Universidad de los Andes
Departamento de Física
High Energy Physics Laboratory
HEPL

Inline X-ray Phase Contrast Imaging

Aplicaciones Interdisciplinarias de Detectores de Particulas

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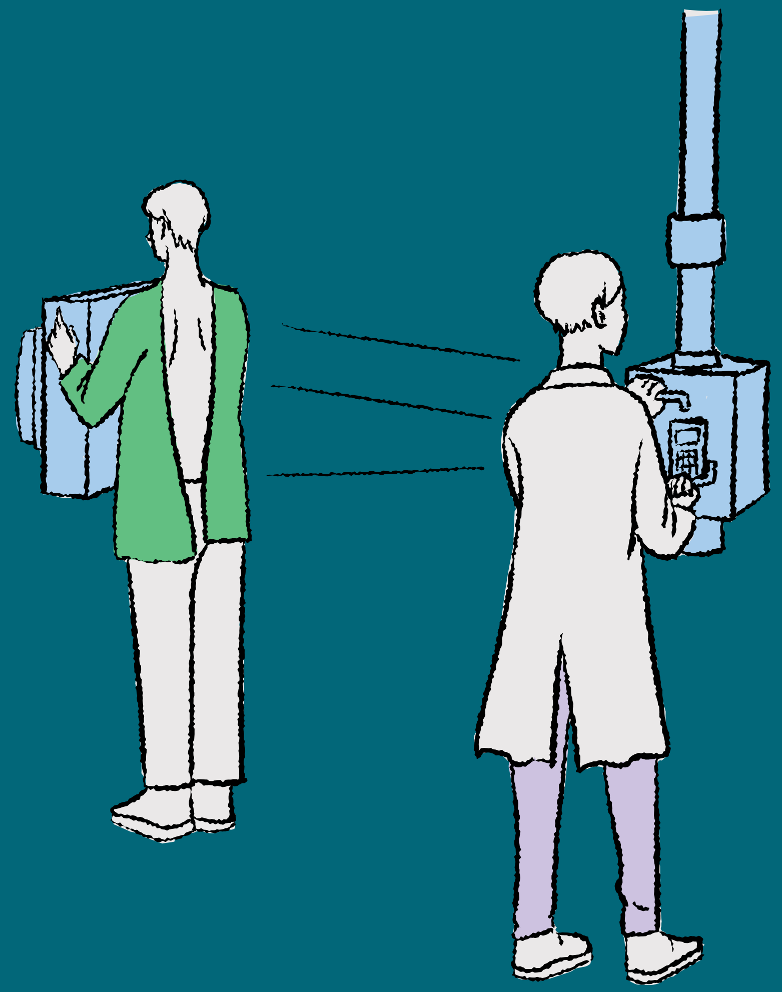
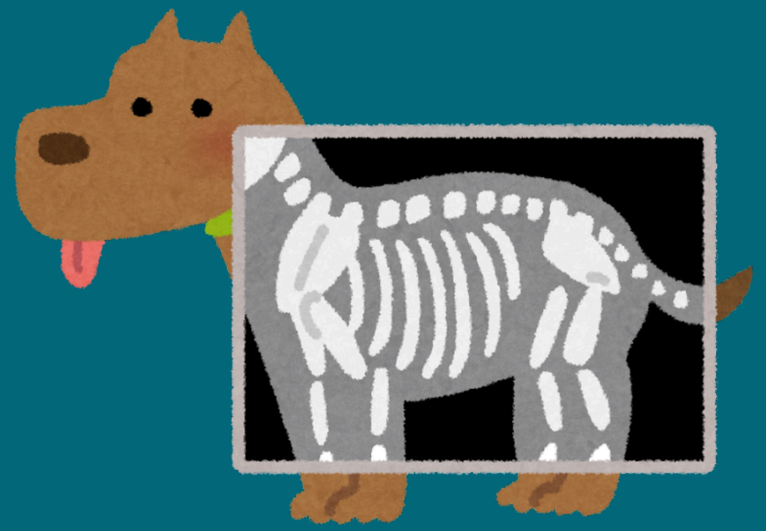
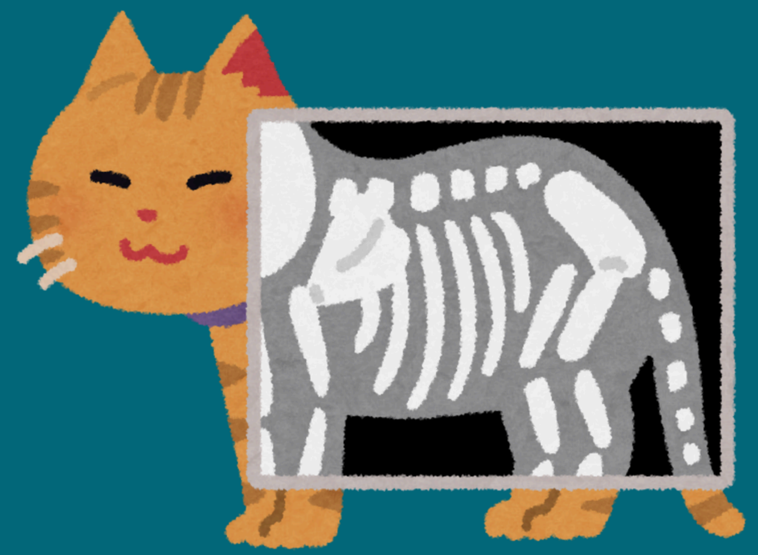
X-ray Imaging

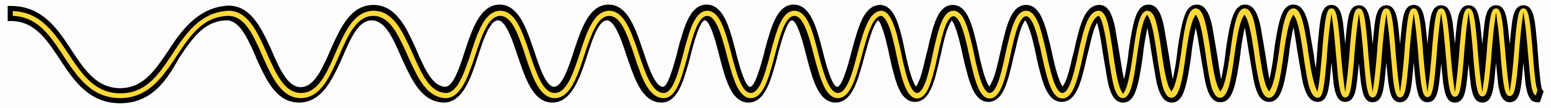






X-ray Imaging

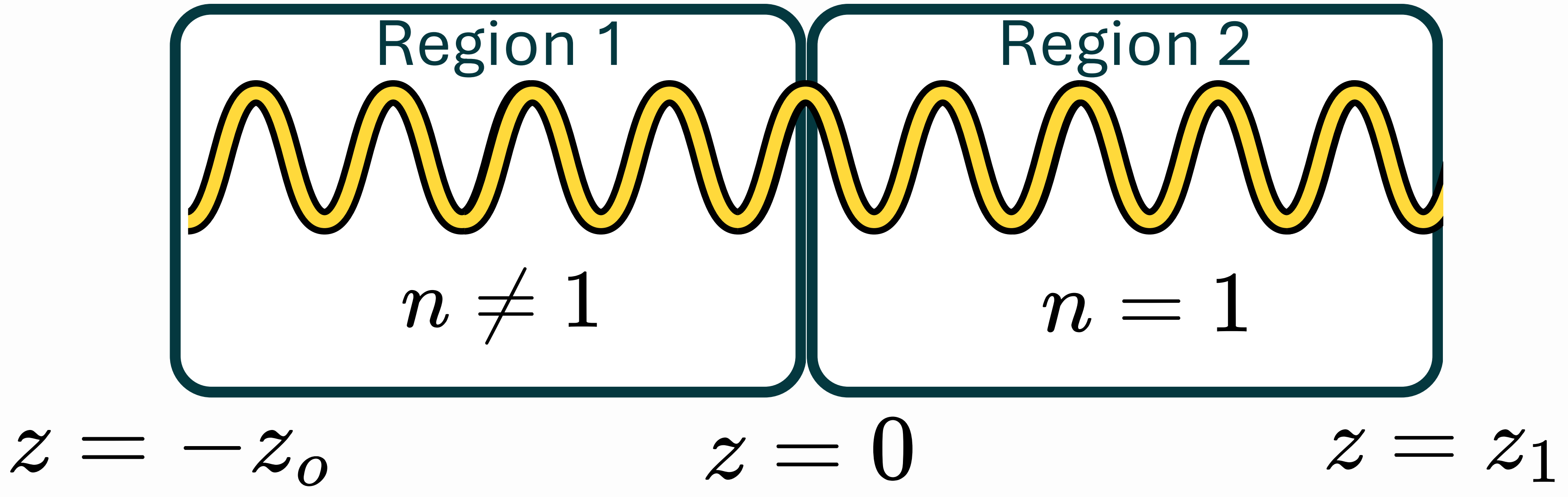
Electromagnetic spectrum



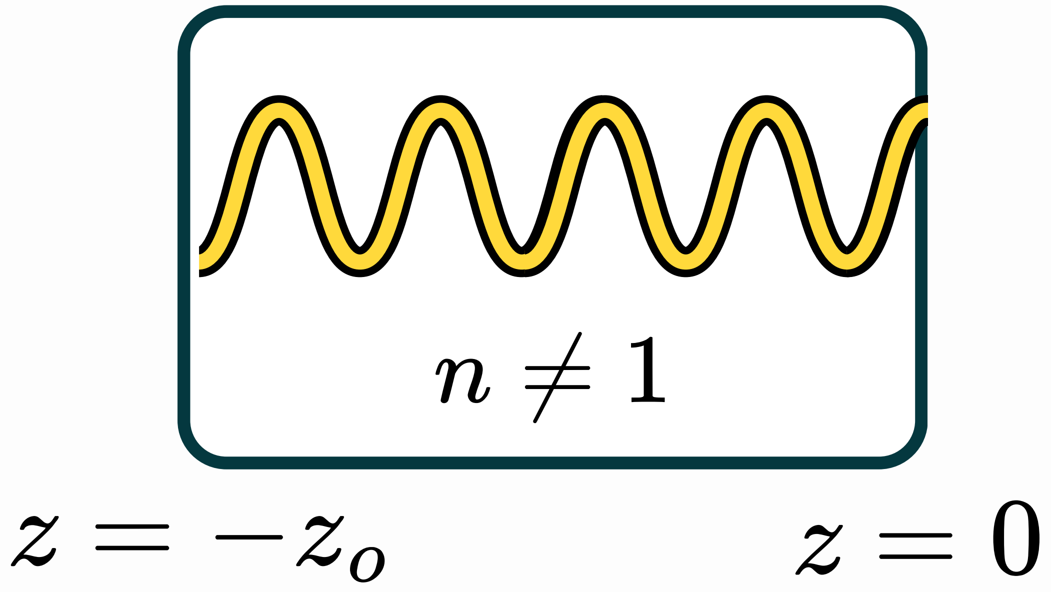
X-ray propagation through matter

Maxwell's wave equations

$$\left[\frac{n^2(x,y,z,\omega)}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right]\varphi(x,y,z,t)=0 \quad \textbf{(S1)}$$



X-ray Imaging



Region 1

Neccesary Assumptions:

1. Paraxial approximation

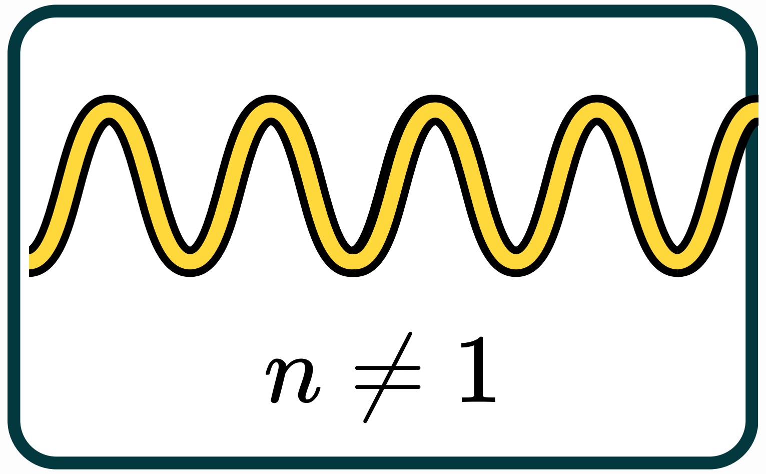
$$\psi(x, y, z, \omega) = \vartheta(x, y, z, \omega) e^{\frac{i\omega z}{c}}$$ ➔ Variation of ϑ is assumed to be linear

2. Projection approximation

X-rays dispersion in the transverse plane within the material ignored

$$\vartheta(x, y, 0, \omega) = \vartheta(x, y, -z_o, \omega) e^{\frac{i\omega}{2c} \int_{-z_o}^{0} \left[n^2(x,y,z,\omega) - 1\right] dz} \quad \textbf{(S2)}$$

