

From motifs to Lévy flights: modeling urban mobility in Bogotá's public transport system

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5th Workshop on statistical Physics
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Content

Motivation

Methods

Results

motivation

- ▶ Use statistical physics to help understand the problem of mobility.
- ▶ How do people move around in Bogotá?

Context of the city



- ▶ 4 modes (BRT system, buses, mixed buses and aerial tramway).
- ▶ Transference between modes (125 minutes).
- ▶ More than 4 million passengers per day.

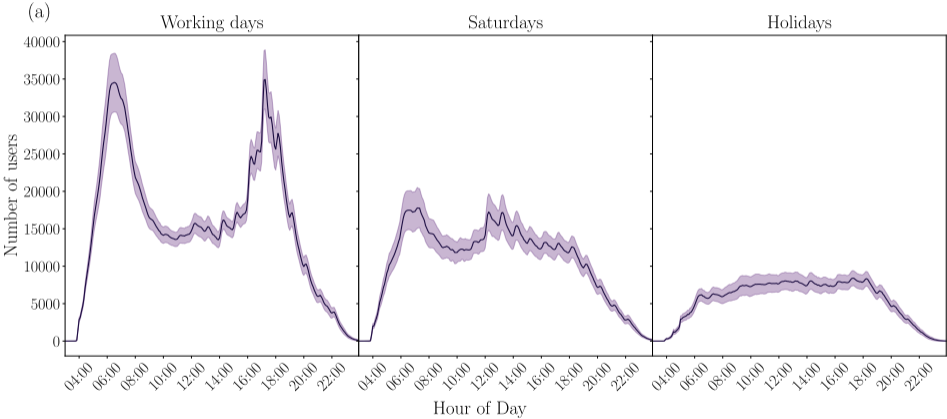
Data ¹

- ▶ 2 years of data, divided in 4 semestres (July 2023-June 2025).
- ▶ More than 2300 millions of records.

Daily use	Semester I		Semester II		Semester III		Semester IV	
	Quantity	%	Quantity	%	Quantity	%	Quantity	%
1 Use	79,318,543	13.53	72,107,310	13.30	77,564,163	12.71	72,259,030	12.76
2 Uses	216,600,274	36.94	201,854,542	37.23	224,987,774	36.86	210,031,776	37.08
3 Uses	121,315,356	20.69	110,462,991	20.37	124,773,723	20.44	115,466,970	20.39
4 Uses	103,600,240	17.67	97,647,848	18.01	111,845,280	18.32	104,336,836	18.42
5 Uses	36,830,805	6.28	34,074,785	6.28	39,891,830	6.54	37,082,375	6.55
6 Uses	17,636,352	3.01	16,169,340	2.98	19,337,514	3.17	17,188,422	3.03
7 Uses or more	11,124,068	1.90	9,871,041	1.82	11,979,044	1.96	10,021,929	1.77
Total	586,425,638		542,187,857		610,379,328		566,387,338	
Transfers	109,056,446	18.60%	103,323,821	19.06%	117,075,238	19.18%	113,875,148	20.11%

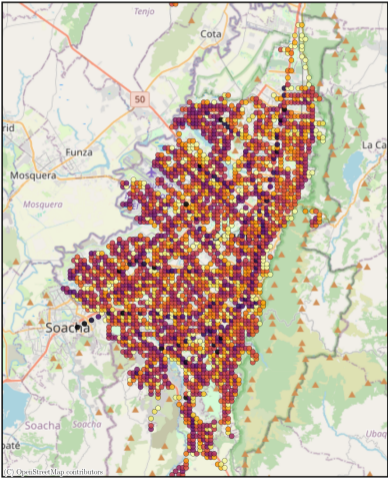
¹Portal de datos abiertos Transmilenio, 2025

Analysis of mobility time patterns of public transport users in Bogotá

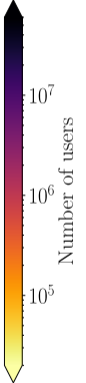
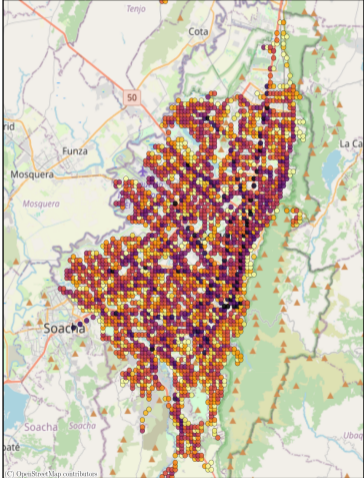


