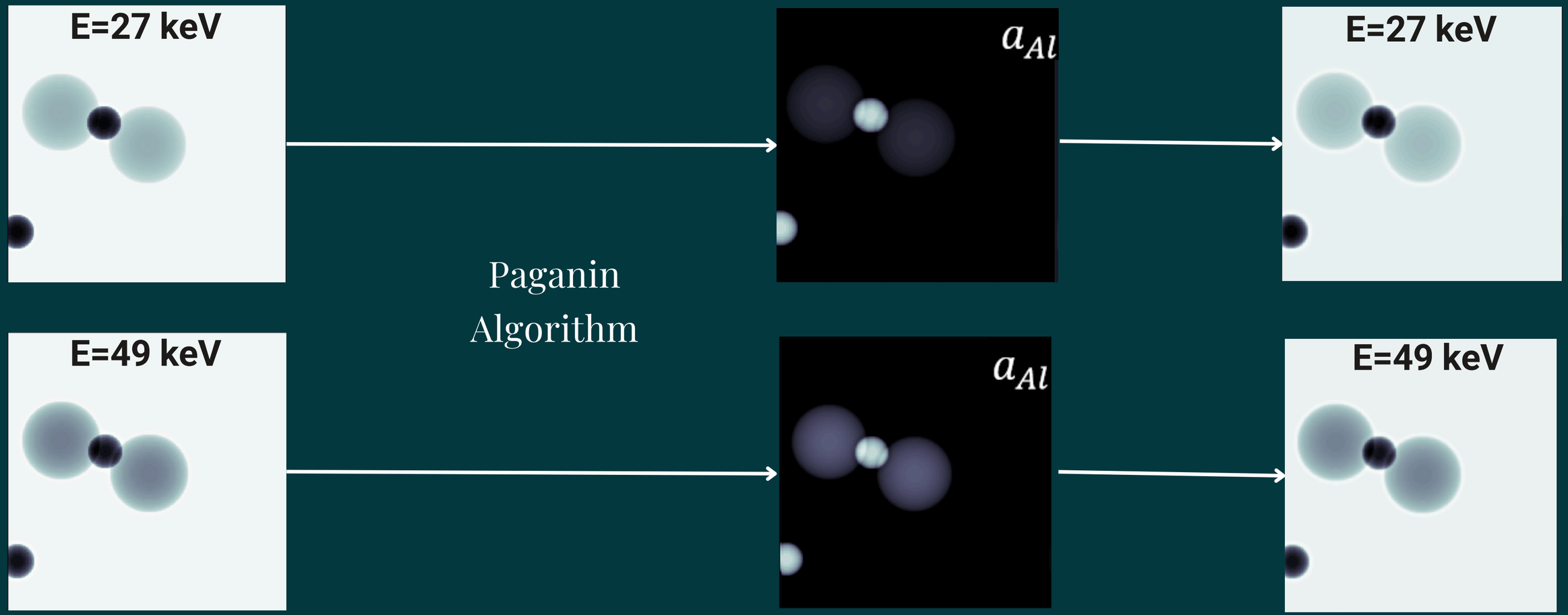
$$\frac{I_{z_1}}{I_o} = \left[1 - \frac{z_1 \gamma}{2k} \nabla_t{}^2\right] e^{\frac{\gamma \Delta\phi}{2}} \qquad (17)$$

$$\Delta\phi = \frac{\gamma}{2} \ln\left[\mathcal{F}^{-1}\left\{\frac{\mathcal{F}\left\{\frac{I_{z_1}}{I_o}\right\}}{1 + \frac{z_1 \gamma}{2k}(u^2 + v^2)}\right\}\right] \qquad (18)$$