X-ray Imaging

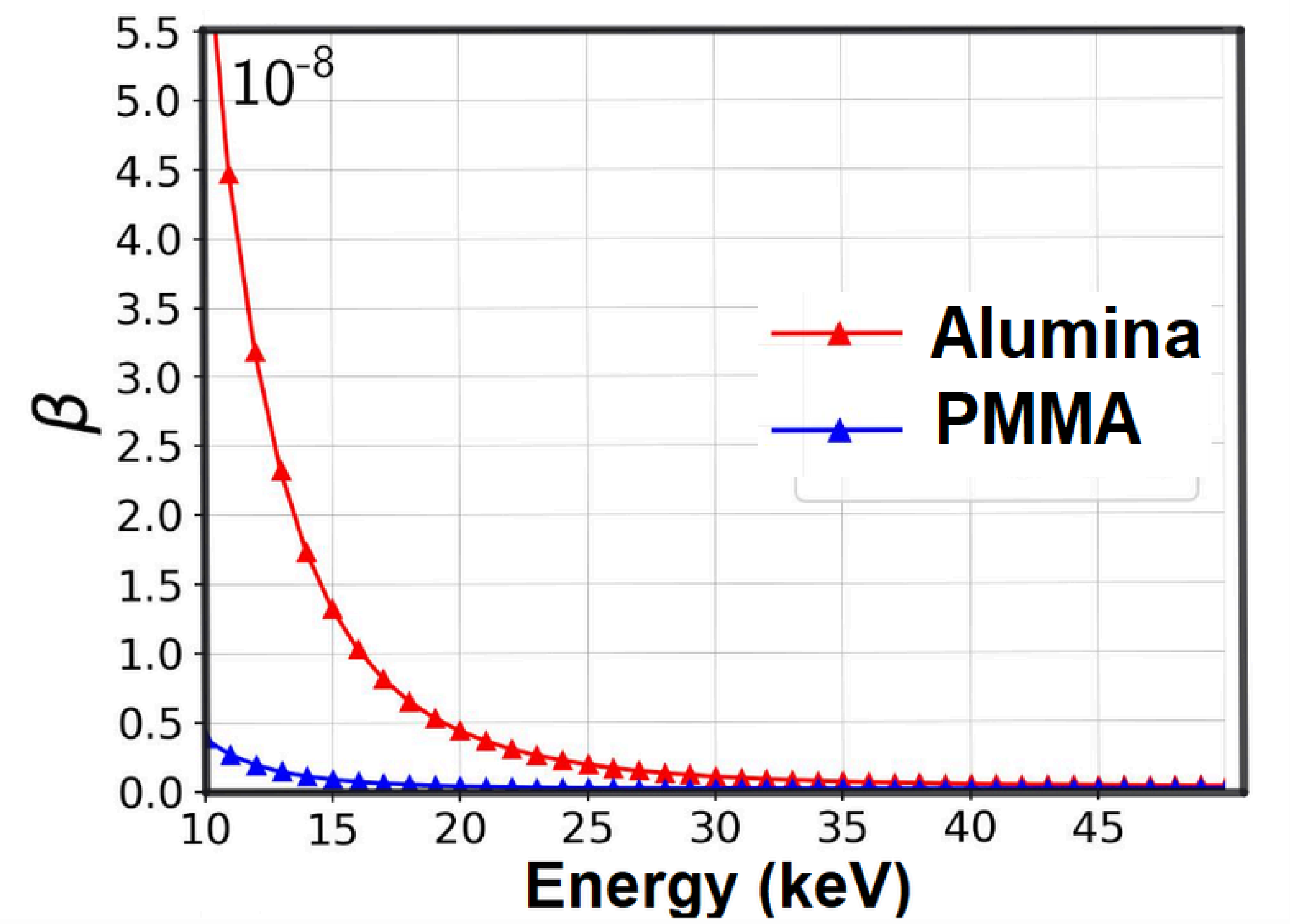


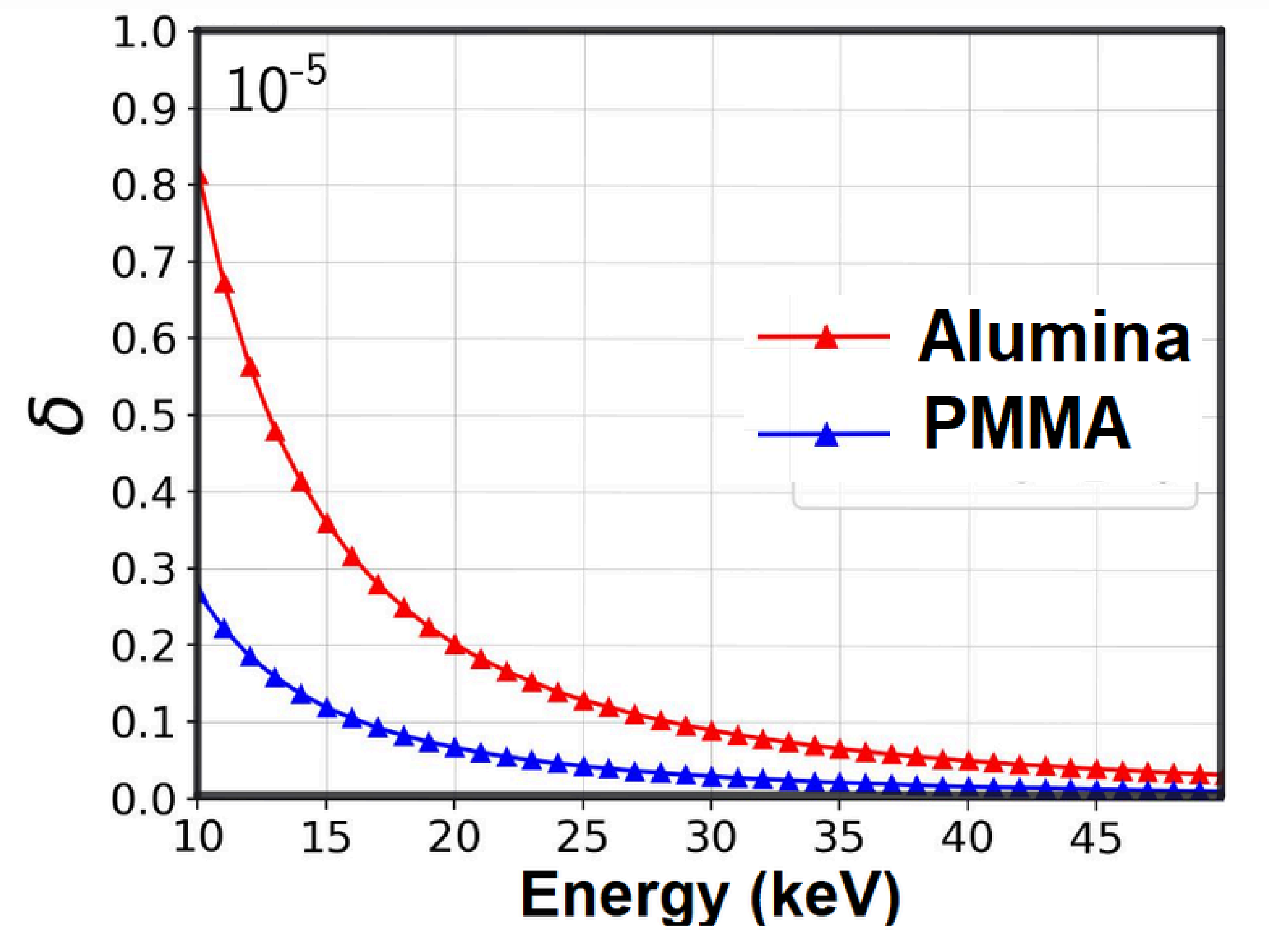
$z = -z_o$ $z = 0$

Region 1

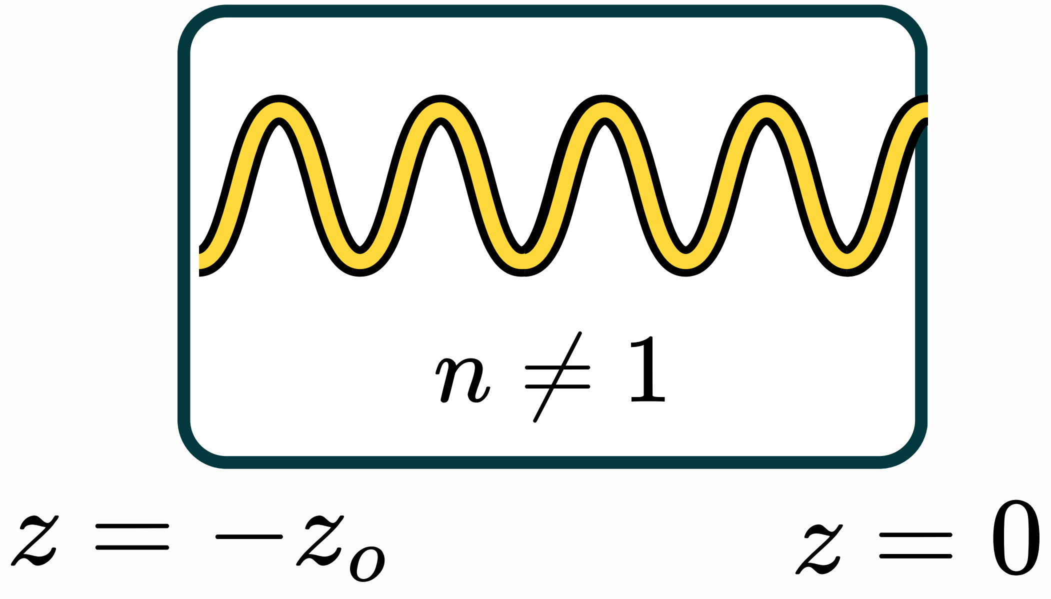
Refractive Index

$$n(x, y, z, \omega) = 1 - \delta(x, y, z, \omega) + i\beta(x, y, z, \omega) \quad \text{(S3)}$$





X-ray Imaging



Region 1

$$\vartheta(x,y,0,\omega) = \vartheta(x,y,-z_o,\omega)e^{-\frac{i\omega}{c}\int_{-z_o}^{0}[\delta(x,y,z,\omega)-i\beta(x,y,z,\omega)]dz} \quad \textbf{(S4)}$$

Spatial and time solution

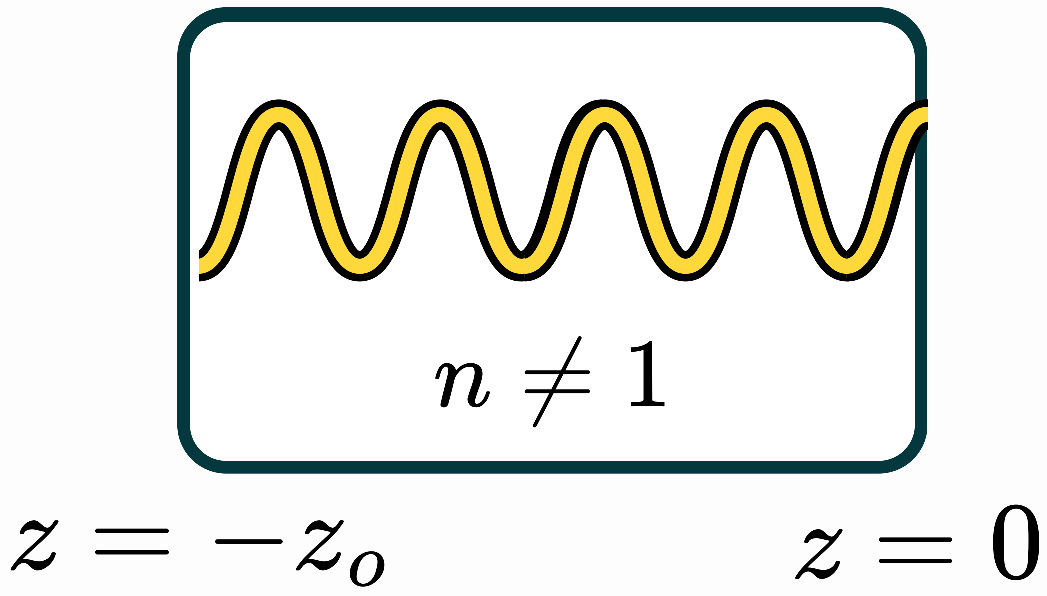
$$\varphi(x,y,z,t) = \int \psi(x,y,z,\omega)e^{-i\omega t}d\omega$$

$$\varphi(x,y,z,t) = \int \left[\vartheta(x,y,-z_o,\omega)e^{-\frac{\omega}{c}\int_{-z_o}^{0}\beta(x,y,z,\omega)dz}\right]e^{i\left[-\frac{\omega}{c}\int_{-z_o}^{0}\delta(x,y,z,\omega)dz+\frac{\omega z}{c}-\omega t\right]}d\omega$$

(S5)

