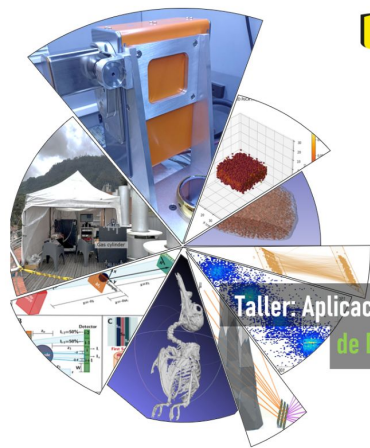
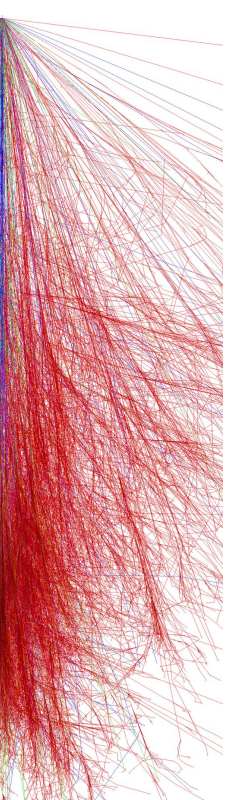


Métodos computacionales en muografía de absorción



Universidad de los Andes Facultad de Ciencias Departamento de Física

2026

Taller: Aplicaciones Interdisciplinarias de Detectores de Partículas

Junio 10 - 12.



Rafael Martínez Rivero

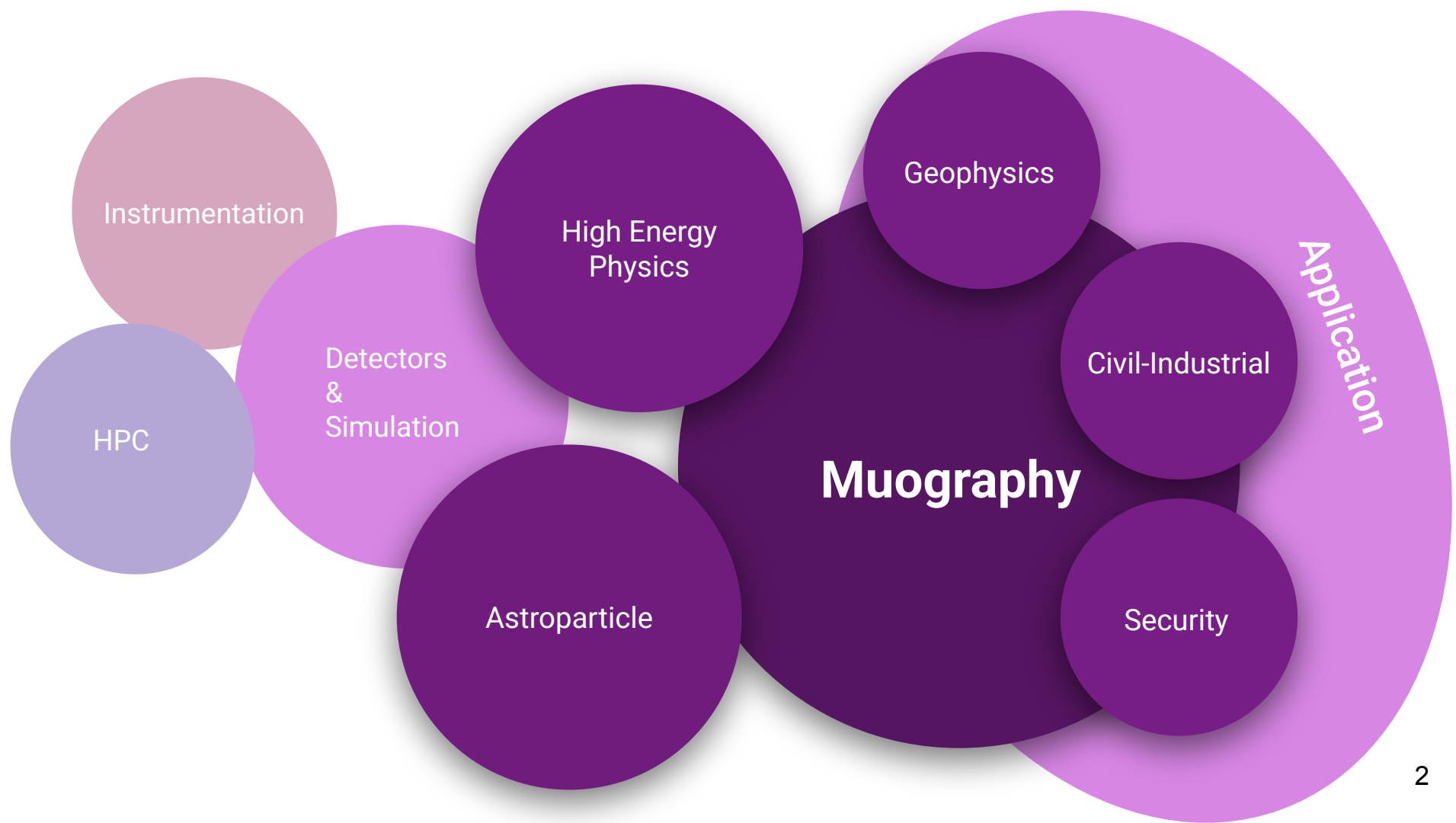
Grupo de Investigación en Relatividad y Gravitación (GIRG)