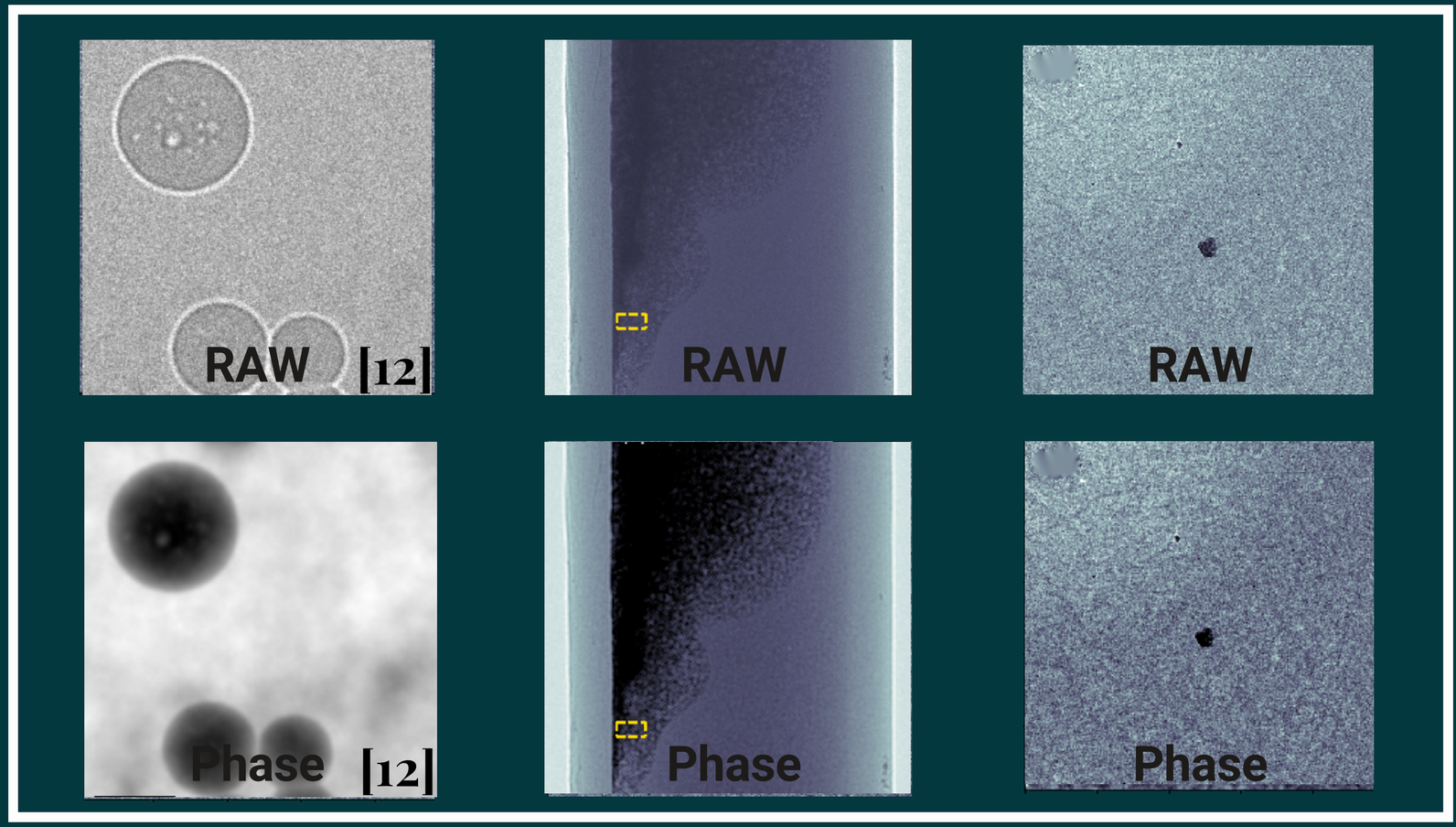
# Phase retrieval: Paganin Approach

Scheme



# Phase retrieval: Paganin Approach

## Applications



# Phase Contrast Imaging

## Inline (Free-Propagation)

**Phase retrieval: Spectral approach**

$$I_{z_1} = I_{z_o}\left[1 - \frac{z_1}{k}\nabla_t{}^2\Delta\phi\right]$$

$$I_{z_1} = I_o e^{-2k\int_0^z \beta(\boldsymbol{r})dz}\left[1 + z_1\nabla_t^2\int_0^z \delta(\boldsymbol{r})dz\right] \quad (19)$$