X-ray Imaging

Region 1



Intensity

$$I(x, y, 0, \omega) = I(x, y, -z_o, \omega) \times e^{-\frac{2\omega}{c}\int_{-z_o}^{0} \beta(x,y,z,\omega)dz} \quad \textbf{(S6)}$$

Beer-Lambert equation

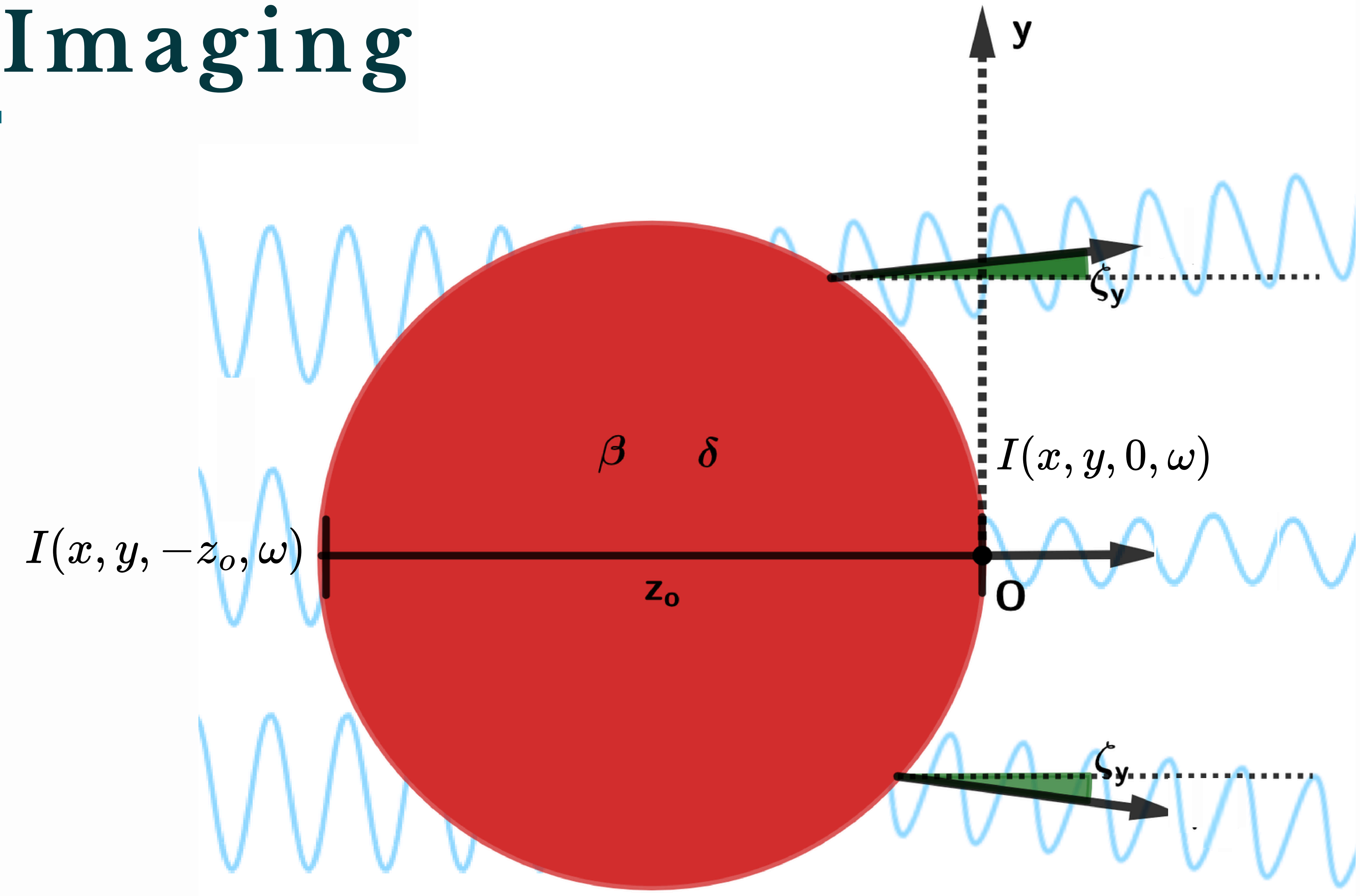
Phase shift

$$\Delta\phi(x, y, 0, \omega) = -\frac{\omega}{c}\int_{-z_o}^{0} \delta(x, y, z, \omega)dz \quad \textbf{(S7)}$$

Refraction

$$\zeta \approx \frac{c}{\omega}\left|\vec{\nabla}_T \Delta\phi\right| \quad \textbf{(S8)}$$

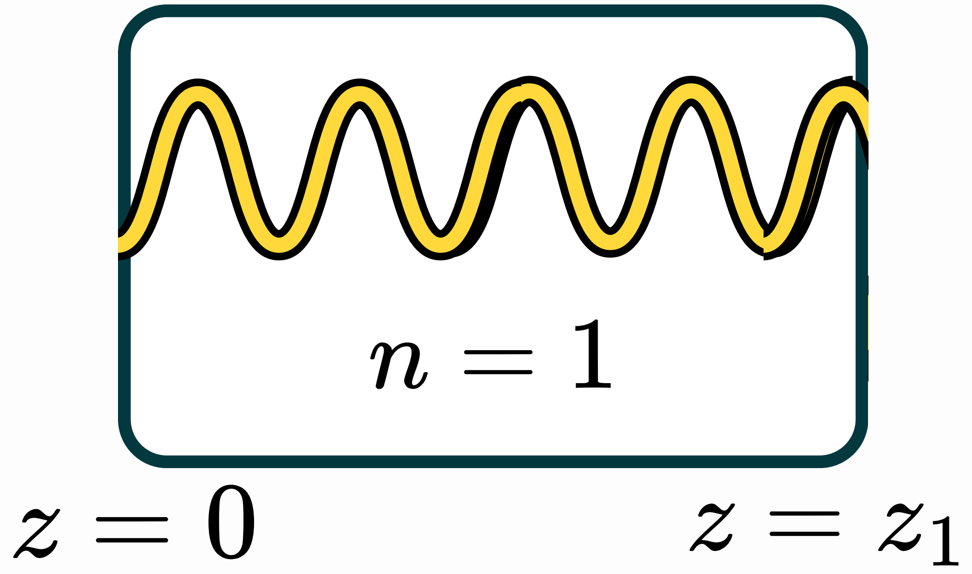
X-ray Imaging

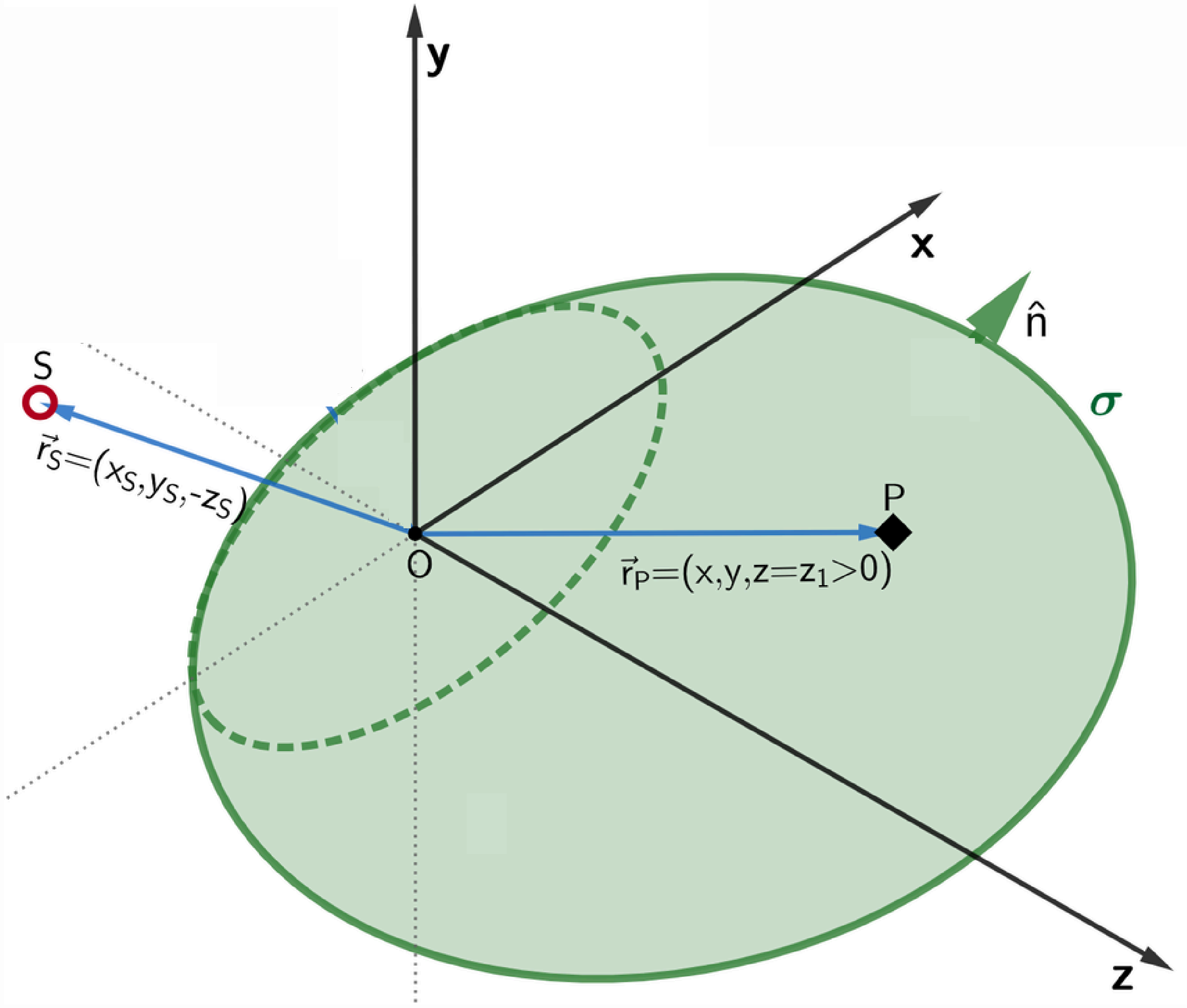


X-ray Imaging

Region 2

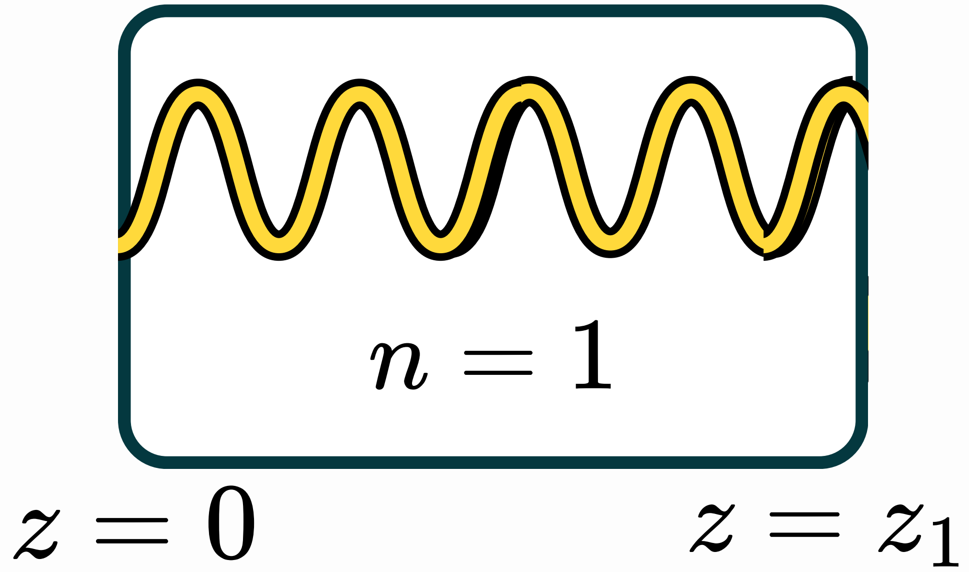
Green theorem





$$\psi(x, y, z > 0) = \frac{1}{4\pi} \oint_\sigma \left[G^D \frac{\partial \psi}{\partial n} - \psi \frac{\partial G^D}{\partial n} \right] dS \quad \text{(S9)}$$

