Universidad de los Andes

5th Workshop on Statistical Physics

13-17 April 2026

Universidad Nacional de Colombia

Centro de Ciencias de la Complejidad

Antifragility of stochastic transport on networks with damage

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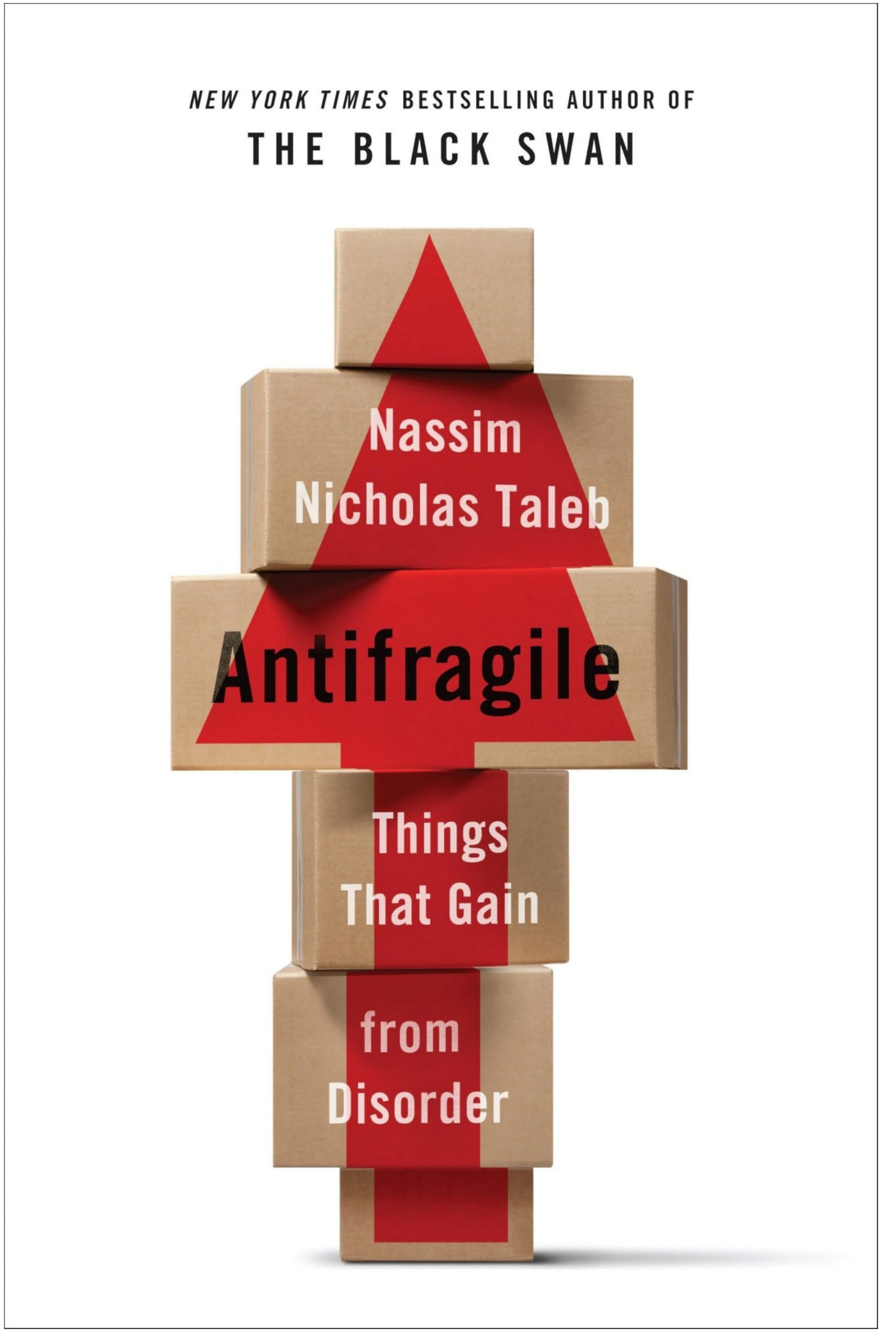
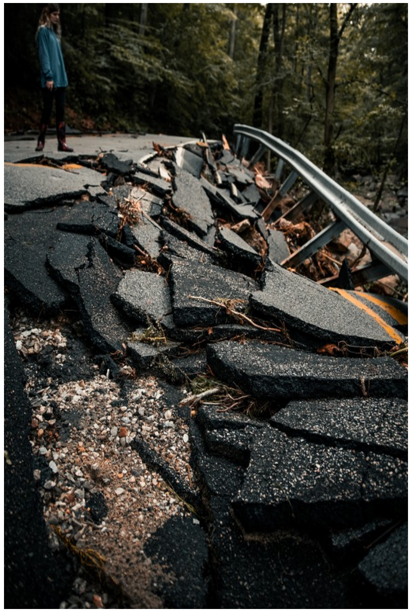
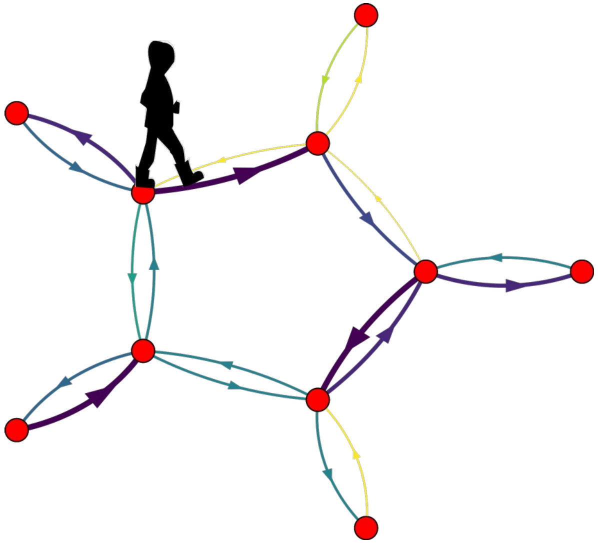
Motivation