Assumption $1 \gg z_1\nabla_t^2\int_0^z \delta(\boldsymbol{r})dz$

# Phase Contrast Imaging

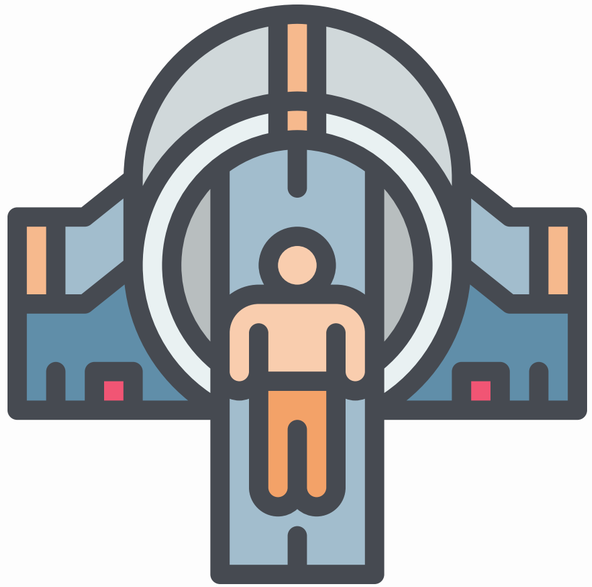
## Inline (Free-Propagation)

**Phase retrieval: Spectral approach**

$$I_{z_1} = I_o e^{-2k\int_0^z \beta(\boldsymbol{r})dz} e^{z_1\nabla_t^2 \int_0^z \delta(\boldsymbol{r})dz} \qquad (20)$$

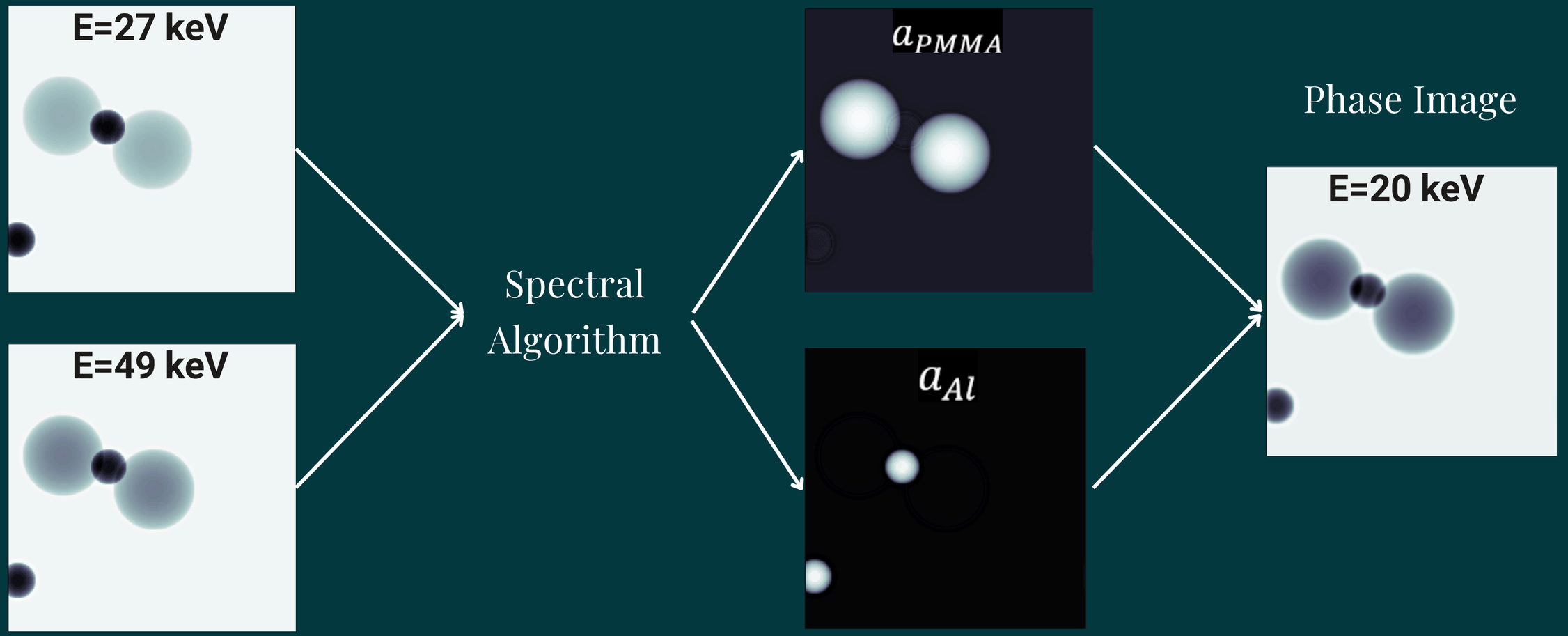
$$\int_0^z \beta/\delta(\boldsymbol{r})dz = \sum_{i=1}^{N} \beta_i/\delta_i \, a_i$$

