Structure of cosmic strings for a gauged $B-L$ symmetry

Standard Model (SM): highly successful why go beyond?
Common motivation: misses gravity, Dark Matter, Dark Energy . . .

Here: conceptual short-coming: baryon number B and lepton number L can change due to top. winding of $\mathrm{SU}(2)_L$, although at tiny rate at accessible energies, never observed.

However: conservation of $B-L$ is an exact, global symmetry: unnatural, only local symmetries (gauge symmetries) have compelling reasons for being exact. We convert it into a U(1) gauge symmetry.

Avoiding a gauge anomaly requires right-handed neutrinos ν_R.

Majorana mass term not allowed, to provide ν_R masses: add an additional Higgs-type field $\chi \in \mathbb{C}$.

Well-motivated, modest SM Extension.

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Cosmic strings

Even if a field theory does not have top. sectors, it may still have local top. defects: $e.g.$ type II superconductors; superfluid ^{4}He; vortices in 2d XY model, can pile up in 3d as closed global strings.

- **Prototype of 4d global cosmic strings**

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu \chi^* \partial_\mu \chi - V(|\chi|^2), \quad V(|\chi|^2) = \tfrac{1}{2}\mu_0^2 |\chi|^2 + \tfrac{1}{4}\lambda_0 |\chi|^4$$

$$\chi(x) \in \mathbb{C}, \quad \mu_0^2 < 0, \quad \lambda_0 > 0.$$

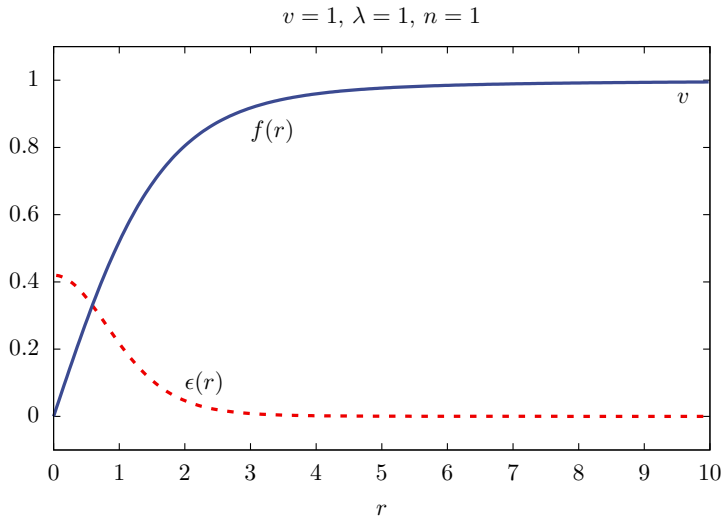
Ansatz for static solution in cylindrical coordinates

$$\chi(r, \varphi, z) = f(r)\exp(\mathrm{i}n\varphi)$$

$f(r)$ profile function, $n \in \mathbb{Z}$ winding number.

$f(r \to \infty) = v = \sqrt{-\mu_0^2/\lambda_0}$, and for $n \neq 0$: $f(0) = 0$ (no phase ambiguity). Non-linear field equation

$$f''(r) + \tfrac{1}{r} f'(r) = \left(\frac{n^2}{r^2} + \mu_0^2 + \lambda_0 f^2(r) \right) f(r)$$



Numerical solution for $v = 1$ (sets scale), $\lambda = 1$, $n = 1$.
$\epsilon(r)$: energy density