- Which are the mechanisms behind antifragility?
- In which systems can antifragilitly emerge ?
- How to characterize antifragility?

In this work we present a model for antifragility using random walks on networks with damage

Edges or links (i,j)

- Graph: $G = (\mathcal{V}, \mathcal{E})$

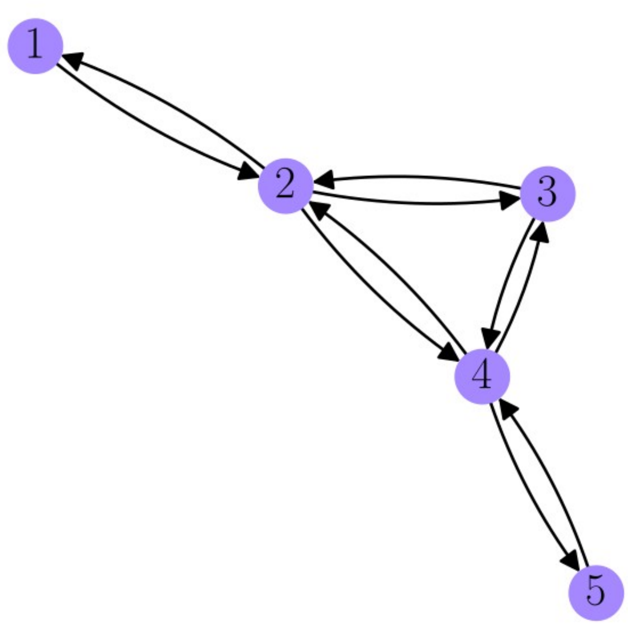
Vertexes or nodes

N Number of nodes

$|\mathcal{E}|$ Number of edges

- Adjacency matrix:

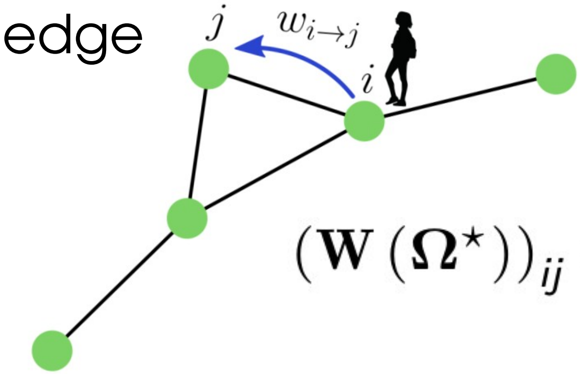
$$A_{ij} = \begin{cases} 1 & \text{si} \quad (i,j) \in \mathcal{E} \\ 0 & \text{si} \quad (i,j) \notin \mathcal{E} \end{cases}$$



$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Random walkers in a network with damage in one edge

Damage is a reduction in the weight of an edge of the network



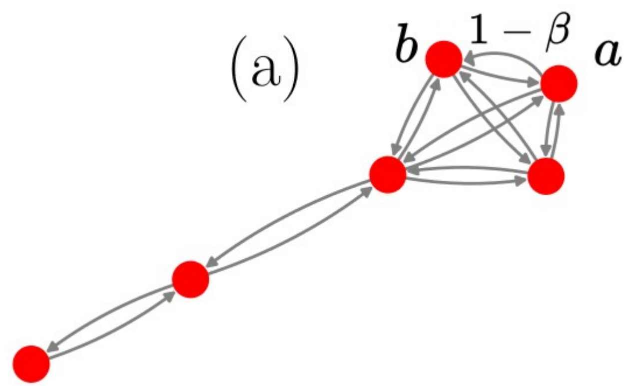
$$(\mathbf{W}(\mathbf{\Omega}^\star))_{ij} \equiv \frac{\Omega^\star_{ij}}{\sum_{\ell=1}^{N} \Omega^\star_{i\ell}}$$

Matrix of weights for a graph with damage in edge (a,b):

$$\Omega^\star_{ij} = \begin{cases} A_{ij} & i \neq a,\ j \neq b \\ (1-\beta)A_{ab} & i = a,\ j = b. \end{cases}$$

Damage Parameter

(a)



(b)

$$\mathbf{\Omega}^\star = \begin{array}{c c} & \begin{array}{cccccc} & & b & & & \end{array} \\ \begin{array}{c} \\ a \\ \\ \\ \\ \\ \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & \boxed{1\text{-}\beta} & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(c)