$$\ln\left(\frac{I_o}{I_{z_1}}\right) = \sum_{i=1}^{N}[2k\beta_i - z_1\delta_i\nabla_t^2]\, a_i \qquad (21)$$



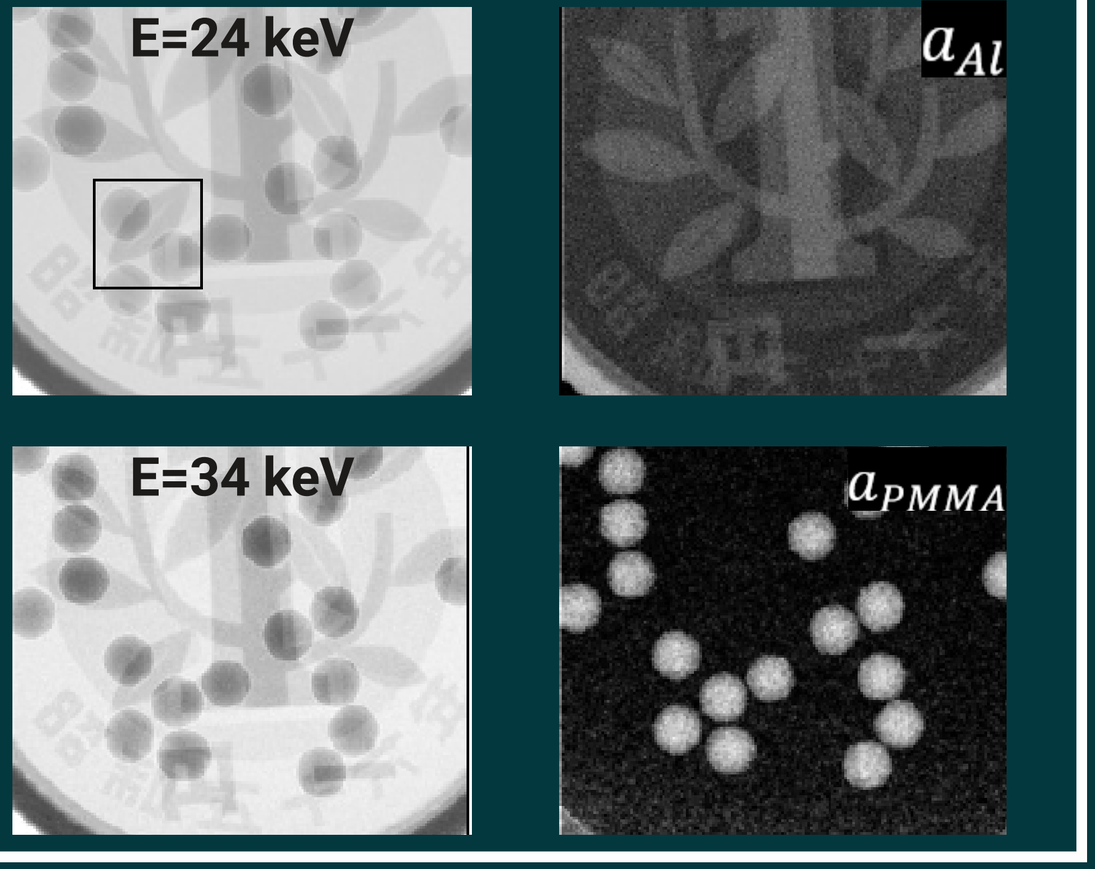
# Phase retrieval: Spectral Approach

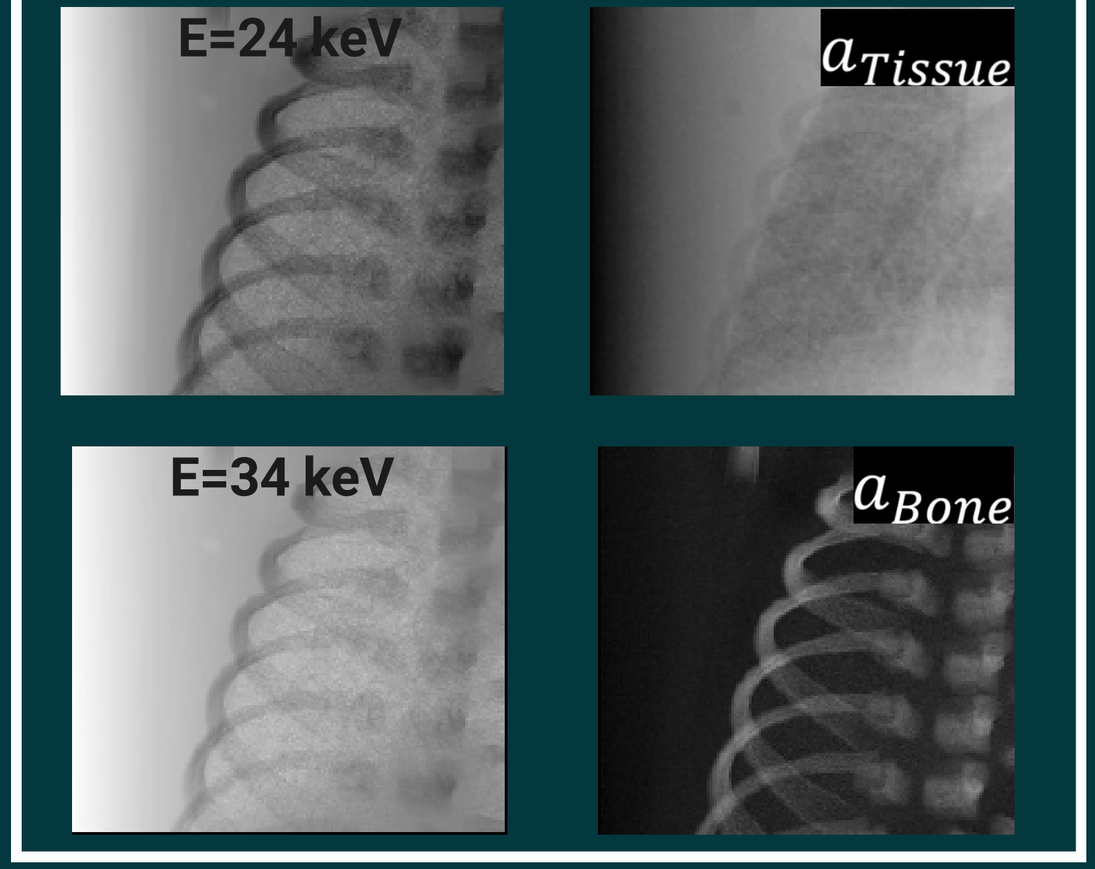
Scheme



# Phase retrieval: Spectral Approach

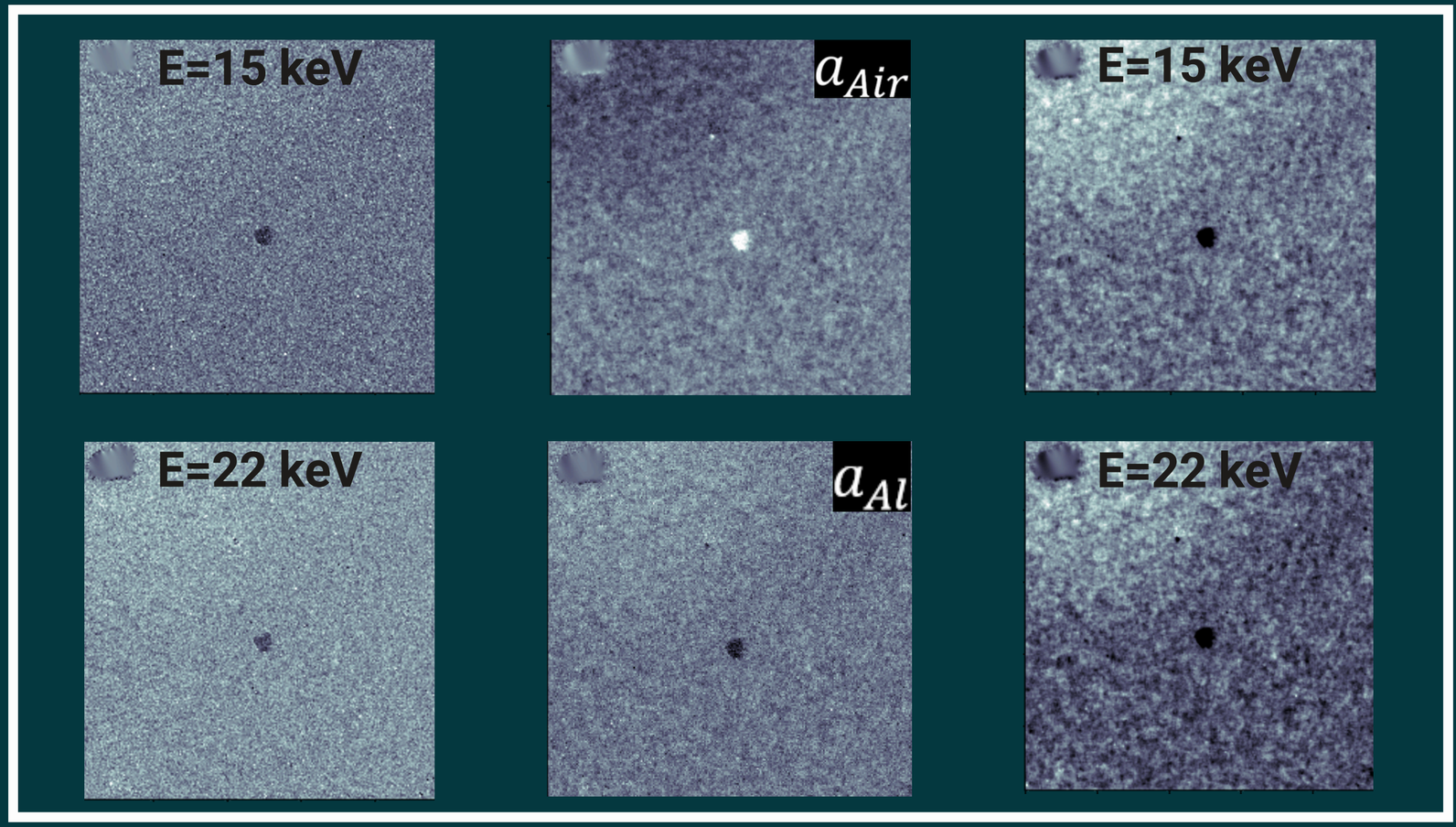
Applications



# Phase retrieval: Spectral Approach

Applications



# Thanks for your Attention

# References


[1],[4],[7] MANUEL SÁNCHEZ BOOK

[2] HTTPS://STOCK.ADOBE.COM/CO/SEARCH?K=SKULL+XRAY

[3] HTTPS://DOI.ORG/10.1148/RYCAN.2021200097

[5] HTTPS://DOI.ORG/10.1038/S41598-021-83330-W

[6] TESIS DAVID JURADO

[8] HTTPS://DOI.ORG/10.1117/12.2060605

[9] HTTPS://DOI.ORG/10.1148/RADIOL.2015141614

[11] HTTPS://DOI.ORG/10.1088/0031-9155/50/21/006

[10] HTTPS://DOI.ORG/10.1364/OE.19.010359

[12]HTTPS://DOI.ORG/10.1046/J.1365-2818.2002.01010.X