X-ray Imaging



Region 2

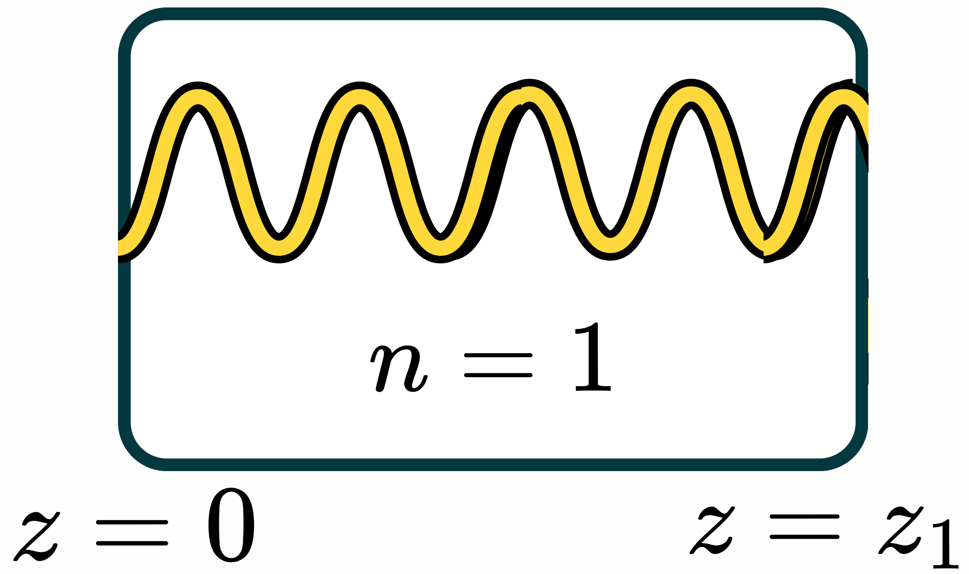
Transmission function

$$\Gamma(x', y', \omega) = e^{-\frac{i\omega}{c}\int_{-z_o}^{0}[\delta(x',y',z',\omega) - i\beta(x',y',z',\omega)]dz'} \quad \textbf{(S10)}$$

Solution

$$\psi(x, y, z_1, \omega) = -\frac{i\psi_o \omega e^{\frac{i\omega}{c}(z_S + z_1)}}{2\pi z_1 c}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Gamma(x', y', \omega) \times e^{\frac{i\omega}{c}\left(\frac{(x_S - x')^2 + (y_S - y')^2}{2z_S} + \frac{(x - x')^2 + (y - y')^2}{2z_1}\right)} dx'dy' \quad \textbf{(S11)}$$

X-ray Imaging



Region 2

Intensity

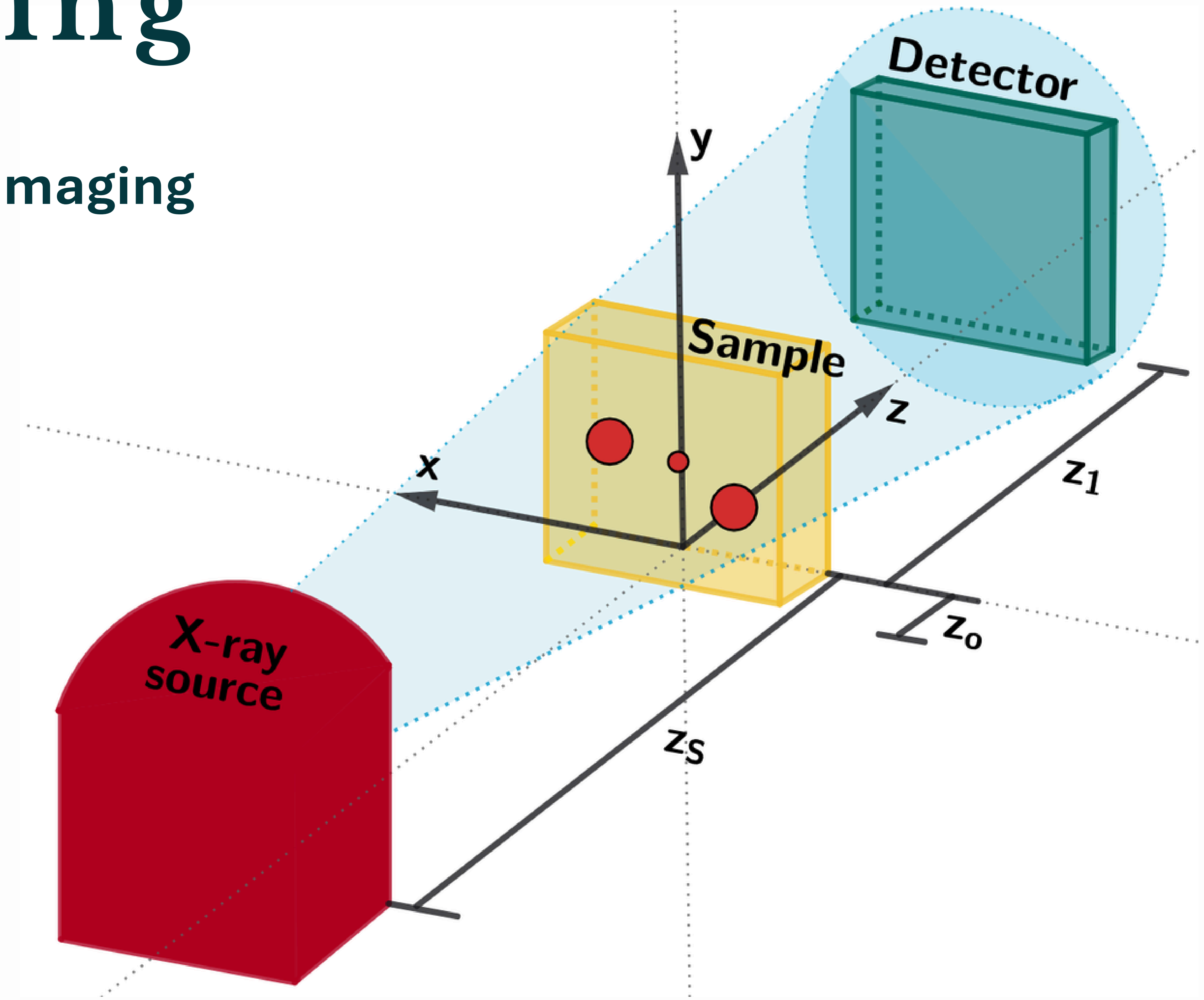
$$I(x, y, z_1, \omega) = \psi^*(x, y, z_1, \omega)\psi(x, y, z_1, \omega)$$

Transport-Intensity equation

$$\frac{I(x, y, z_1, \omega)}{I\left(\frac{x}{M}, \frac{y}{M}, -z_o, \omega\right)} = \frac{1}{M^2} e^{-\frac{2\omega}{c}\int_{-z_o}^{0} \beta\left(\frac{x}{M}, \frac{y}{M}, z', \omega\right) dz'} + \frac{z_1}{M^2} \vec{\nabla}_T \cdot \left[e^{-\frac{2\omega}{c}\int_{-z_o}^{0} \beta\left(\frac{x}{M}, \frac{y}{M}, z', \omega\right) dz'} \vec{\nabla}_T \left[\int_{-z_o}^{0} \delta\left(\frac{x}{M}, \frac{y}{M}, z', \omega\right) dz' \right] \right] \quad \textbf{(S12)}$$

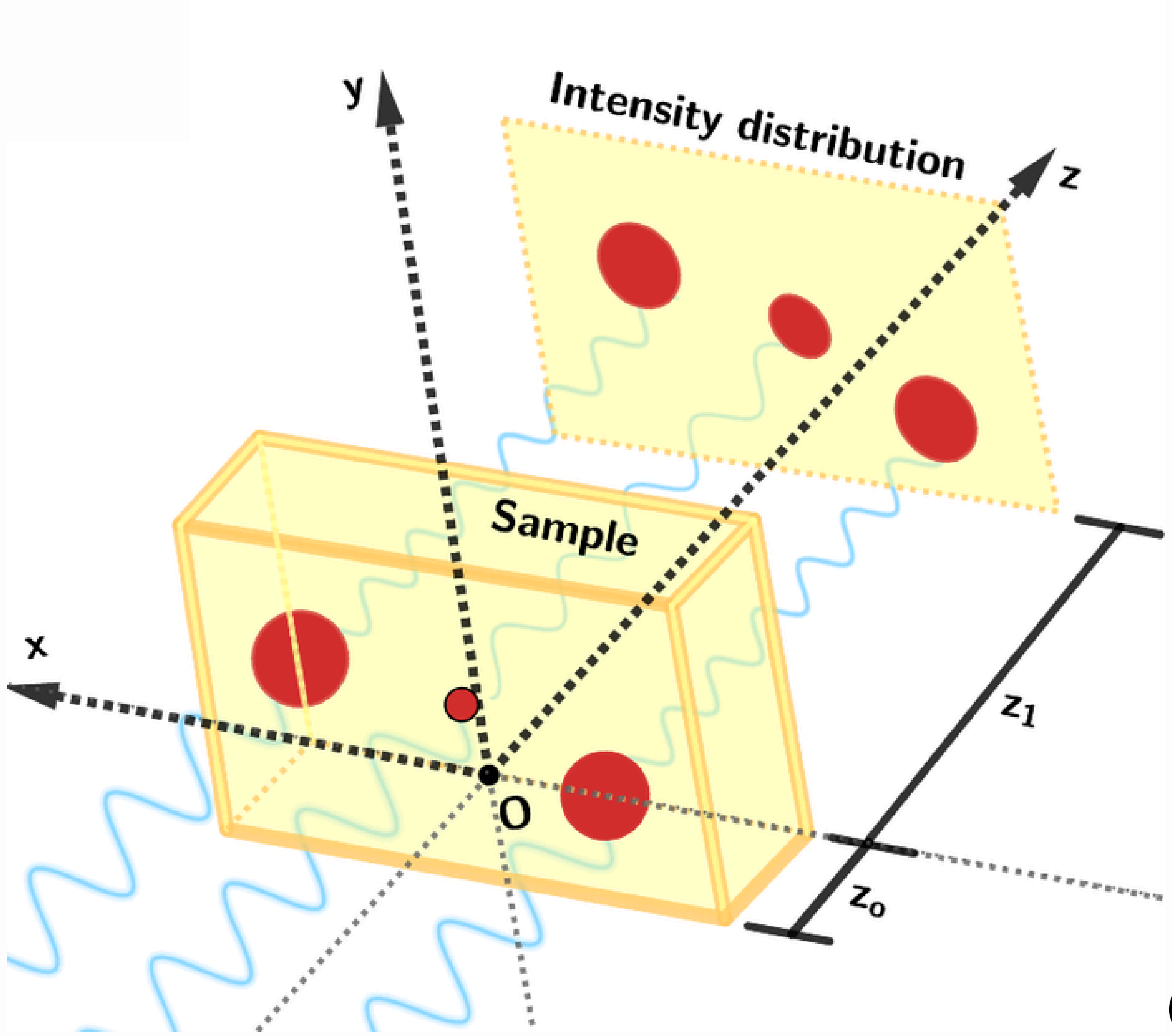
X-ray Imaging

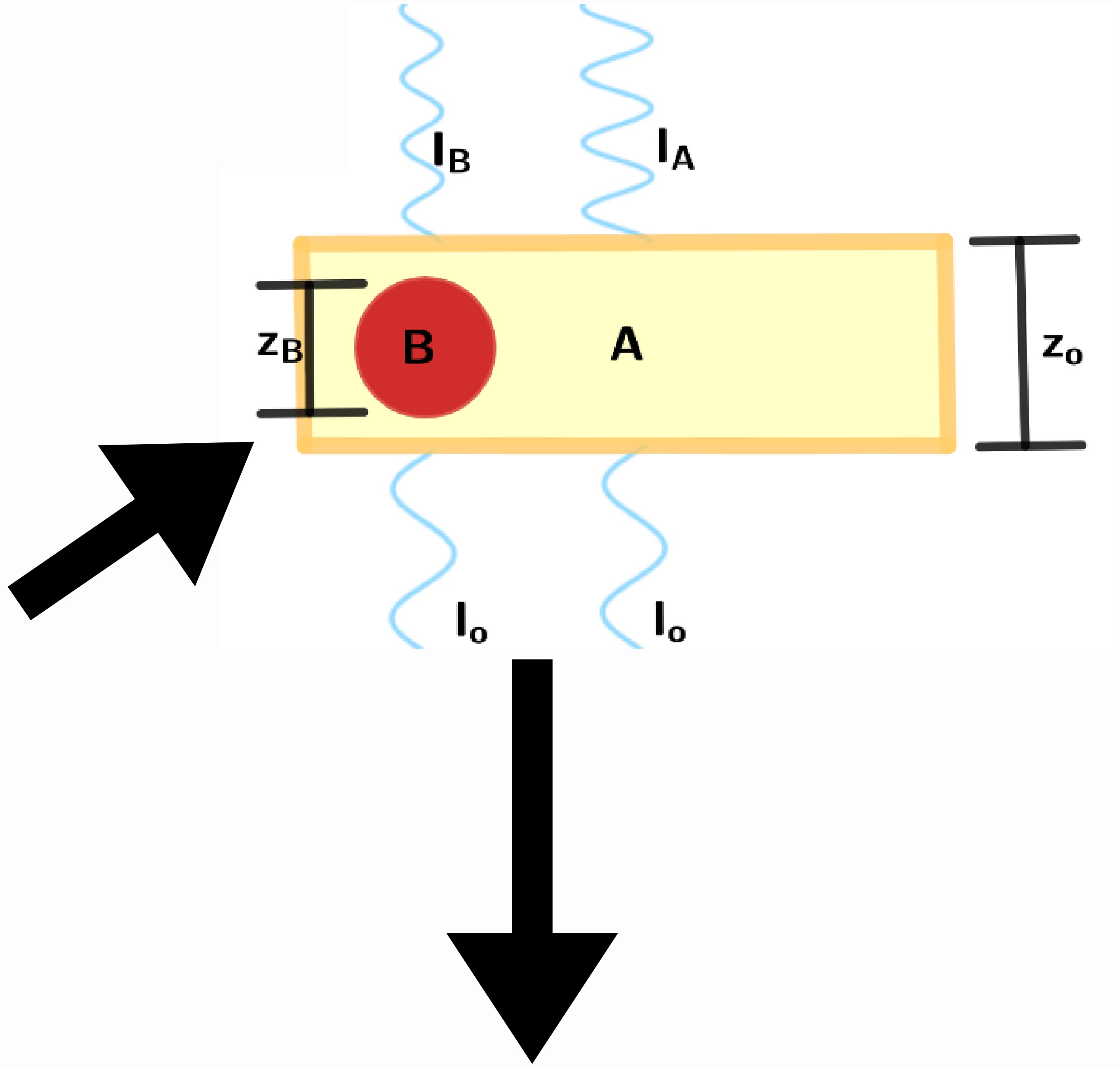
X-ray Absorption Imaging



$$\frac{I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)} = \frac{1}{M^2} e^{-\frac{2\omega}{c}\int_{-z_o}^{0}\beta\left(\frac{x}{M},\frac{y}{M},z',\omega\right)dz'} \quad \text{(S13)}$$