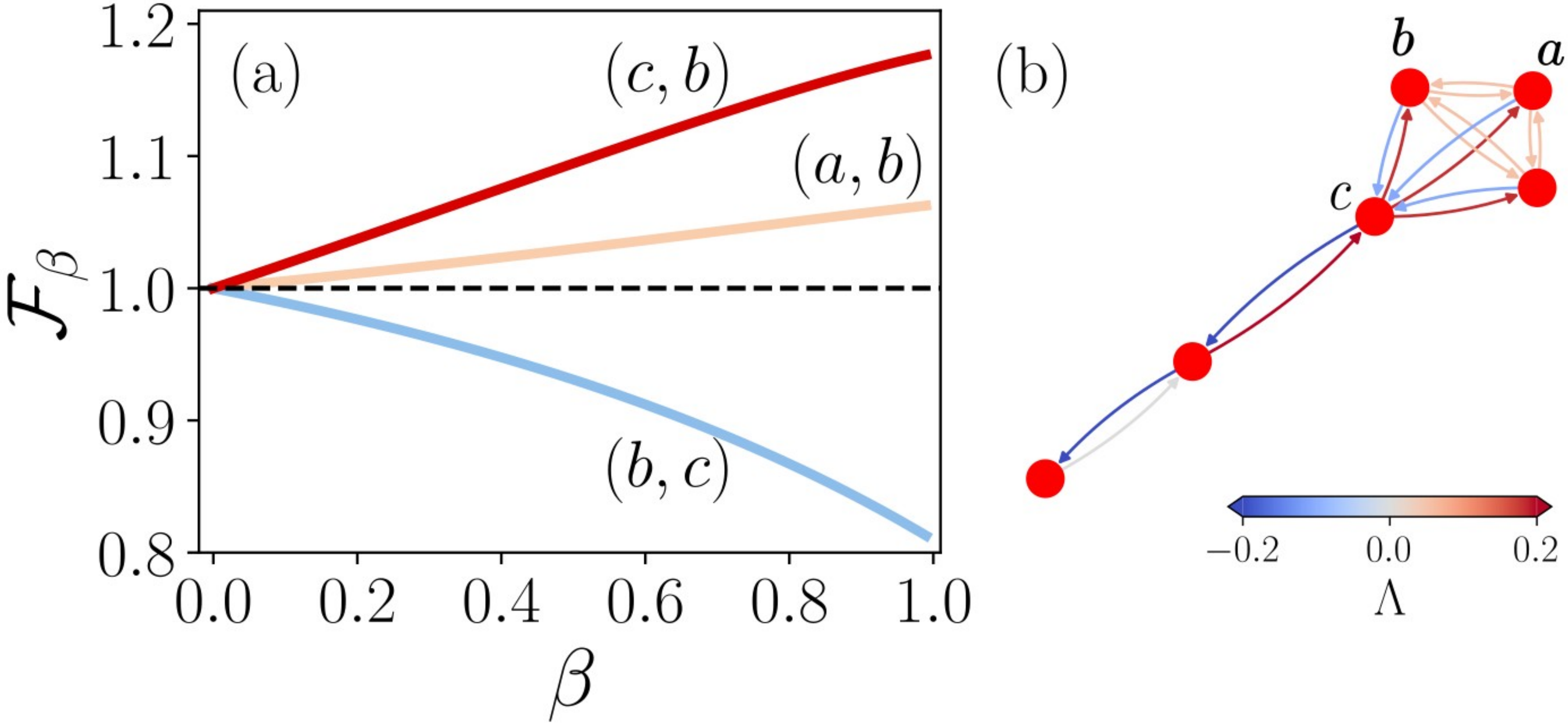
$$\mathbf{W}(\mathbf{\Omega}^\star) = \begin{array}{c c} & \begin{array}{cccccc} & & b & & & \end{array} \\ \begin{array}{c} \\ a \\ \\ \\ \\ \\ \end{array} & \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ \frac{1}{3-\beta} & 0 & \frac{1-\beta}{3-\beta} & \frac{1}{3-\beta} & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Antifragility Measure

Functionality

$$\boxed{\mathcal{F}_\beta \equiv \frac{\tau(0)}{\tau(\beta)}}$$

τ Global time: the average time a random walker needs to reach any node in the network