Energy per length in z-direction, E/z diverges $\propto \ln r_{\max}$

- **Prototype of 4d local cosmic strings**

Couple $\chi \in \mathbb{C}$ to a U(1) gauge field A_μ,

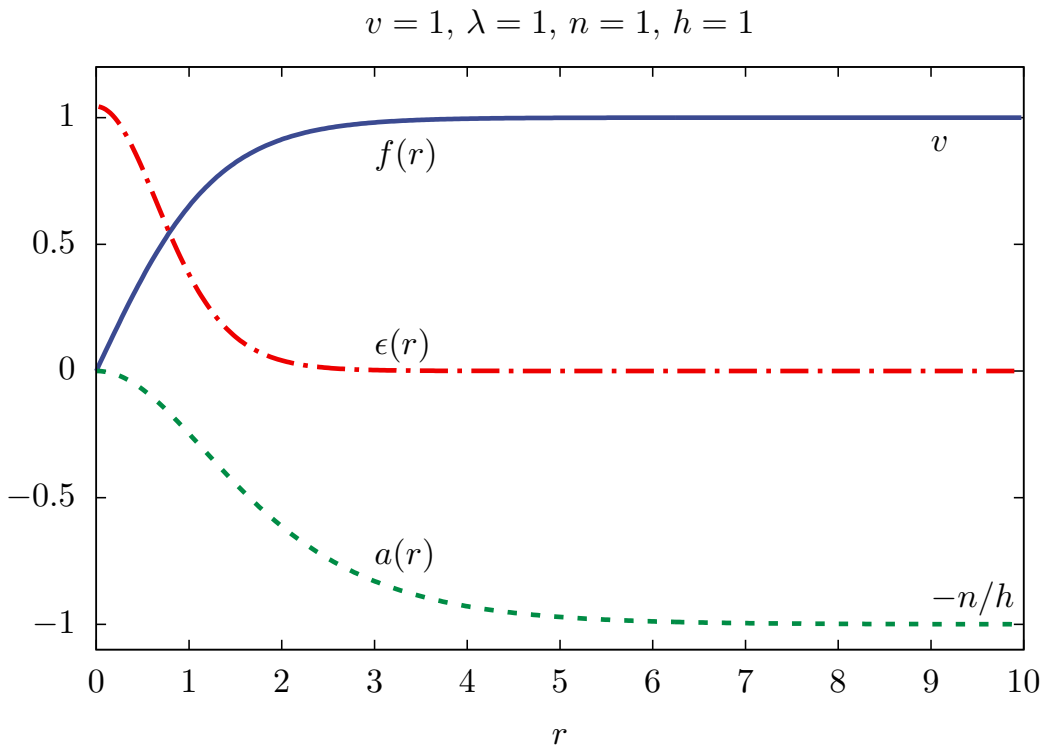
$$\mathcal{L} = \tfrac{1}{2}(D^\mu\chi)^* D_\mu\chi - V(|\chi|^2) - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad D_\mu = \partial_\mu - \mathrm{i}gA_\mu$$

Coupled non-linear field equations

$$\begin{aligned} (D^\mu D_\mu + \mu_0^2\chi + \lambda_0|\chi|^2)\chi &= 0 \\ D^\mu F_{\mu\nu} + \tfrac{\mathrm{i}g}{2}((D_\nu\chi)^*\chi - \chi^* D_\nu\chi) &= 0 \end{aligned}$$

Again cylindric ansatz, with $A_0 = A_r = 0$, tangential $\vec{A}$,

$$\chi = f(r)\exp(\mathrm{i}n\varphi)\ , \quad \vec{A} = \frac{a(r)}{r}\hat{\varphi}, \quad a(0) = 0$$



Solution for $v = 1,\ \lambda = 1,\ n = 1,\ h = 1$

E/z is finite for local strings

Kibble mechanism for the formation of cosmic strings

Top. defects omnipresent in the Early Universe.

Under rapid cooling, part of them persisted.

About 10^{-12} sec after Big Bang: Higgs field took non-zero VEV.

In separated regions, causally disconnected, the VEV took different complex phases. Kibble: vortices in interfaces between these regions, piled up to strings, which might still persist, due to top. stability.

Cosmic strings are not observed yet, but the Kibble mechanism is confirmed experimentally for non-linear optical systems [Ducci et al '99], Josephson junctions of superconductors [Monaco et al '06], manganites [Griffin et al '12, Lin et al '14] and superfluids [Rysti et al '21, Lee et al '24], and numerically in the 2d XY model [Jelić and Cugliandolo, '11] and 3d XY model [López-Contreras et al, '24].

Extended Standard Model with a gauged $B - L$ symmetry and a massive ν_R

We introduce a U(1) gauge field $\mathcal{A}_\mu$, which couples to the conserved charges Y (weak hypercharge) and $B - L$ as

$$2hY + \tfrac{1}{2}h'(B - L)$$

Gauge anomaly cancellation requests balanced left and right chiralities, which is achieved with a right-handed neutrino ν_R in each generation. This condition leave two degrees of freedom, which corresponds to the couplings h and h', even if we include the graviton.

SO(10) GUT incorporates the case $h'/h = -5$ [Buchmüller, Greub, Minkowski '91]

Since ν_R is not sterile, the usual Majorana mass term breaks gauge symmetry.