Laboratorio de Investigación para la Detección de Radiación y Astropartículas (LiDERA)

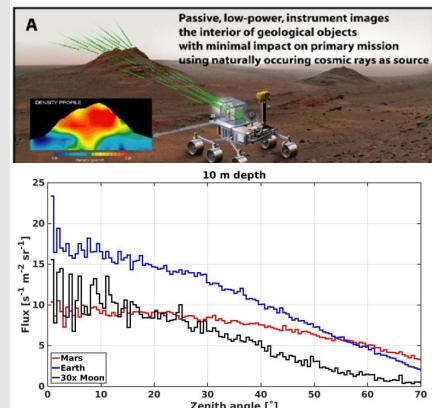
Universidad Industrial de Santander. Bucaramanga Colombia

10 June, 2026 Bogotá

rafael2248058@correo.uis.edu.co 1

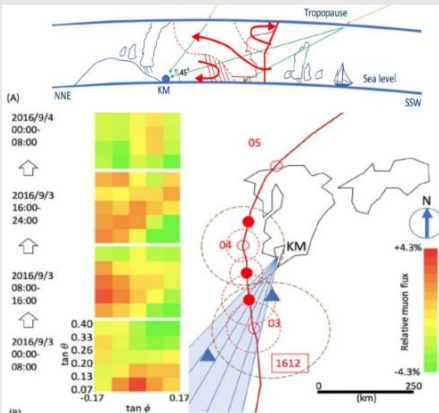


Muography on Mars and on the Moon



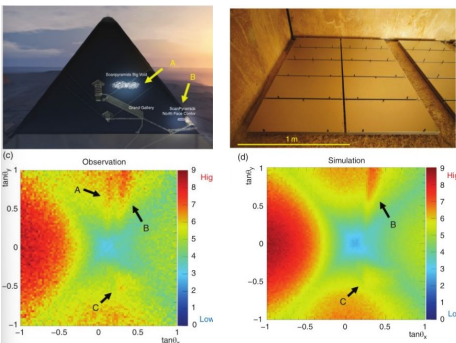
özsef Gábor Szűcs Muographers 2026-Talk

Muography of Tropical Cyclones



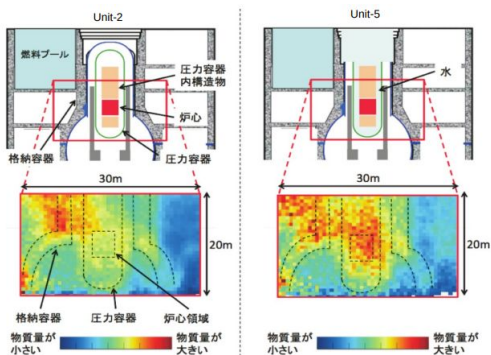
Tanaka et al. (2022) Sci. Rep. 12, 16710
<https://doi.org/10.1038/s41598-022-20039-4>