X-ray Absorption Imaging

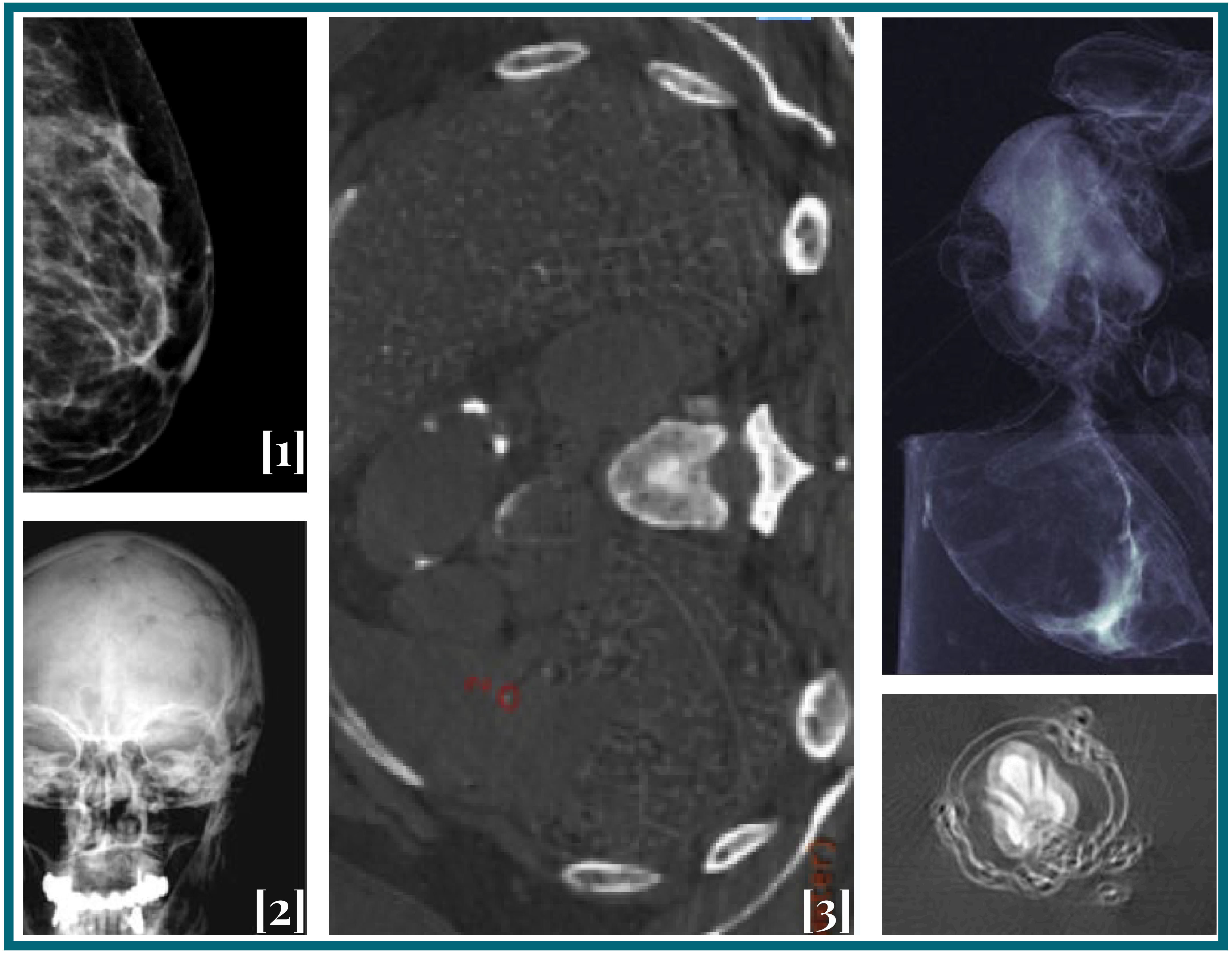




$$C(\omega) = 1 - e^{-\frac{2\omega}{c}(\beta_B(\omega) - \beta_A(\omega))z_B}$$

X-ray Absorption Imaging

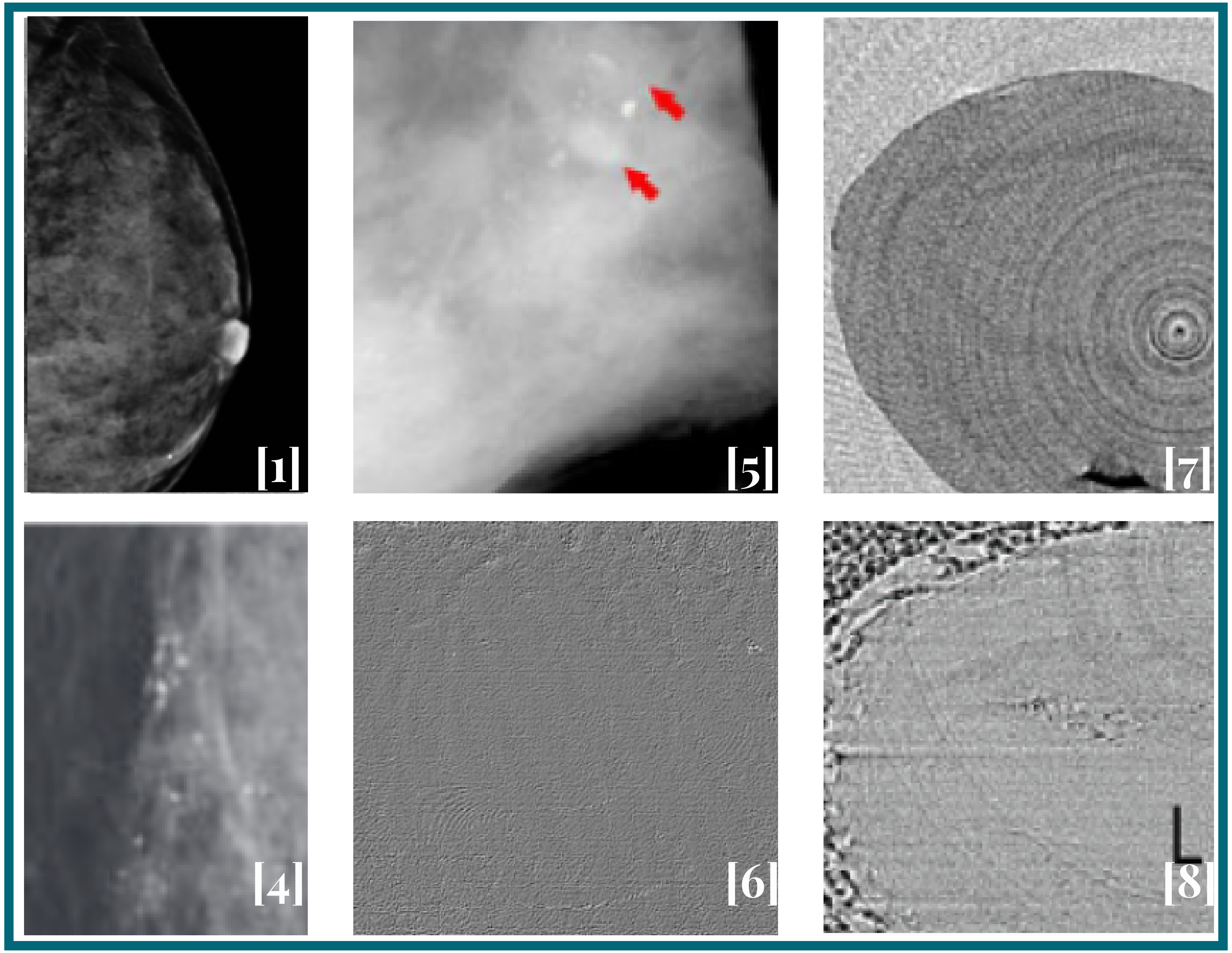
Applications



References

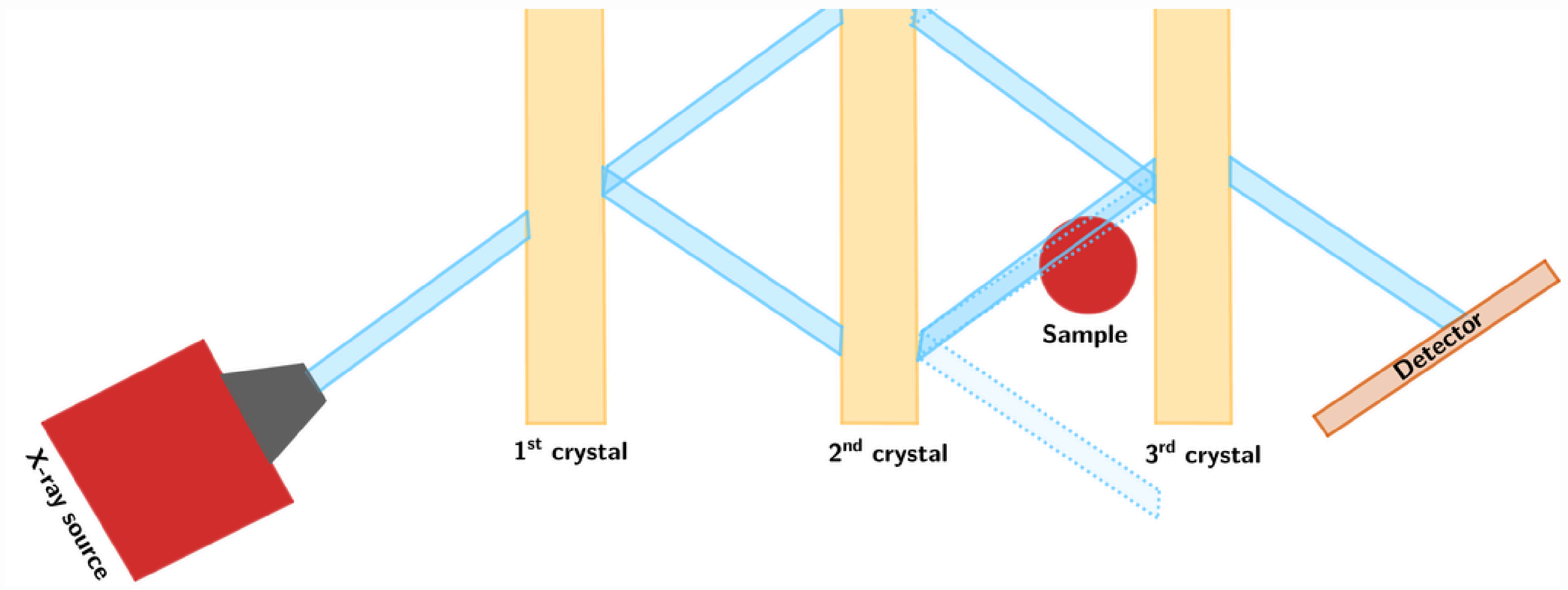
X-ray Absorption Imaging

Limitations

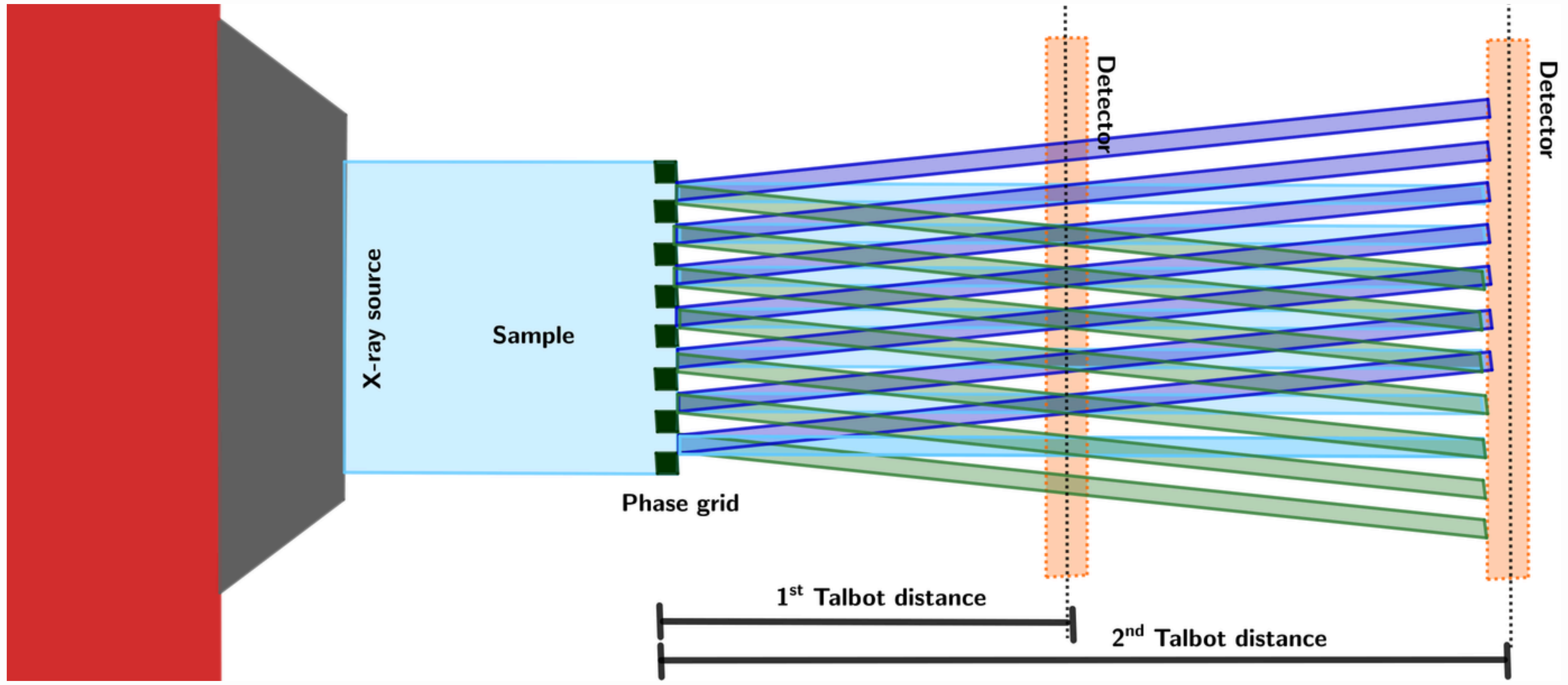