We provide a mass for ν_R with an additional Higgs-type field $\chi \in \mathbb{C}$, and consider

$$\begin{aligned}
\mathcal{L} &= \tfrac{1}{2}(D^\mu\Phi)^\dagger)D_\mu\Phi + \tfrac{1}{2}(d^\mu\chi)^* d_\mu\chi - V(\Phi,\chi) - \tfrac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} \\
D_\mu &= \partial_\mu + \mathrm{i}h\mathcal{A}_\mu, \quad d_\mu = \partial_\mu + \mathrm{i}h'\mathcal{A}_\mu \\
V(\Phi,\chi) &= \tfrac{1}{2}\mu^2\Phi^\dagger\Phi + \tfrac{1}{4}\lambda(\Phi^\dagger\Phi)^2 \\
&+ \tfrac{1}{2}\mu'^2\chi^*\chi + \tfrac{1}{4}\lambda'(\chi^*\chi)^2 + \tfrac{1}{2}\kappa\Phi^\dagger\Phi\chi^*\chi
\end{aligned}$$

where $\Phi \in \mathbb{C}^2$ is the (standard) Higgs field, $Y_\Phi = 1/2$, and the non-standard Higgs field has charge $(B-L)_\chi = 2$.

We do not include SM fermion or gauge fields, assuming that they do not significantly affect the cosmic strings.

The VEVs v, v' should make $m_\mathcal{A} = hv + h'v'$ sufficiently large to explain that $\mathcal{A}_\mu$ has not been observed.

Existence of a ground state requires $\kappa^2 < \lambda\lambda'$.

Cylindrical ansatz

$$\Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi(r) e^{in\varphi} \ , \quad \chi = \xi(r) e^{in'\varphi} \ , \quad \vec{A} = \frac{a(r)}{r} \hat{\varphi}$$

Coupled non-linear field equations

$$\begin{aligned}
\phi''(r) + \tfrac{1}{r}\phi'(r) &= \left[\tfrac{(n+ha)^2}{r^2} + \mu^2 + \lambda\phi^2 + \kappa\xi^2 \right] \phi \\
\xi''(r) + \tfrac{1}{r}\xi'(r) &= \left[\tfrac{(n'+h'a)^2}{r^2} + \mu'^2 + \lambda'\xi^2 + \kappa\phi^2 \right] \xi \\
a''(r) - \tfrac{1}{r}a'(r) &= h\phi^2(n+ha) + h'\xi^2(n'+h'a)
\end{aligned}$$

Radial boundary conditions

$$\phi(0) = 0 \ (\text{if } n \neq 0), \quad \xi(0) = 0 \ (\text{if } n' \neq 0), \quad a(0) = 0$$

Assuming all profile functions to be asymptotically constant at large r

$$\begin{aligned}
\phi(r\to\infty) &= v = \tfrac{\kappa\mu'^2-\mu^2\lambda'}{\lambda\lambda'-\kappa^2},\\
\xi(r\to\infty) &= v' = \tfrac{\kappa\mu^2-\mu'^2\lambda}{\lambda\lambda'-\kappa^2},\\
a(r\to\infty) &= -\tfrac{n}{h} = -\tfrac{n'}{h'}
\end{aligned}$$

The constraint in the last line implies for the SO(10) GUT $n'/n = -5$.

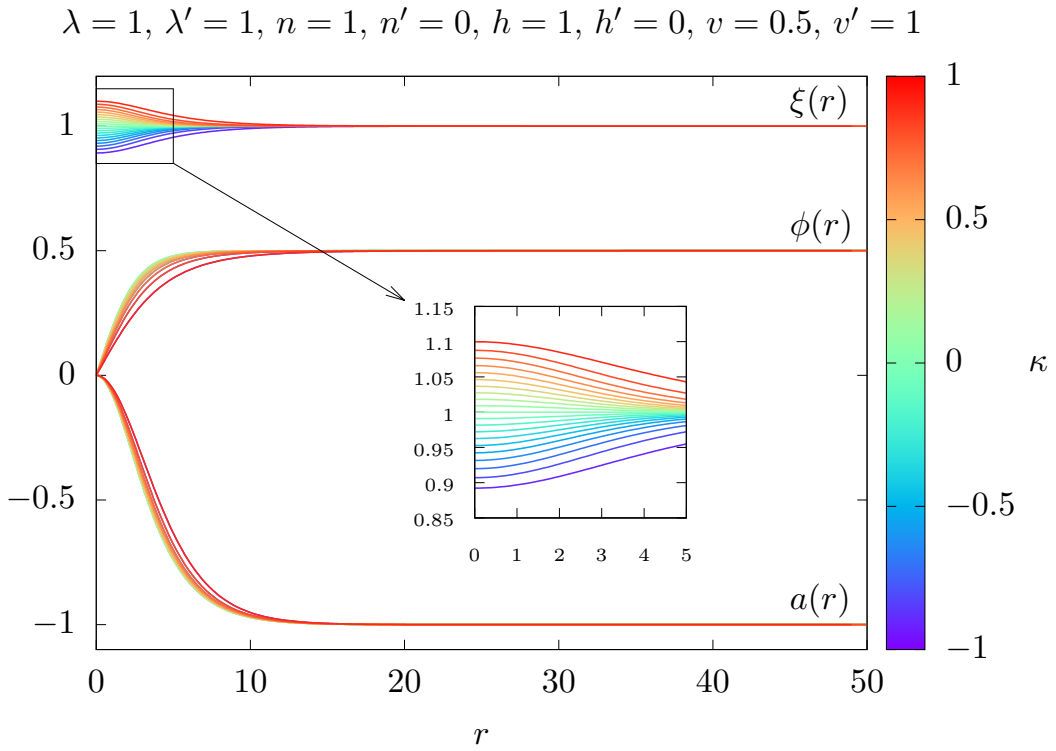
In the numerical study, we choose two values of v.
Based on $v = 246\ \mathrm{GeV}$ they set the scale, *e.g.* for the radius