Analysis of mobility space patterns of public transport users in Bogotá

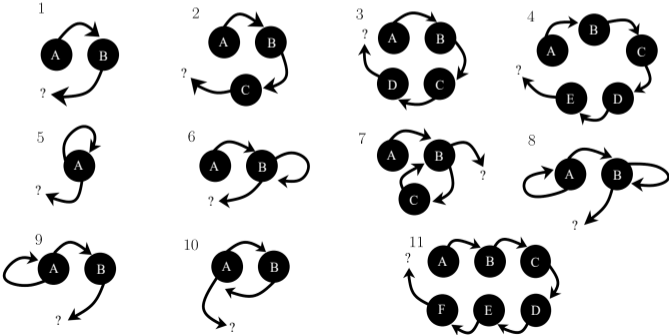
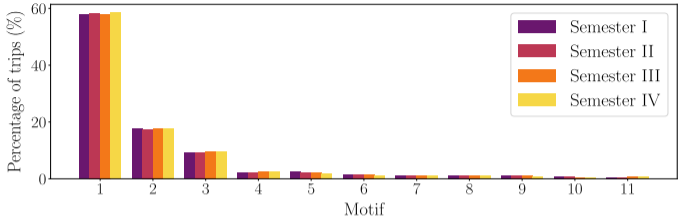
(b) Morning



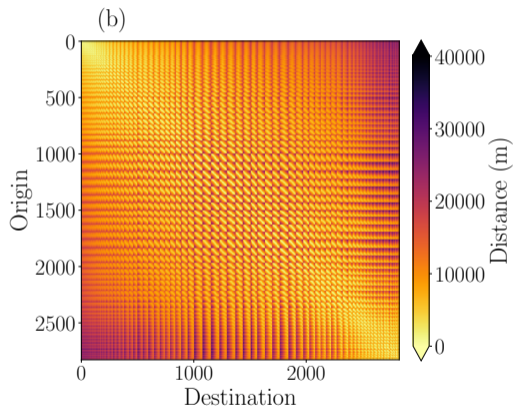
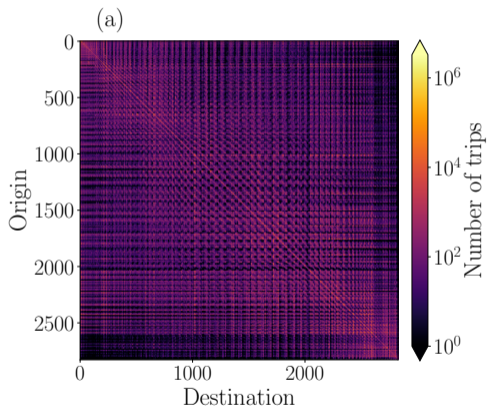
(c) Afternoon



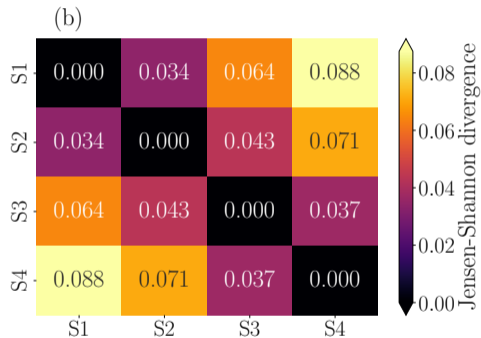
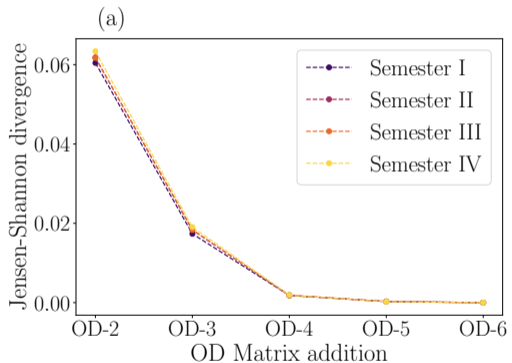
Statistical representation of the 11 most frequent motifs



Matrix representation of public transport mobility in Bogotá

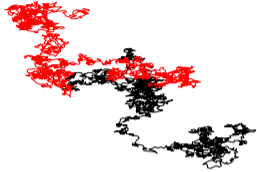


Comparative study of OD matrices using the Jensen–Shannon criterion

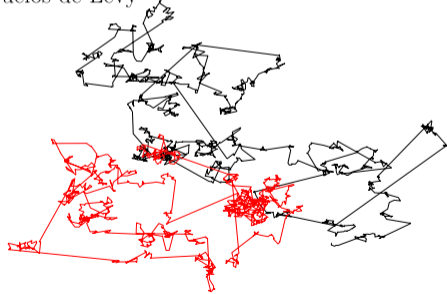


Lévy flights²

Movimiento Browniano



Vuelos de Lévy



- ▶ Generalisation of the normal diffusion associated with Brownian motion
- ▶ Spatial dynamics of the system can be approximated by a Lévy flight model using geographic distance. d_{ij}

$$\omega_{i \rightarrow j}^{(OD)} \propto d_{ij}^{-\gamma} \quad \text{for} \quad d_{ij} > 0$$

²R. Metzler and J. Klafter, The restaurant at the end of the random walk: recent developments in the description of anomalous transport by fractional dynamics, 2004

Relationship between the transition probability $\omega_{i \rightarrow j}$ and geographic distance d_{ij}

Defined in terms of OD matrix entries

T_{ij}

$$\blacktriangleright \omega_{i \rightarrow j}^{(\text{OD})} = \frac{T_{ij}}{k_i^{(\text{out})}}$$

where the out-degree $k_i^{(\text{out})} = \sum_{l=1}^N T_{il}$ with N denoting the number of zones, ensures the normalization.