Exploring Hidden Cavities



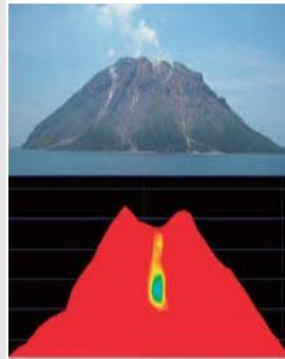
Morishima, K., et al. Nature 552, 386–390 (2017).
<https://doi.org/10.1038/nature24647>

Inspection of Fukushima Daiichi Reactor



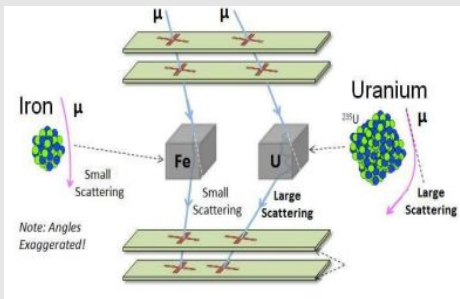
K. Morishima: Journal of the Photographic Society of Japan, 2016, Vol. 79, No. 1: 42-47

Volcano Muography



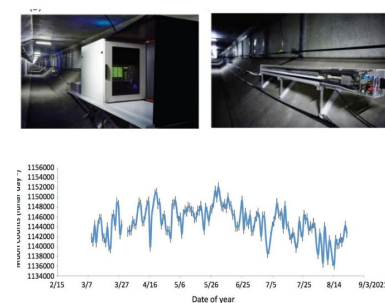
Oláh, L., Tanaka, H.K.M. (2025). Muography of Volcanoes.
https://doi.org/10.1007/978-3-031-86841-2_16

Detection of High-Z Materials



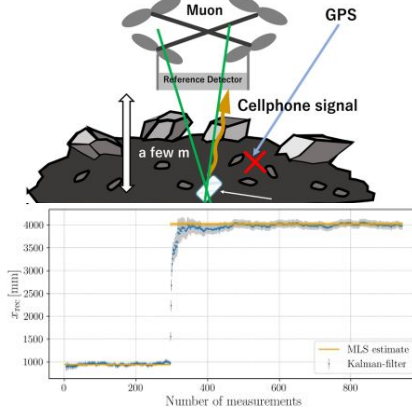
Presente, S., et al.: NIM A 604, 738 (2009)
<https://doi.org/10.1016/j.nima.2009.03.017>

Undersea Muography



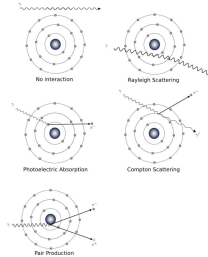
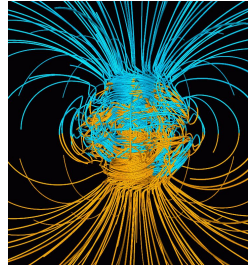
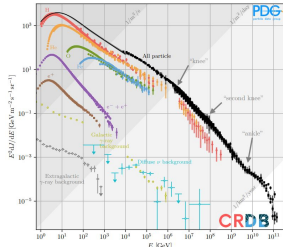
Tanaka, et al. Sci Rep 12, 6097 (2022).
<https://doi.org/10.1038/s41598-022-10078-2>

Muon Positioning (muPS)

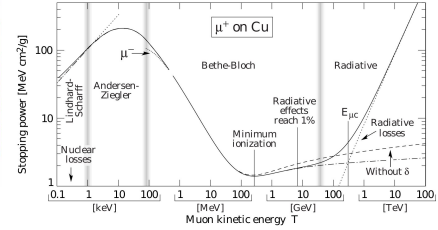


Varga, D., Tanaka, H.K.M. Sci Rep 14, 7605 (2024).
<https://doi.org/10.1038/s41598-024-57857-7>