References

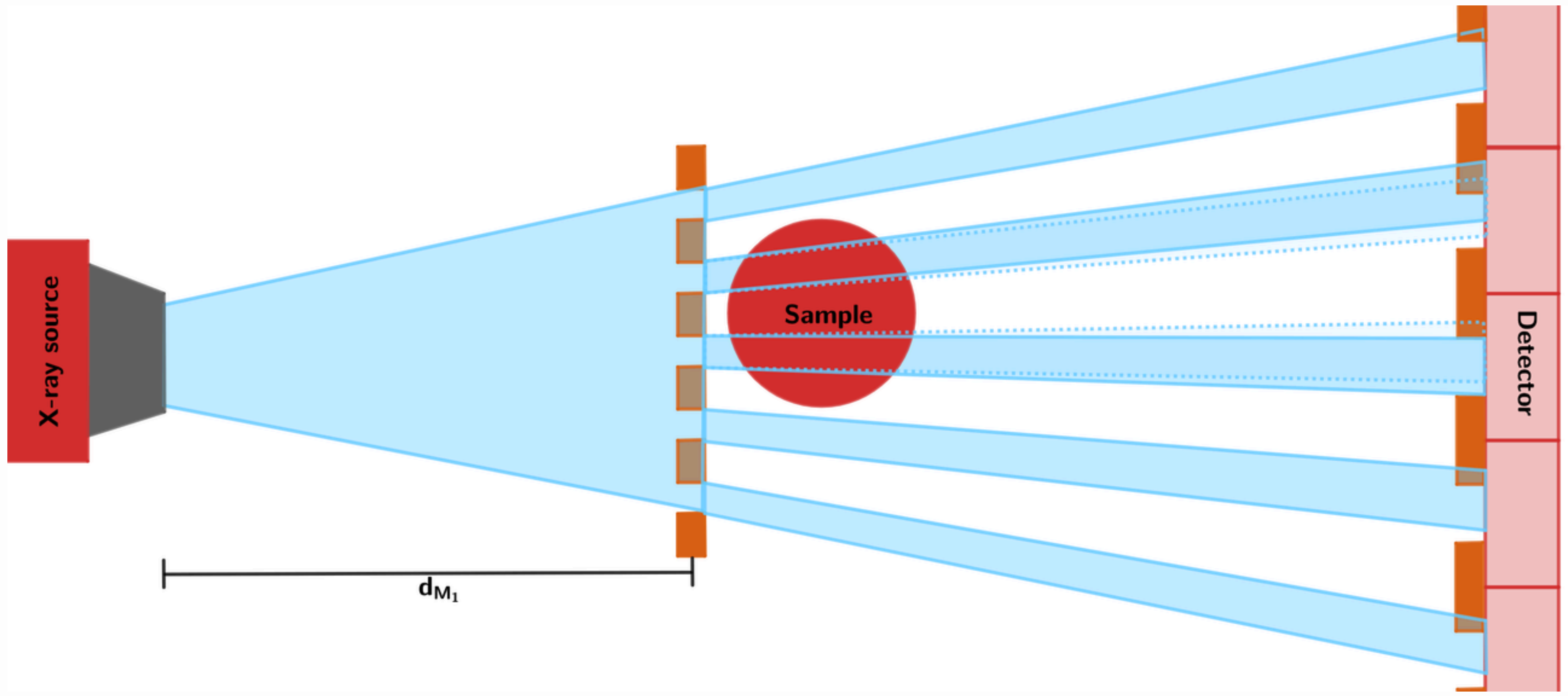
X-ray Phase Contrast Imaging (XPCI)



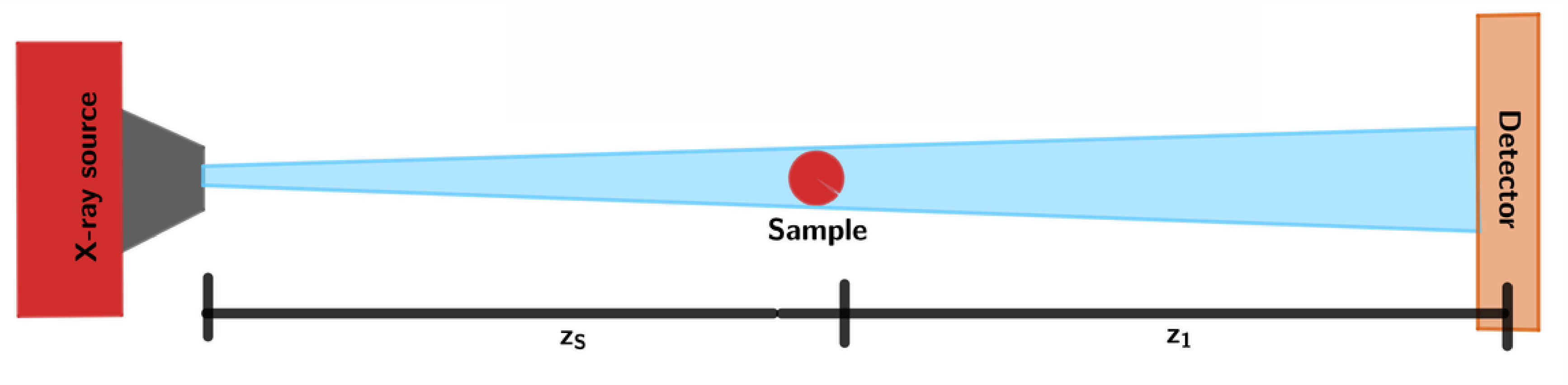
Schematic configuration of the Bonse–Hart method.



Schematic configuration of the Grating-based method.



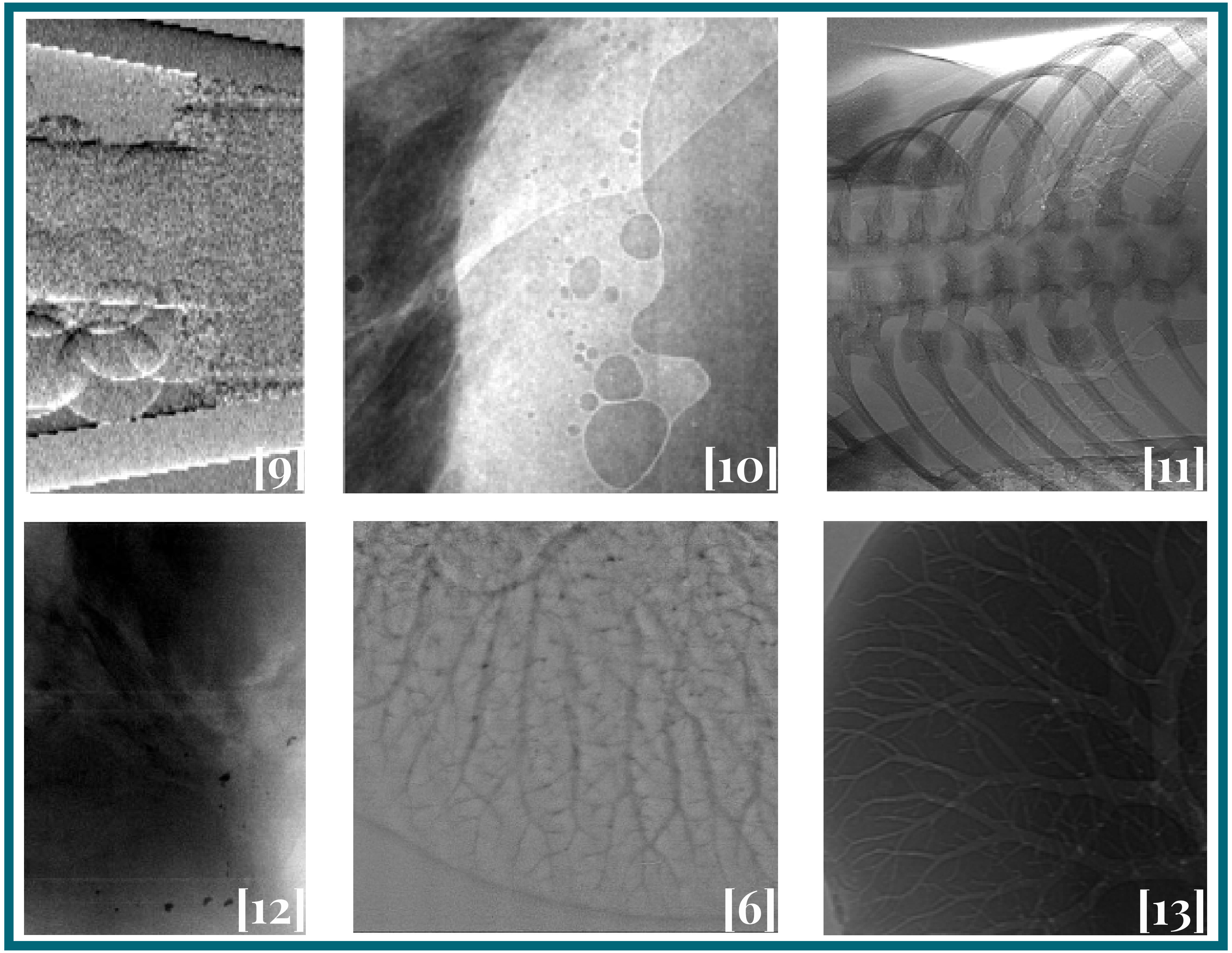
Schematic configuration of Edge Illumination.



Schematic configuration of Inline XPCI.