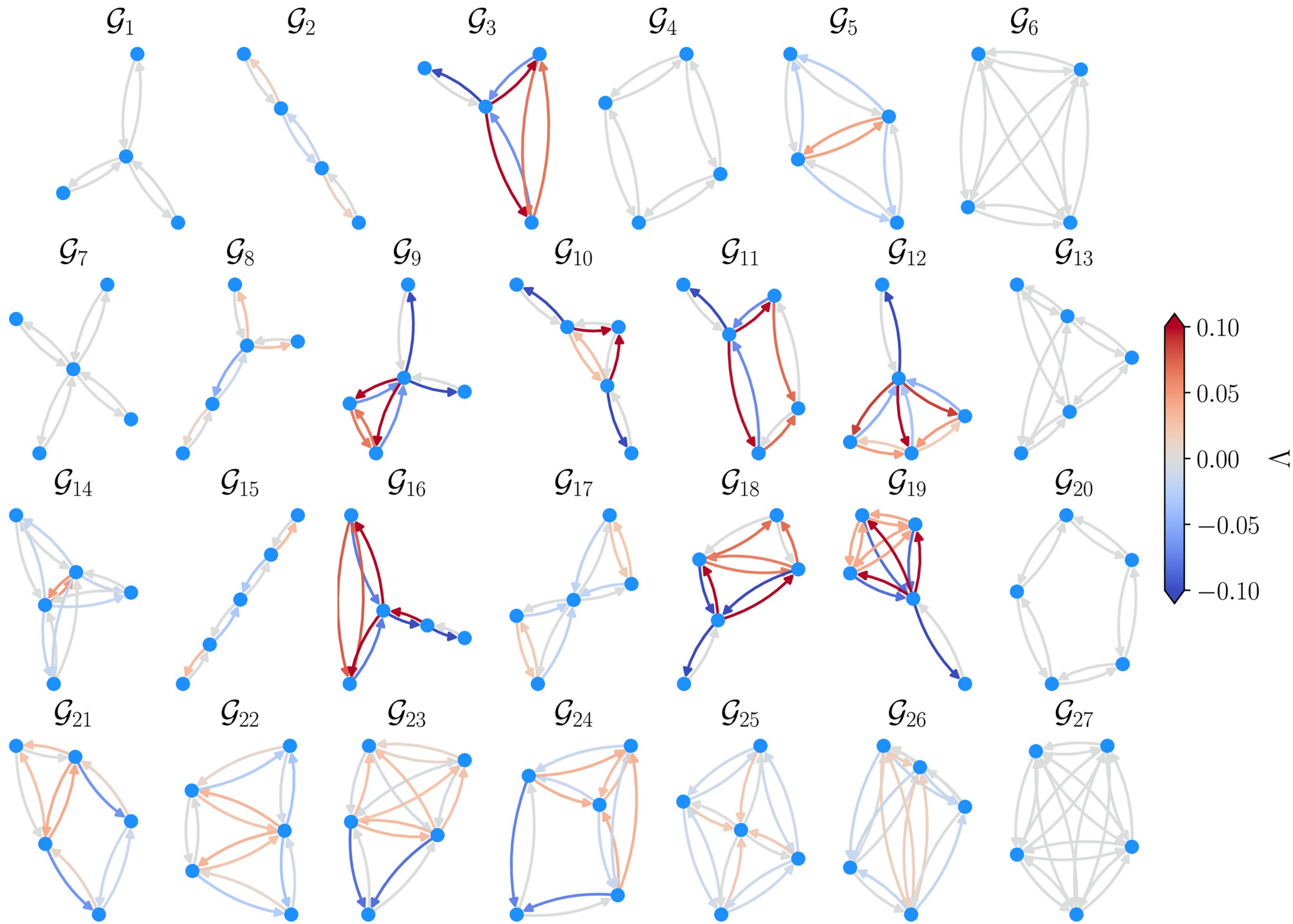
$$\Lambda \equiv \left. \frac{d\mathcal{F}_\beta}{d\beta} \right|_{\beta\to 0}$$