Problema directo en muografía



las generaciones de materia (fermiones)				partículas de fuerza (bosones)			
QUARKS		LEPTONES		BOSONES ESCALARES		BOSONES DE GAUGE	
u up	c charm	t top	b bottom	g gluon	h Higgs	W	Z
d down	s strange	b bottom	t tau	W	Z	photon	graviton
e electrón	μ muon	τ tau	ν _e neutrino electrónico	ν _μ neutrino muónico	ν _τ neutrino tau	W	Z
quarks	leptons	quarks	leptons	quarks	leptons	quarks	leptons



Astropartículas

Primarios, energías, composición química, campo geomagnético.

EAS/Flujo de entrada

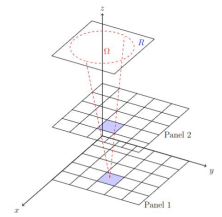
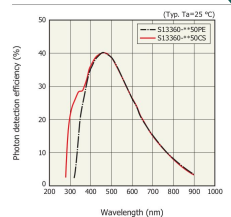
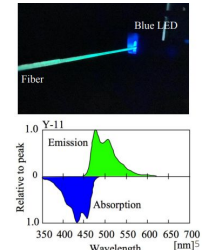
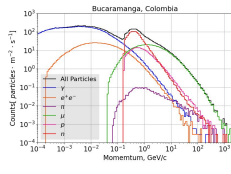
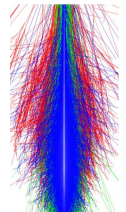
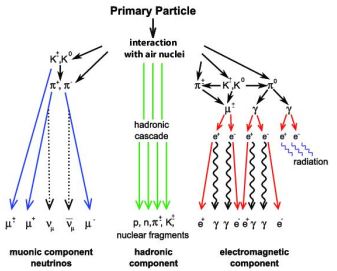
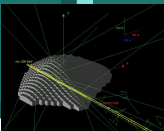
Atmósfera, Interacciones UHECR, generación de secundarios.

Objeto/Flujo de salida

Interacciones y transporte en la materia

Respuesta del detector

Geometría, Interacciones, eficiencias.



Estimación robusta, preservando los sistemáticos: Monte Carlo frameworks.

ARTI

Astropartículas

Primarios, energías,
composición
química, campo
geomagnético.

EAS/Flujo de entrada

Atmósfera,
Interacciones
UHECR, generación
de secundarios.

CORSIKA

MEIGA

Objeto/Flujo de salida

Interacciones y
transporte en la
materia

Respuesta del detector

Geometría,
Interacciones,
eficiencias.

Geant4

SPOILER: Puede hacerse un atajo!

Cálculo del flujo: Una fuente poco controlada pero muy estudiada.

Astropartículas

Primarios, energías, composición química, campo geomagnético.

EAS/Flujo de entrada

Atmósfera, Interacciones UHECR, generación de secundarios.