X-ray Phase Contrast Imaging (XPCI)

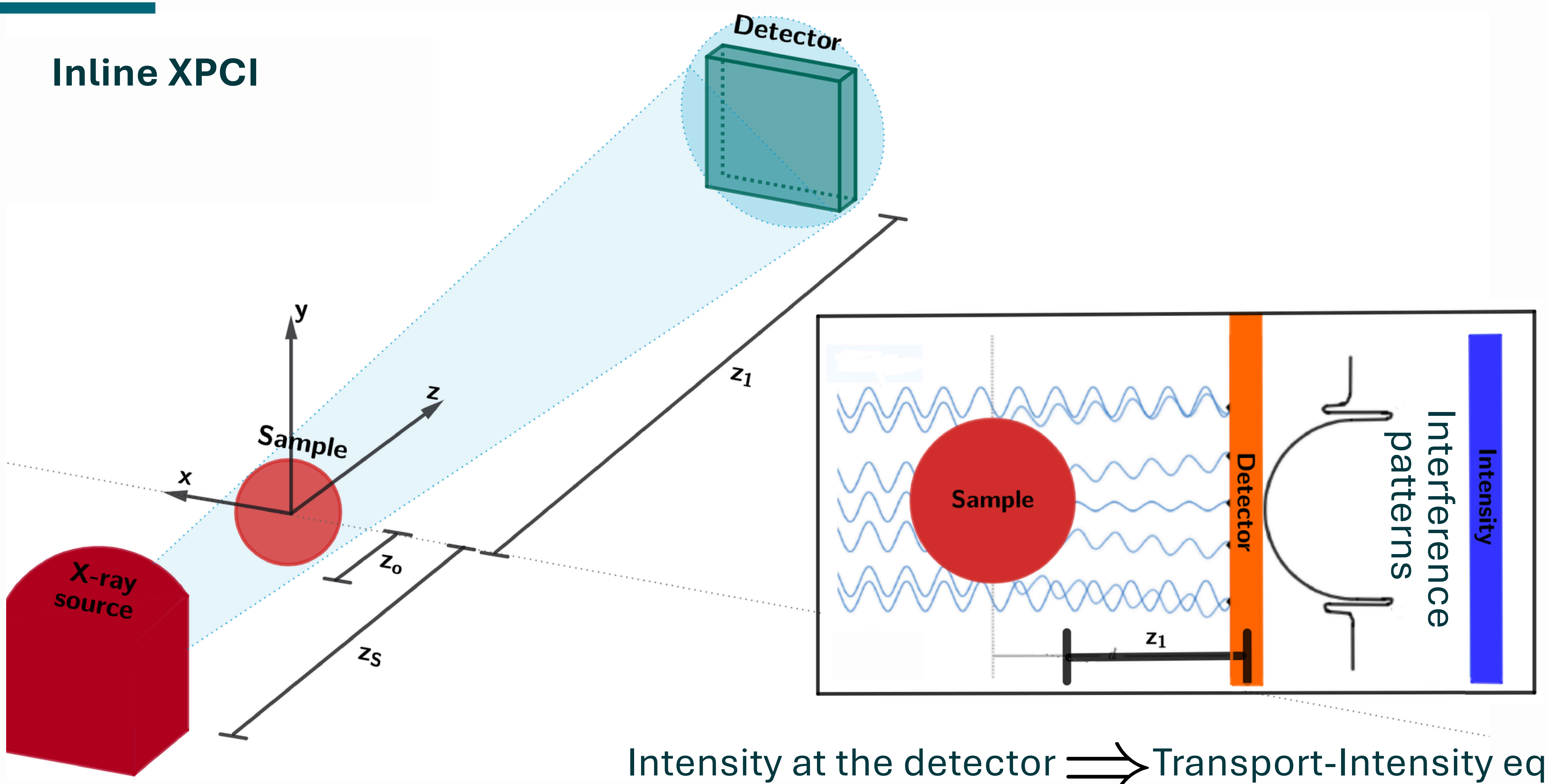
Applications



References

X-ray Phase Contrast Imaging (XPCI)

Inline XPCI



Intensity at the detector $\Longrightarrow$ Transport-Intensity equation

X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ Inline XPCI

Transport-Intensity equation

(equation S12)

↓

Solution

↓

Projected thickness maps

↓

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right)$$

Solutions of equation S12

Paganin approach

$$Z_1\left(\frac{x}{M},\frac{y}{M}\right) = -\frac{c}{2\omega\beta_1(\omega)} \times \ln\left(\mathscr{F}^{-1}\left(\frac{\mathscr{F}\left(\frac{M^2 I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)}\right)}{\Gamma\left(\beta_1,\delta_1,\omega,z_1,u,v\right)}\right)\right) \quad \textbf{(S14)}$$

Beltran approach

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right) = -\frac{c}{2\omega\Delta\beta(\omega)} \times \ln\left(\mathscr{F}^{-1}\left(\frac{\mathscr{F}\left(\frac{M^2 I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)e^{-\frac{2\omega}{c}\beta_1(\omega)Z_T\left(\frac{x}{M},\frac{y}{M}\right)}}\right)}{\Theta\left(\Delta\beta,\Delta\delta,\omega,z_1,u,v\right)}\right)\right) \quad \textbf{(S15)}$$

X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ **Inline XPCI**

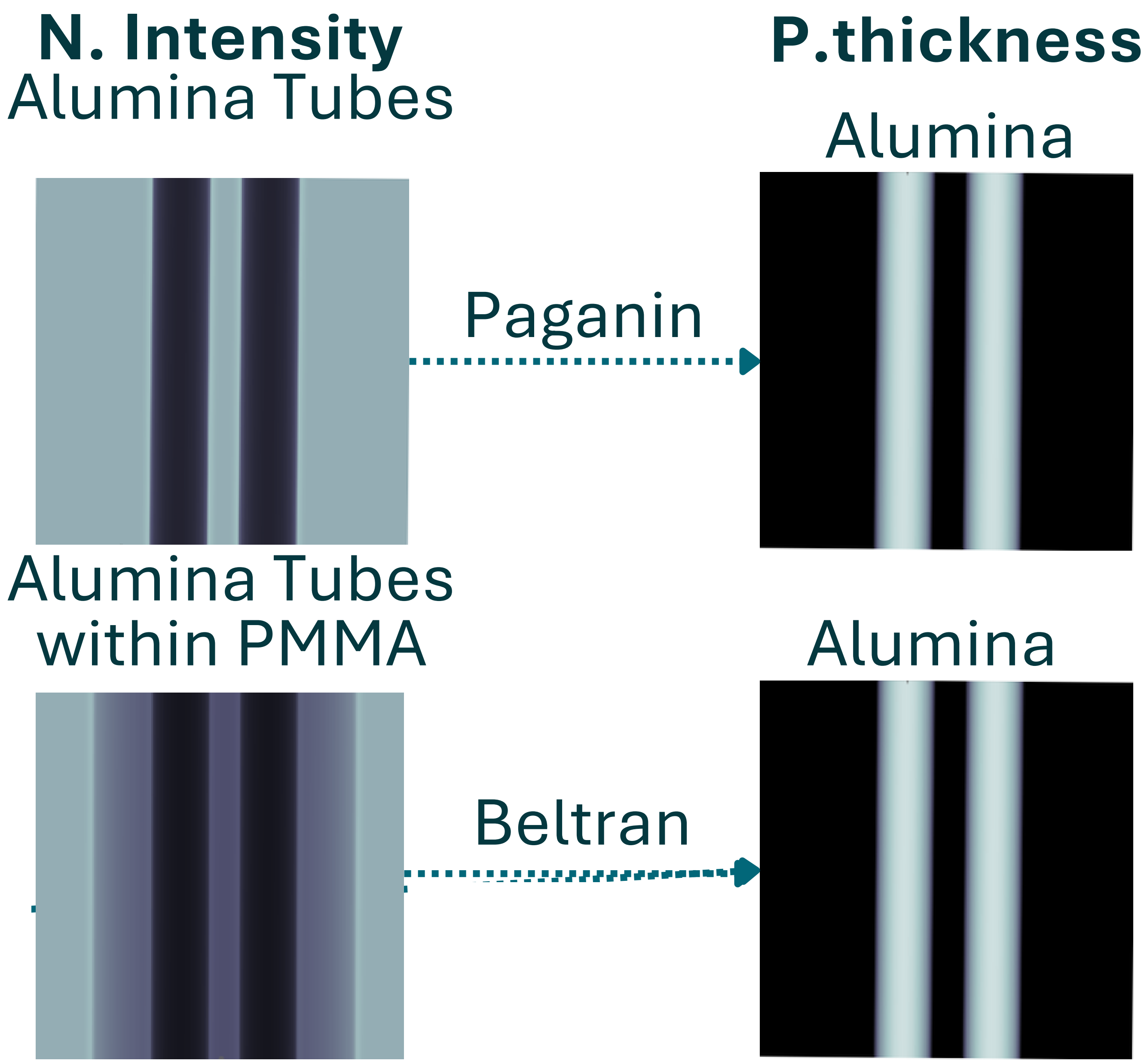
Solutions of equation S12

Paganin approach

$$Z_1\left(\frac{x}{M},\frac{y}{M}\right) = -\frac{c}{2\omega\beta_1(\omega)} \times \ln\left(\mathscr{F}^{-1}\left(\frac{\mathscr{F}\left(\frac{M^2 I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)}\right)}{\Gamma\left(\beta_1,\delta_1,\omega,z_1,u,v\right)}\right)\right) \quad \textbf{(S14)}$$

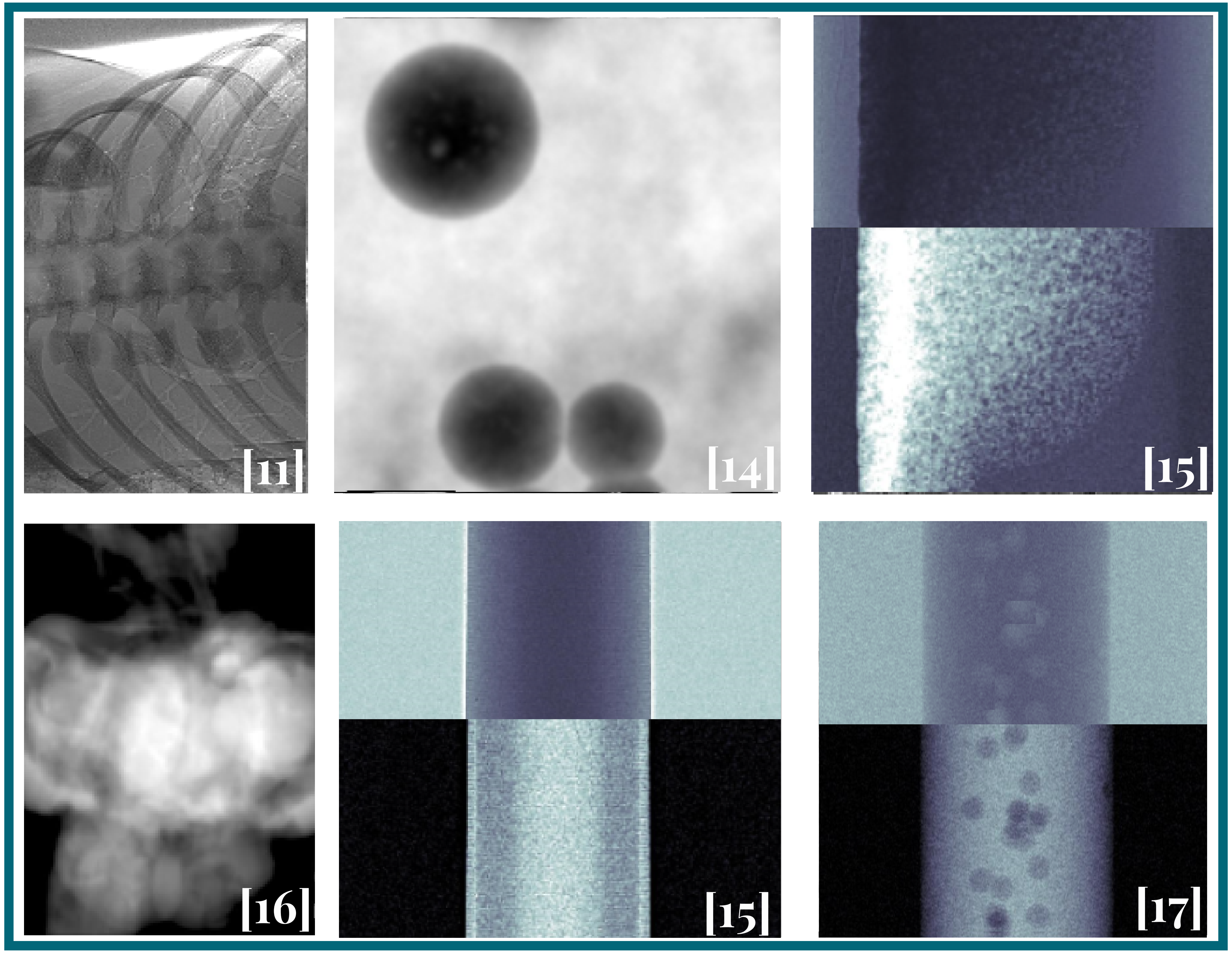
Beltran approach

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right) = -\frac{c}{2\omega\Delta\beta\left(\omega\right)} \times \ln\left(\mathscr{F}^{-1}\left(\frac{\mathscr{F}\left(\frac{M^2 I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)e^{-\frac{2\omega}{c}\beta_1(\omega)Z_T\left(\frac{x}{M},\frac{y}{M}\right)}}\right)}{\Theta\left(\Delta\beta,\Delta\delta,\omega,z_1,u,v\right)}\right)\right) \quad \textbf{(S15)}$$