$\Lambda > 0$: Antifragility

$\Lambda < 0$: Fragility

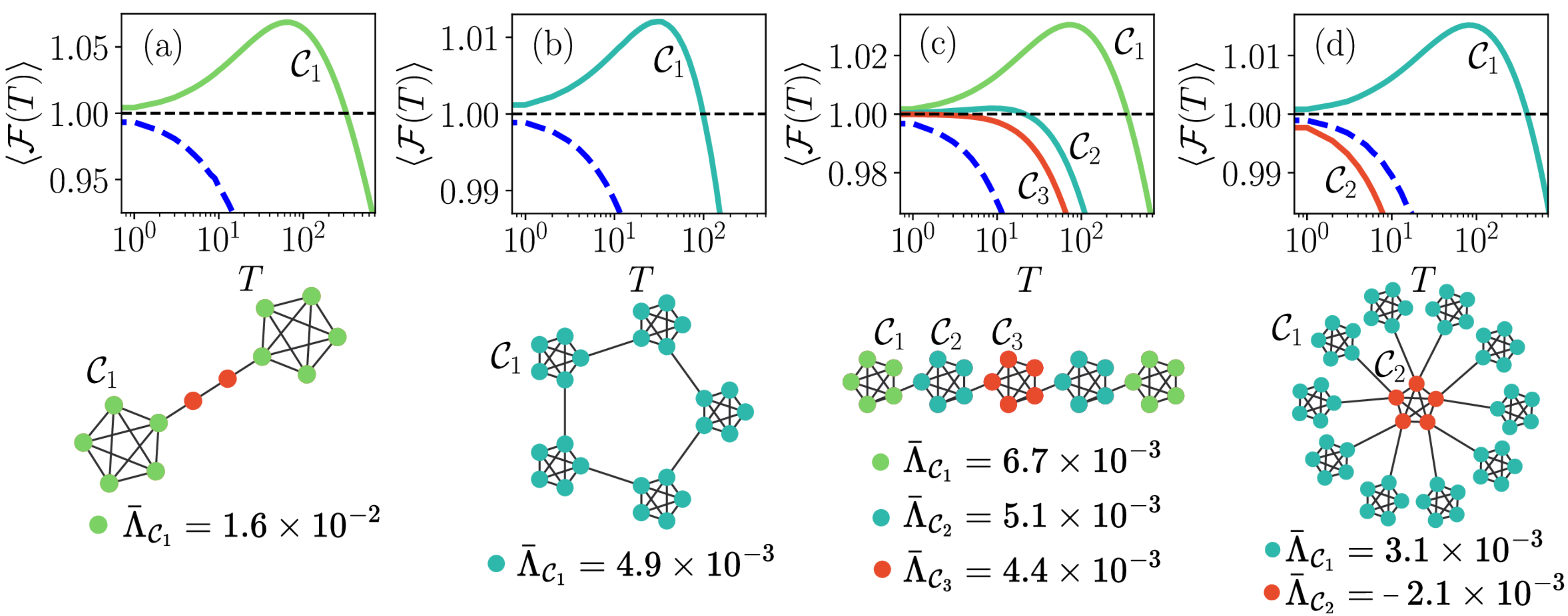
$\Lambda = 0$: Robustness

Antifragility in graphs with N=4 and N=5 nodes



Antifragility can appear in heterogeneous topologies with densely connected parts!

Antifragility at the level of communities



$$\bar{\Lambda}_{\mathcal{C}_i} \equiv \frac{1}{|\mathcal{E}_i|} \sum_{(a,b)\in\mathcal{E}_i} \Lambda_{(a,b)}$$