$$\begin{aligned}
v = 0.01 &: \quad r = 1 \text{ corresponds to } 8\cdot 10^{-6}\ \mathrm{fm}\\
v = 0.5 &: \quad r = 1 \text{ corresponds to } 4\cdot 10^{-4}\ \mathrm{fm}
\end{aligned}$$

Solution with damped Newton method, checked with iterative relaxation and Runge-Kutta integration.

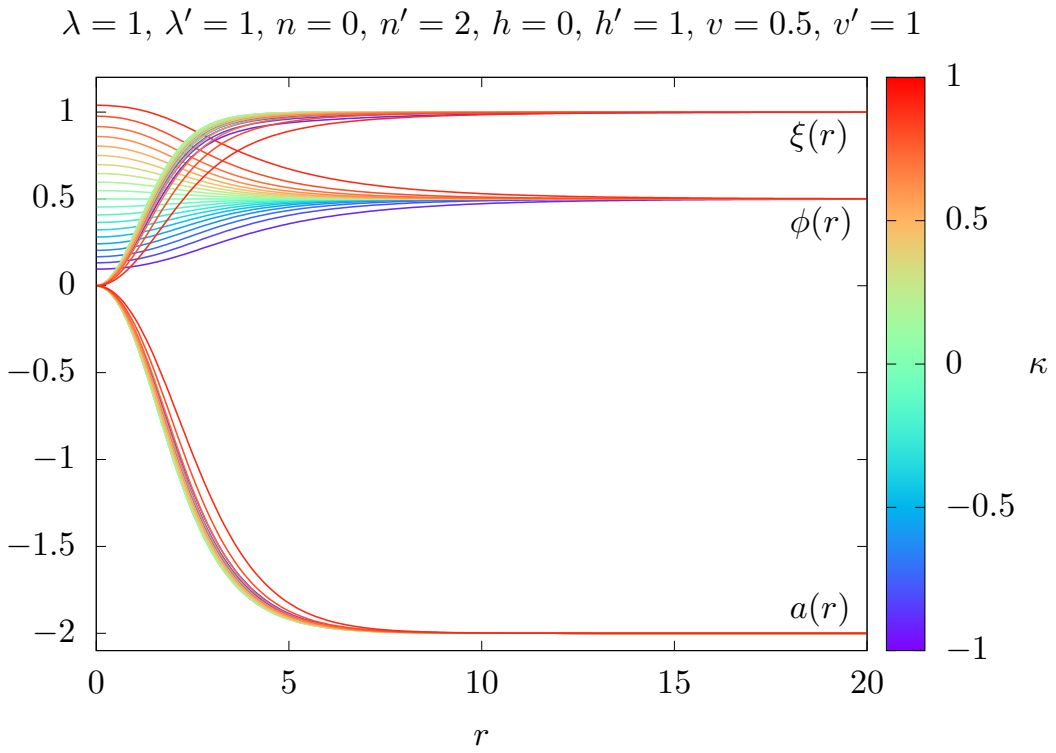
Results for the string profiles