CORSIKA

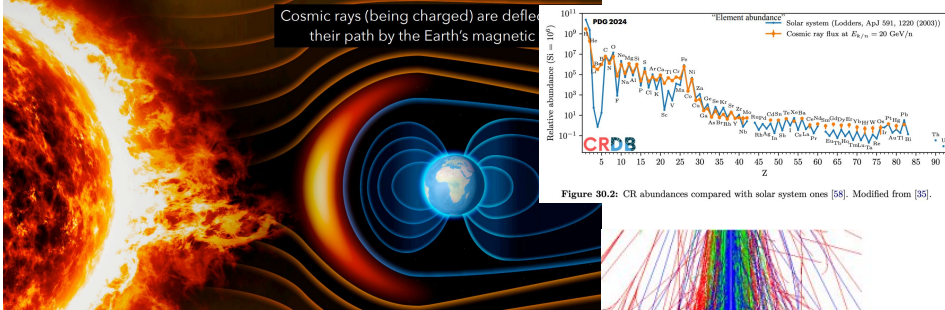
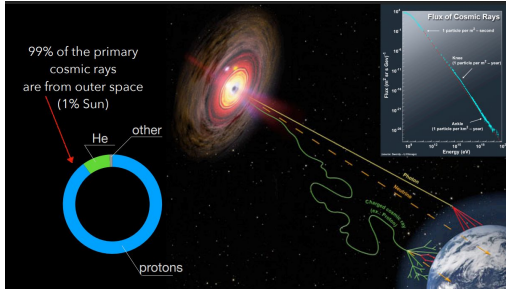
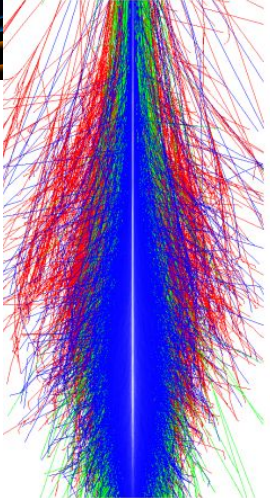
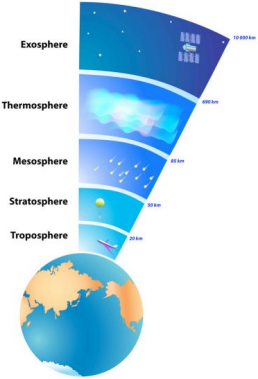
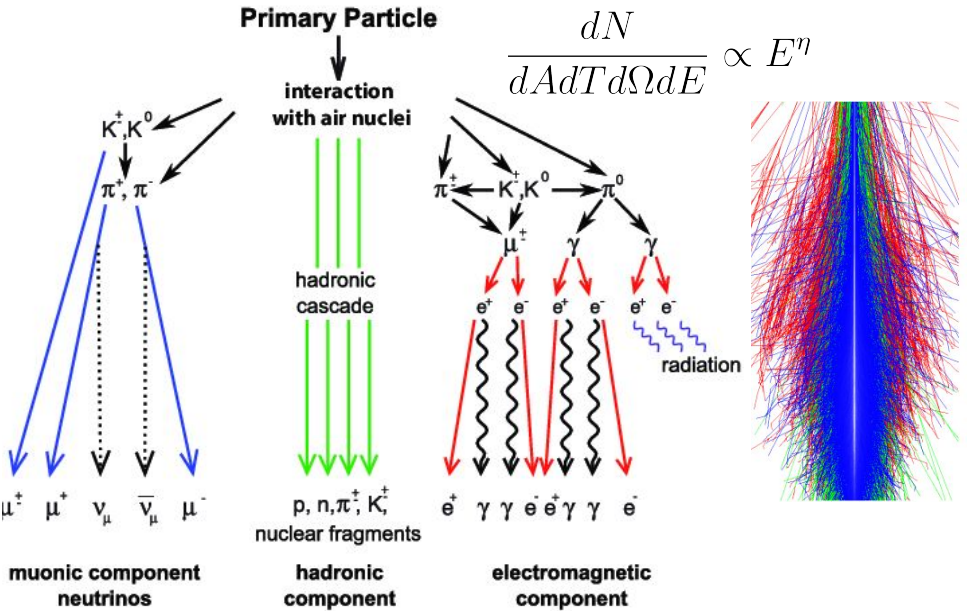
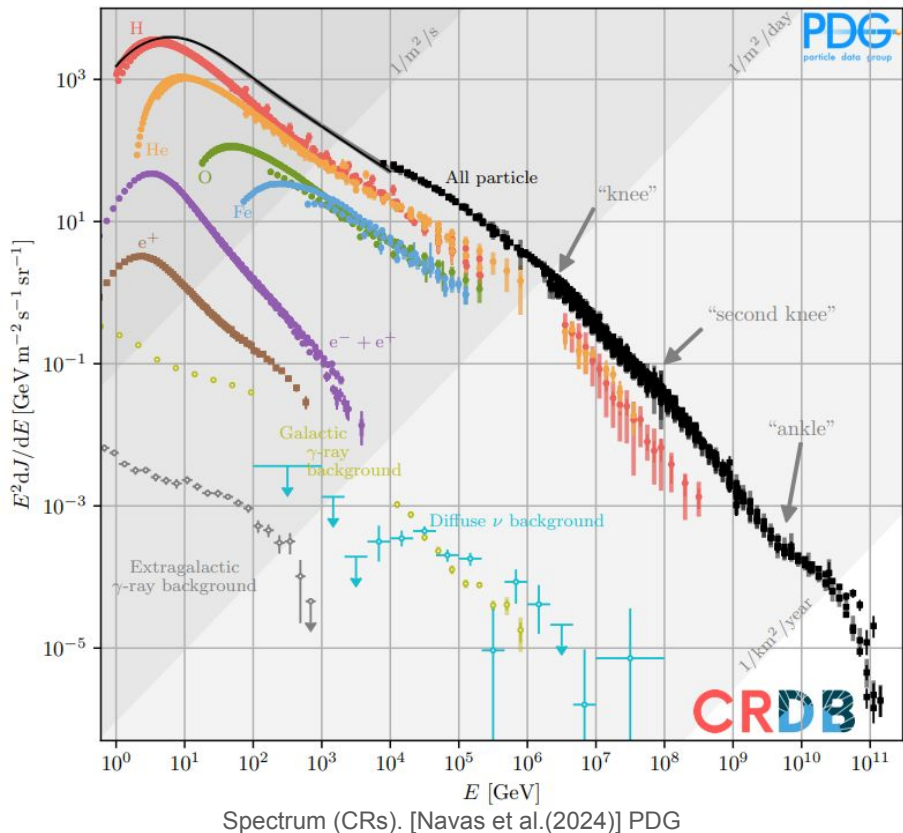