number of edges in comunity i

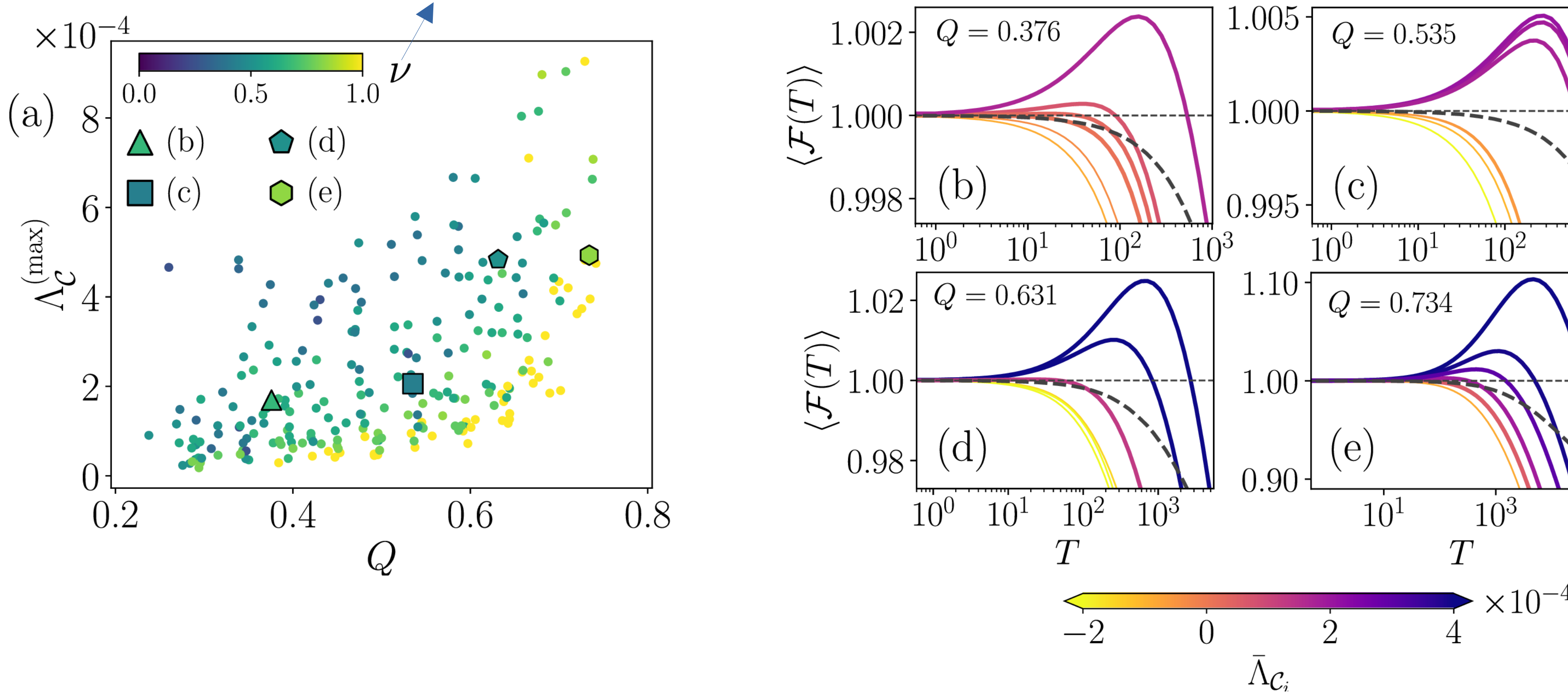
Antigragility measure of (a,b)