Defined a transition probability for a hop i to j (model)

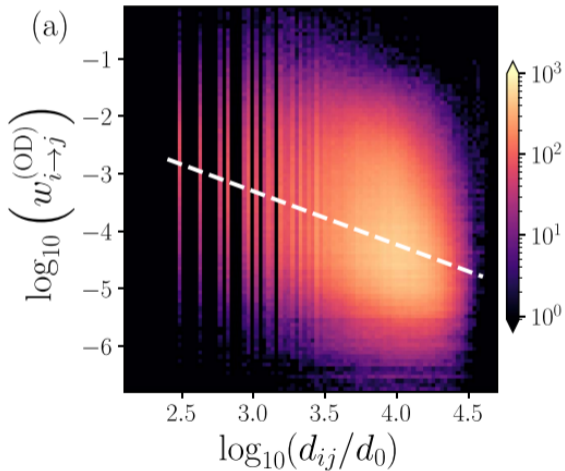
$$\blacktriangleright \omega_{i \rightarrow j}^{(\gamma)} = \frac{\Omega_{ij}^{(\gamma)}}{\sum_{l=1}^N \Omega_{ij}^{(\gamma)}}$$

With $\Omega_{ij}^{(\gamma)} = (d_0/d_{ij})^\gamma$

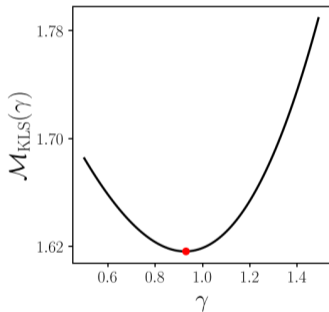
Relationship between the transition probability $\omega_{i \rightarrow j}^{(\text{OD})}$ and geographic distance d_{ij}

$$\log_{10} \left(\omega_{i \rightarrow j}^{(\text{OD})} \right) = C - \gamma \log_{10} (d_{ij}/d_0)$$

With $\gamma = 0.92$



$M_{KLS}(\gamma)$ to identify the exponent γ^*

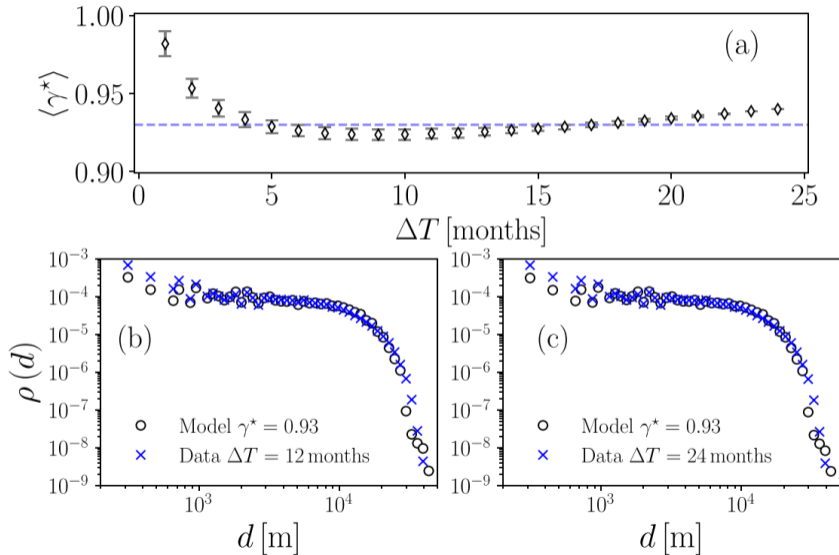


We define a Symmetrized measure $M_{KLS}(\gamma)$ based on the Kullback-Leibler divergence and transition probability models

$$M_{KLS}(\gamma) = \frac{1}{2} [D_{KL}(P(\gamma) \| Q) + [D_{KL}(Q \| P(\gamma))]]$$

provides a quantitative measure of the discrepancy between empirical and modeled transition probabilities to identify the exponent γ^*

Statistical analysis of displacements between zones in the system



Conclusions

- ▶ Motif-based analysis reveals complex mobility patterns beyond simple home–work trips, improving the reconstruction and resolution of OD matrices from entry-only data.
- ▶ User displacements follow Lévy flight dynamics, with stable parameters across time, indicating robust and scale-invariant mobility behavior in Bogotá's public transport system.
- ▶ The methodology provides a reliable framework for urban mobility analysis, offering practical value for transport planning while highlighting limitations and opportunities for improving OD estimation methods.

More Information



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PAPER

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