We scan the allowed values of κ, $|\kappa| < \lambda\lambda'$



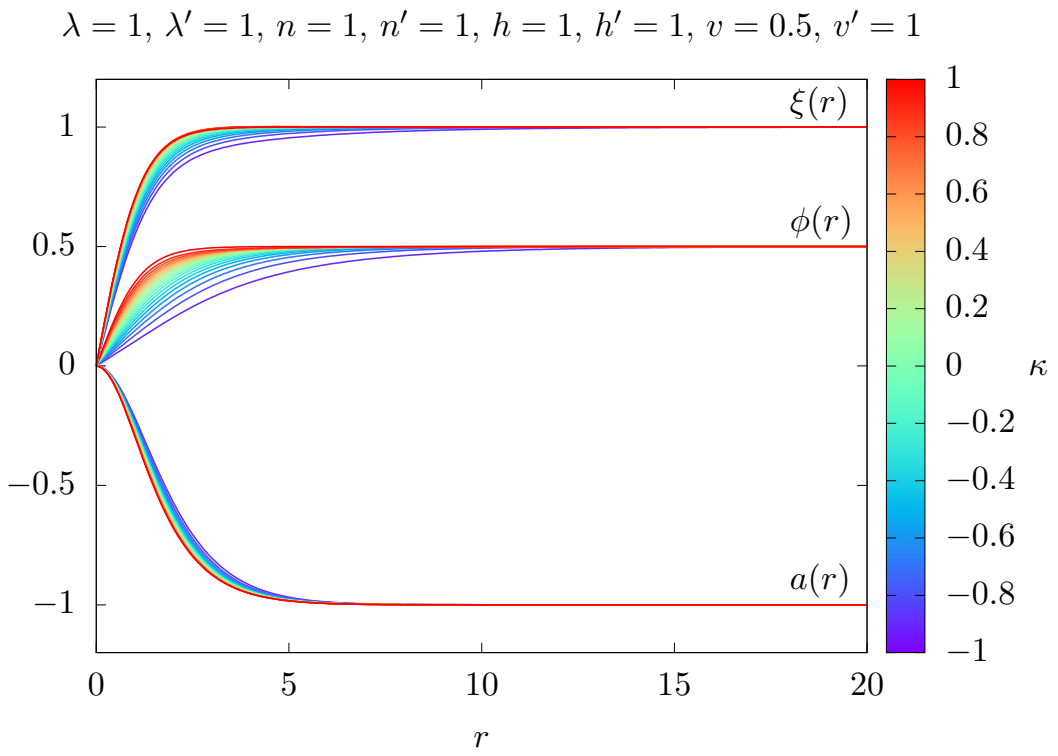
Due to $n' = 0$, $\xi(0)$ does not vanish, but there is the condition $\xi'(0) = 0$.

$\phi(r \to \infty)$ corresponds to $v = 246$ GeV.



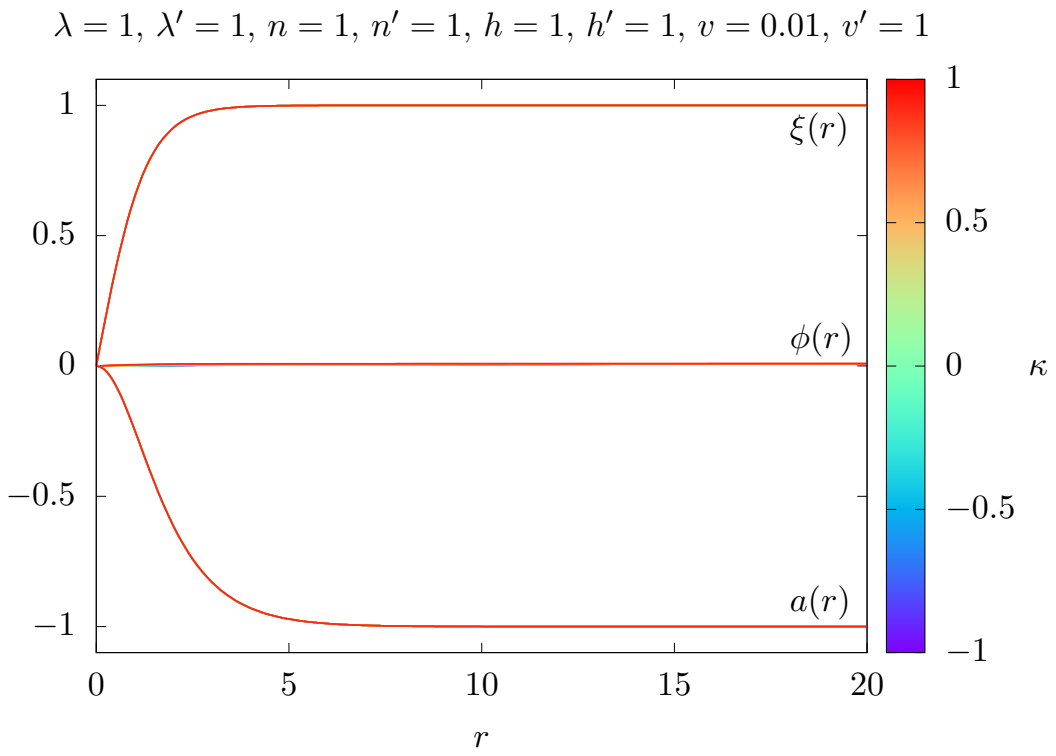
Due to $n = 0$, $\phi(0)$ does not vanish, but there is the condition $\phi'(0) = 0$.