Figure 30.2: CR abundances compared with solar system ones [58]. Modified from [36].



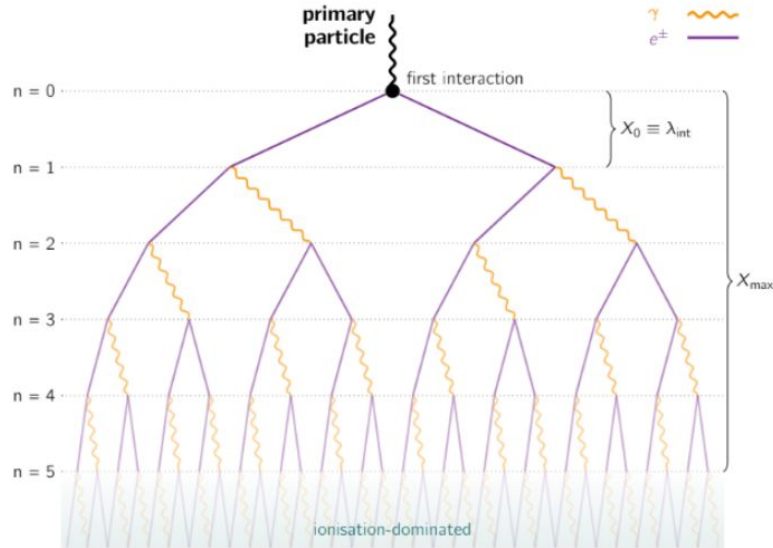
Radiación Cósmica: Partamos de nuestras observaciones.