A community is a group of nodes strongly connected

The communities are affected with random damage distributed in their edges

Antifragility of networks with heterogeneous communities

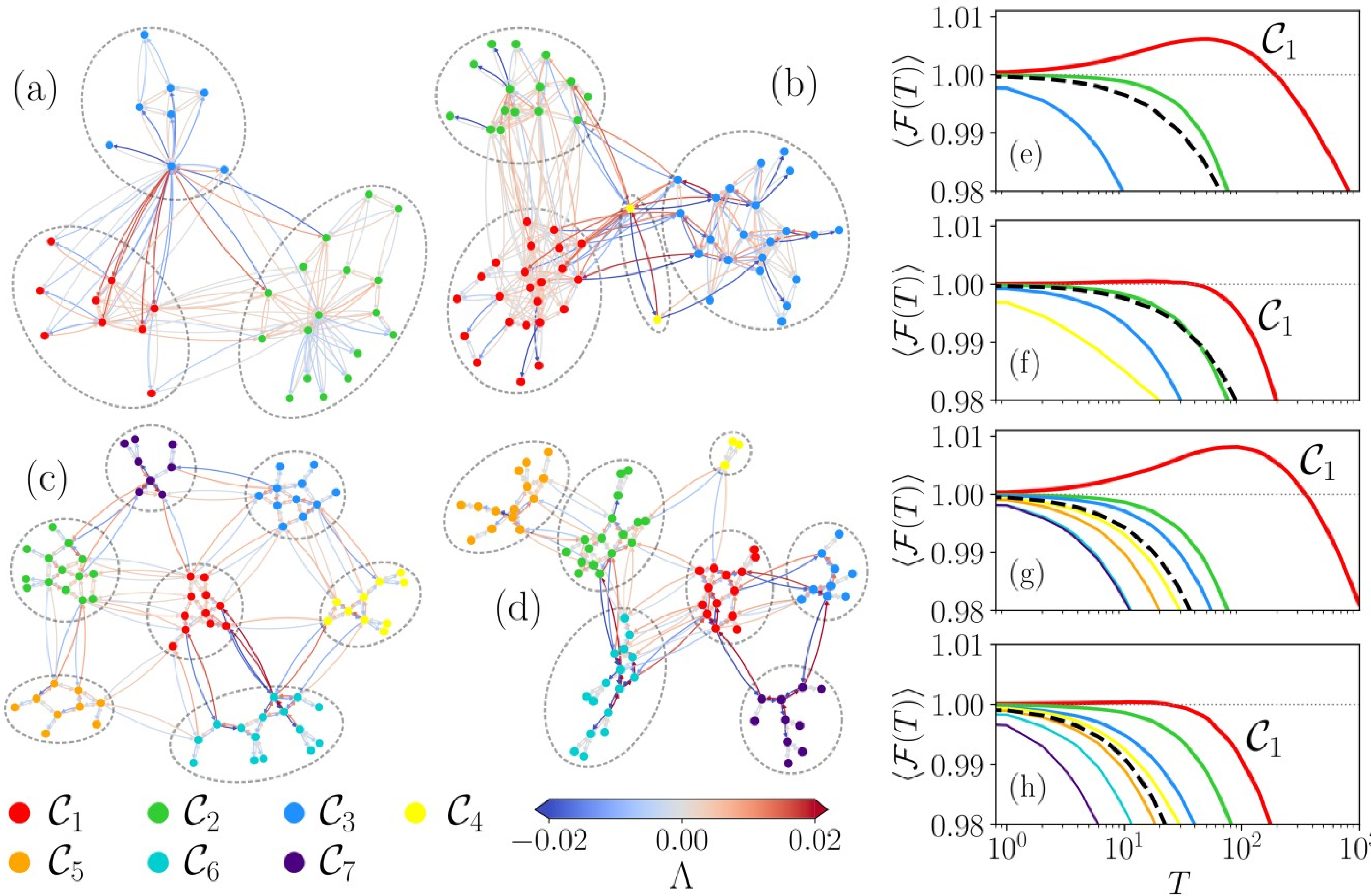
Fraction of antifragile communities



Q: how defined the communities are in the network

The networks with more defined communities have the most antifragile responses

Antifragility in real networks



(a) Karate club, (b) Bottlenose Dolphins, (c) Metro network in Paris, (d) Metro network in London

Final remarks

Our model shows the emergence of antifragility in stochastic transport

Antifragility emerges in networks with communities

Λ and $\bar{\Lambda}_{\mathcal{C}_i}$ identify the antifragile parts of a network to the level of edges and communities respectively.

We present a framework to study the emergence of antifragility in other dynamical systems

PHYSICAL REVIEW E **110**, 044309 (2024)

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(Received 13 May 2024; accepted 24 September 2024; published 21 October 2024)

A system is called antifragile when damage acts as a constructive element improving the performance of a global function. In this paper, we analyze the emergence of antifragility in the movement of random walkers on networks with modular structures or communities. The random walker hops considering the capacity of transport of each link, whereas the links are susceptible to random damage that accumulates over time. We show that in networks with communities and high modularity, the localization of damage in specific groups of nodes leads to a global antifragile response of the system improving the capacity of stochastic transport to more easily reach the nodes of a network. Our findings give evidence of the mechanisms behind antifragile response in complex systems and pave the way for their quantitative exploration in different fields.