So far all solutions are smooth and monotonous.



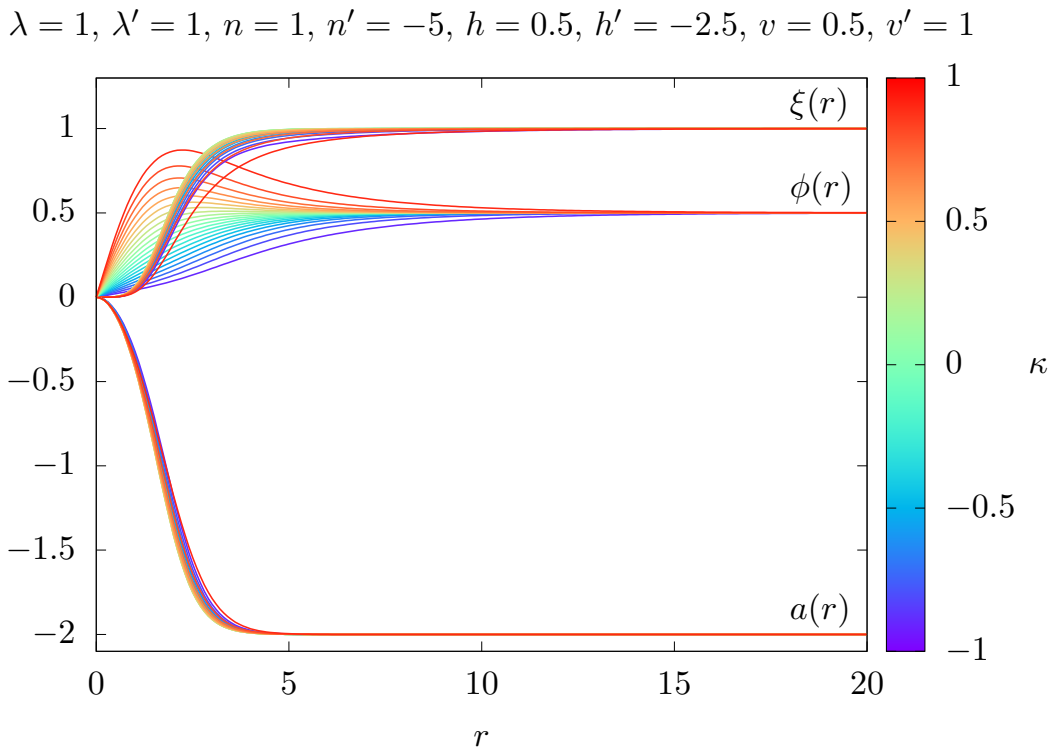
Now turn on both Higgs field windings, $n = 1$, $n' = 1$.

Still smooth and monotonous.



We reduce the standard Higgs VEV to $v = 0.01$: very thin string.