X-ray Phase Contrast Imaging (XPCI)

Applications



References

X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ Inline XPCI

Transport-Intensity equation

(equation S12)

↓

Solution

↓

Projected thickness maps

↓

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right)$$

Transport-Intensity equation

(equation S12)

↓

Assumptions

- Transverse gradient of $e^{-\frac{2\omega}{c}\int_{-z_o}^{0}\beta\left(\frac{x}{M},\frac{y}{M},z,\omega\right)dz}$ not too strong
- Phase shift effects moderate
- Slow variation of the total projected thickness along the transverse coordinates

X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ **Inline XPCI**

Transport-Intensity equation

(equation S12)

↓

Solution

↓

Projected thickness maps

↓

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right)$$

Solution

$$f\,(u,v,\omega)=\sum_{j=2}^{\Omega}\eta_j(u,v,\omega)A_j(u,v)+\sum_{j=\Omega+1}^{n}\varkappa_j(u,v,\omega)A_j(u,v) \qquad \textbf{(S15)}$$

↓

$$f\,(u,v,\omega)\xrightarrow{\text{DFT}}\ln\left[\frac{M^2 I(x,y,z_1,\omega)}{I\left(\frac{x}{M},\frac{y}{M},-z_o,\omega\right)e^{-\frac{2\omega}{c}\beta_1(\omega)Z_T\left(\frac{x}{M},\frac{y}{M}\right)}}\right]$$

$$A_j(u,v)\xrightarrow{\text{DFT}}Z_j\left(\frac{x}{M},\frac{y}{M}\right)$$

$$\eta_j(\vec{u},\omega)=-\frac{2\omega}{c}\Delta\beta_j(\omega)+\frac{2z_1M^2\Delta\delta_j(\omega)}{W^2}\left[\cos\left(\frac{2\pi Wu}{NM}\right)+\cos\left(\frac{2\pi Wv}{NM}\right)-2\right]$$

$$\varkappa_j(\vec{u},\omega)=-\frac{2\omega}{c}\beta_j(\omega)+\frac{2z_1M^2\delta_j(\omega)}{W^2}\left[\cos\left(\frac{2\pi Wu}{NM}\right)+\cos\left(\frac{2\pi Wv}{NM}\right)-2\right]$$

X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ Inline XPCI

Transport-Intensity equation

(equation S12)

↓

Solution

↓

Projected thickness maps

↓

$$Z_j\left(\frac{x}{M},\frac{y}{M}\right)$$

Solution

$$\Phi = \left[\Upsilon^T \Xi^T \Xi \Upsilon\right]^{-1} \Upsilon^T \Xi^T \Xi \Pi \qquad \textbf{(S16)}$$

↓

$$\Pi = \begin{pmatrix} f(u,v,\omega_1) \\ \vdots \\ f(u,v,\omega_m) \end{pmatrix} \quad \Phi = \begin{pmatrix} A_2(u,v) \\ \vdots \\ A_n(u,v) \end{pmatrix} \quad \Xi = \mathrm{diag}\left(\frac{\bar{S}_B(\omega_1)}{\sigma_B(\omega_1)},\ldots,\frac{\bar{S}_B(\omega_m)}{\sigma_B(\omega_m)}\right)$$

$$\Upsilon = \begin{pmatrix} \eta_2(\vec{u},\omega_1) & \cdots & \eta_\Omega(\vec{u},\omega_1) & \varkappa_{(\Omega+1)}(\vec{u},\omega_1) & \cdots & \varkappa_n(\vec{u},\omega_1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \eta_2(\vec{u},\omega_m) & \cdots & \eta_\Omega(\vec{u},\omega_m) & \varkappa_{(\Omega+1)}(\vec{u},\omega_m) & \cdots & \varkappa_n(\vec{u},\omega_m) \end{pmatrix}$$

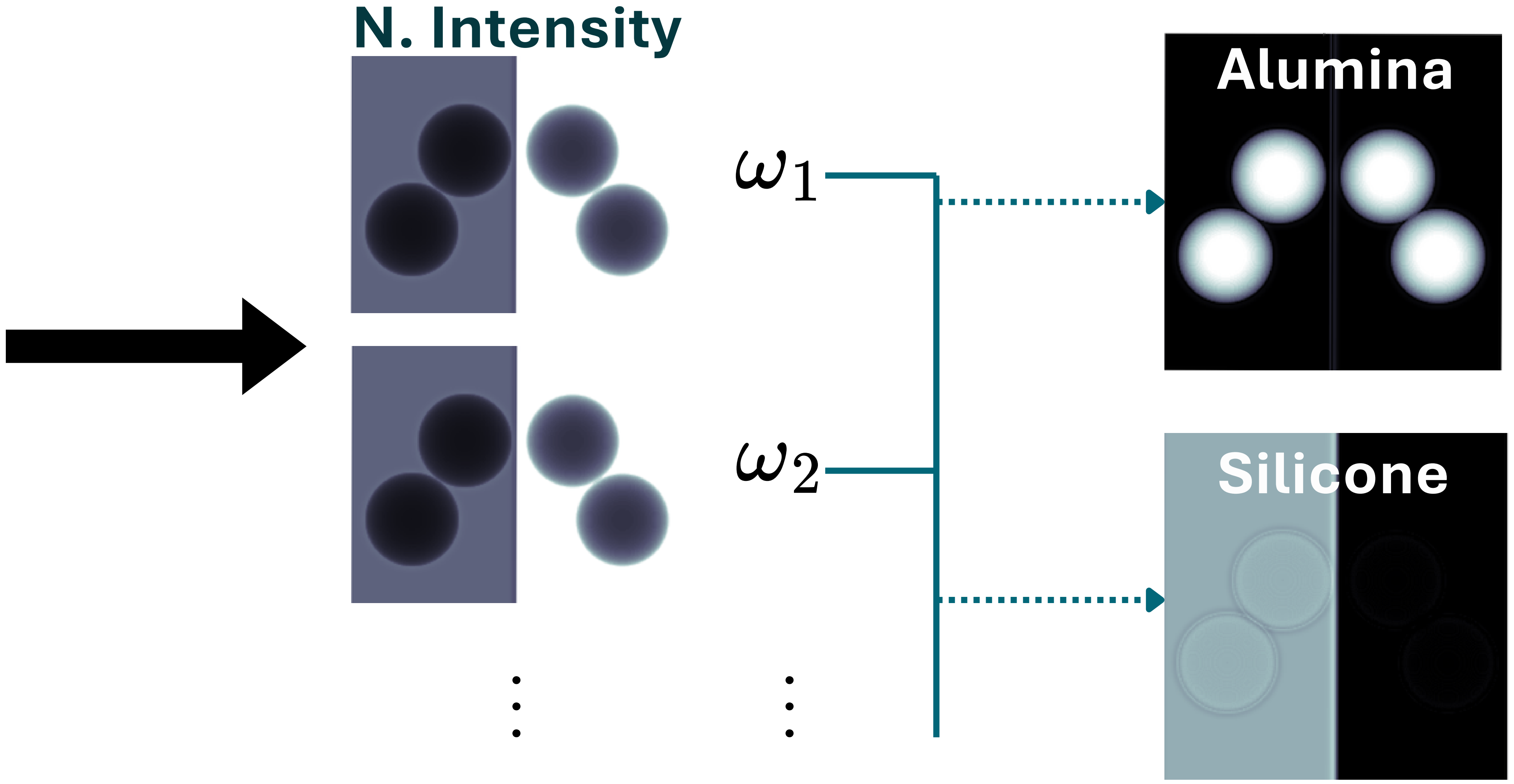
X-ray Phase Contrast Imaging (XPCI)

Phase retrieval process $\Longrightarrow$ Inline XPCI

Solution

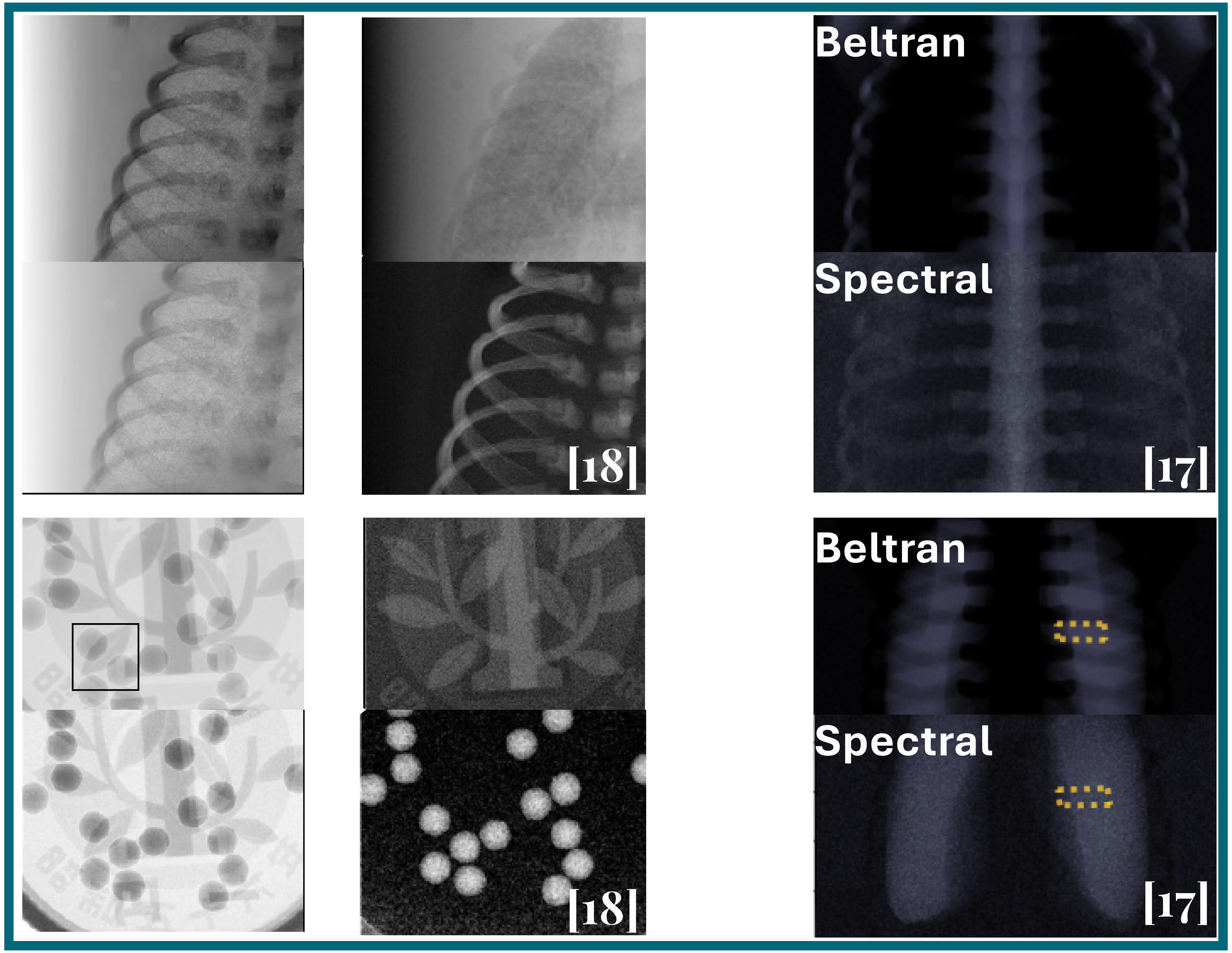
$$\Phi = \left[\Upsilon^T \Xi^T \Xi \Upsilon\right]^{-1} \Upsilon^T \Xi^T \Xi \Pi \quad \textbf{(S16)}$$

Alumina Spheres within a PMMA box and partially shielded by a Silicone block



X-ray Phase Contrast Imaging (XPCI)

Applications



References

Thank You for Your Attention