$\xi(r)$ similar to local prototype, v' very heavy. Hardly sensitive to κ

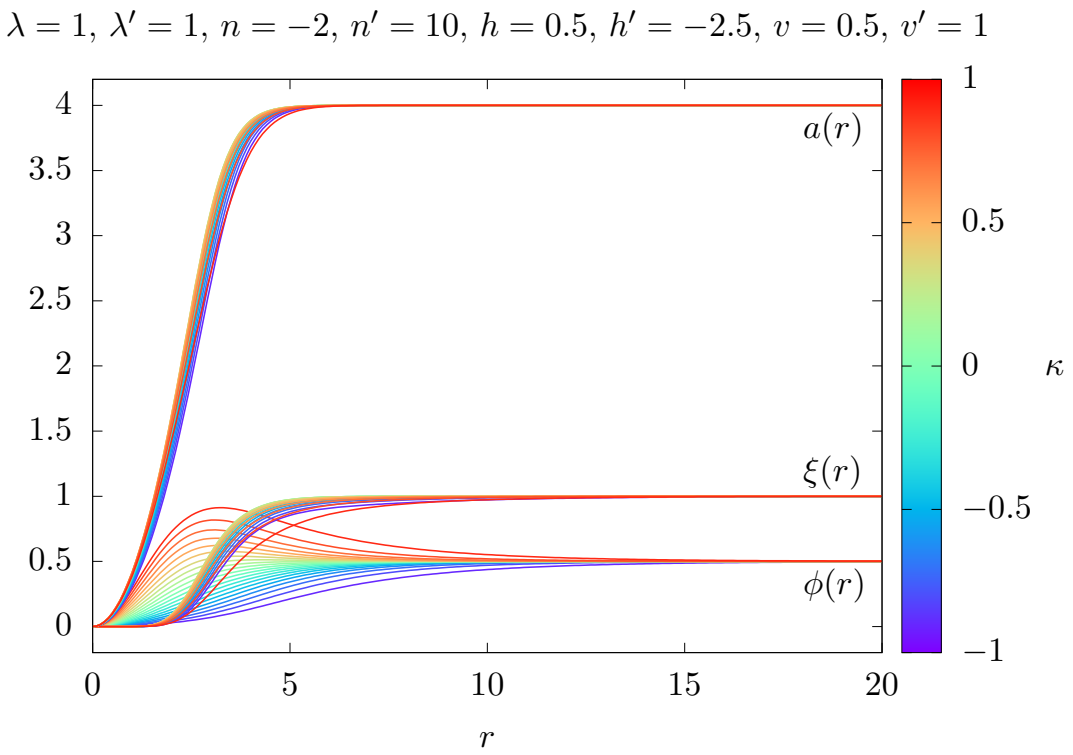


SO(10) scenario with strongly winded non-standard Higgs field, $n' = -5$

Not monotonous anymore:
for $\kappa > 0$, $\phi(r)$ overshoots v near the string core.

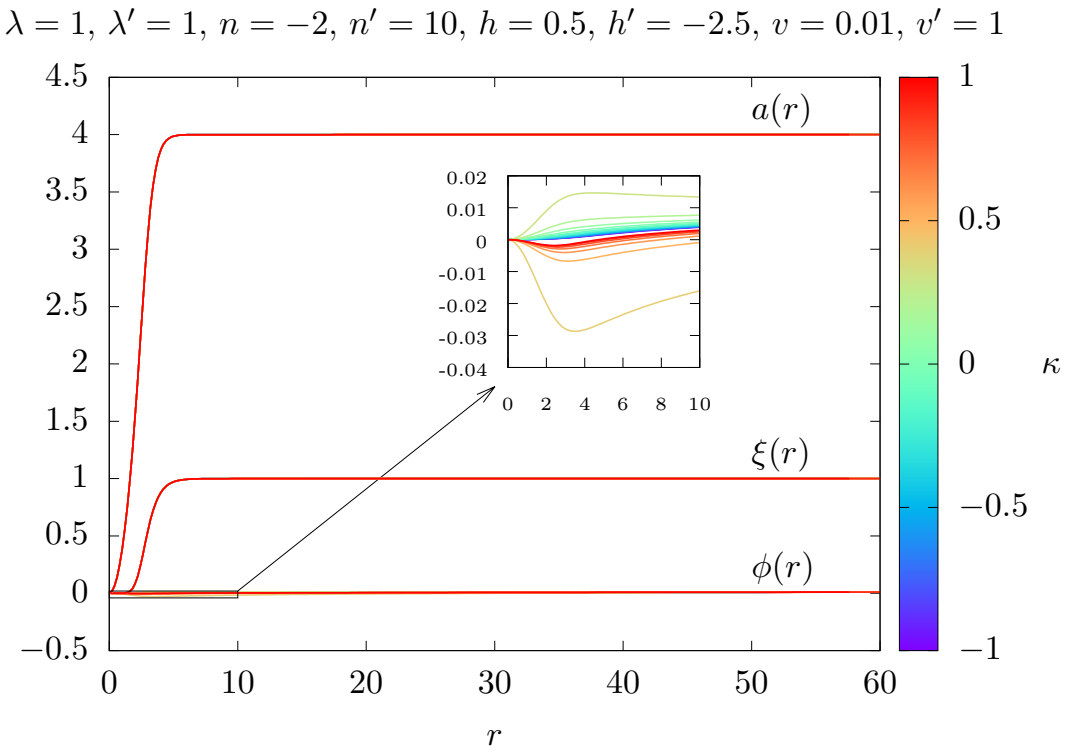
In other examples, also $\xi(r)$ can overshoot.

Effect on particles passing through string



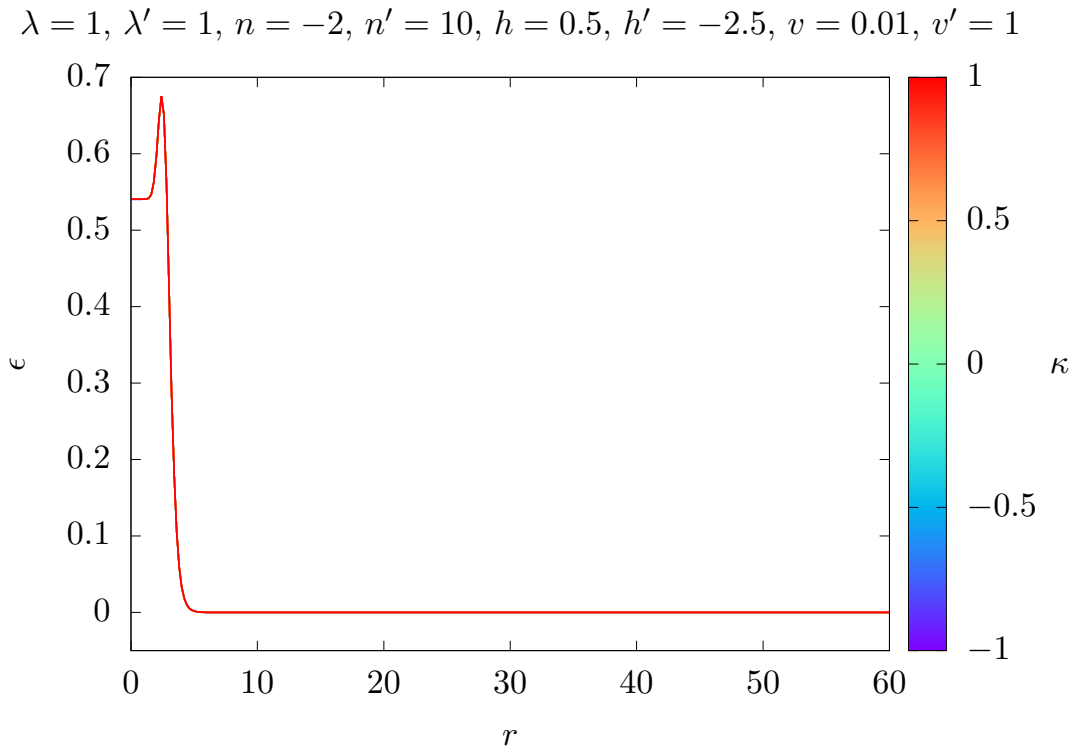
SO(10) scenario with even more extreme winding: likely to be unstable

For $\kappa > 0$, $\phi(r)$ overshoots again.
$a(r)$ flips sign: $\vec{A}$ in direction of $\hat{\varphi}$.



Back to $v = 0.01$: new phenomenon for $\kappa \gtrsim 0.25$

For increasing κ, change from overshooting to co-axial profile $\phi(r)$; effect first discovered by Bogomol'nyi '76 for unstable, standard cosmic strings



Radial energy density for solution in previous plot.

Sharp peak in the co-axial region near the core.
In physical units, $\epsilon = 1$ corresponds to $\approx 5 \cdot 10^{19}\ \mathrm{GeV/fm}^3$

Conclusions and prospects

We considered a modest and well motivated SM Extension: $B - L$ is the charge coupled to a U(1) gauge field $\mathcal{A}_\mu$, ν_R cancels gauge anomalies, and a non-standard Higgs field $\chi \in \mathbb{C}$ provides a ν_R-mass.

We studied the formation of cosmic strings in the gauge-Higgs sector with Φ, χ, $\mathcal{A}_\mu$. For our parameters: characteristic radius $< 10^{-3}$ fm, approximate Nambu-Goto strings.

Low winding numbers: smooth and monotonous profile functions; higher windings: overshooting and co-axial features, including the case embedded in an SO(10) GUT. Stability is left for future investigation.

In that case: string tension

$$\mu \simeq 10^{10}\ \mathrm{GeV}^2, \quad G\mu \simeq 10^{-28} \qquad (G:\ \text{gravitation constant})$$

String across the visible Universe would have 10^{26} kg $\lesssim M_{\text{Neptune}}$.

Cosmic Microwave Background (CMB) observations: cosmic strings could have caused 1d discontinuities (Kaiser-Stebbins-Gott effect). Unsuccessful direct search $\Rightarrow G\mu < 4 \cdot 10^{-6}$.

Angular power spectrum of CMB anisotropies: WMAP, SDSS, Planck25: $G\mu < 10^{-7}$.

Gravitational waves: LIGO-Virgo-KARGA (2021) referring to oscillating loops of cosmic strings: $G\mu < 4 \cdot 10^{-15}$.

LISA expects to arrive at precision $G\mu < \mathcal{O}(10^{-17})$.

Even then our scenario is not excluded, but hard to verify.