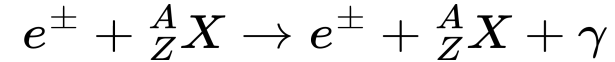
Partícula	Canales de desintegración parcial	BR [%]	Vida media [s]
π^\pm	$\mu^\pm + (\nu_\mu)$	99.99	$2.603 \cdot 10^{-8}$
K^\pm	$\mu^\pm + (\nu_\mu)$	63.43	$1.238 \cdot 10^{-8}$
	$\pi^0 + \mu^\pm + (\nu_\mu)$	3.27	
τ^\pm	$\mu^\pm + (\nu_\mu) + (\bar{\nu}_\tau)$	17.36	$2.906 \cdot 10^{-13}$
D^\pm	$K^0 + \mu^\pm + (\nu_\mu)$	7.0	$1.040 \cdot 10^{-12}$
	$\mu^\pm + \text{Hadrones}$	6.5	
D^0	$K^- + \mu^+ + \nu_\mu$	3.19	$4.103 \cdot 10^{-13}$

Modelos para EAS: Revisemos 2 aproximaciones al desarrollo de las EAS.

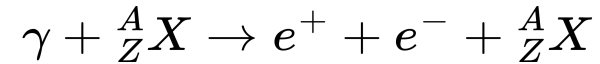
- ▶ Heitler model (1944)
- ▶ electromagnetic showers in the atmosphere



- ◆ *bremsstrahlung:*



- ◆ *Bethe-Heitler pair production:*



- ◆ *ionisation losses*

maximum number of particles:

$$N_{\text{max}} = \frac{E_0}{E_c}$$

depth of the shower maximum:

$$X_{\text{max}} = X_0 \log_2 \left(\frac{E_0}{E_c} \right)$$

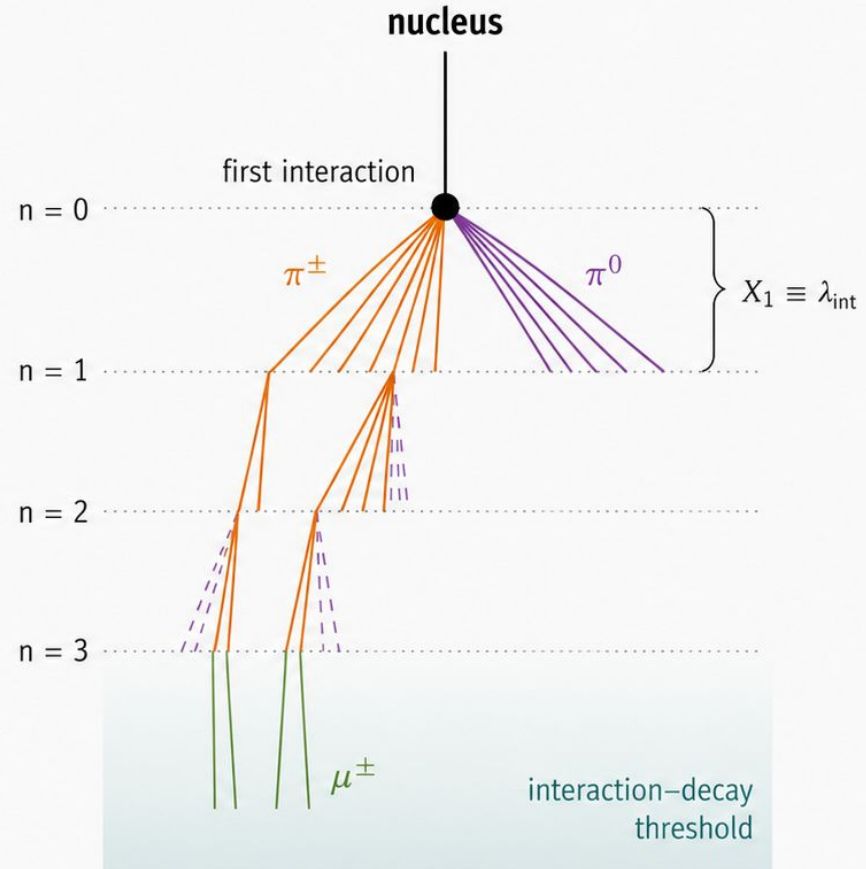
the Heitler-Matthews model for air showers

- ▶ **Heitler-Matthews model** (2005)
- ▶ **hadronic showers** in the atmosphere
- ▶ interplay of:
 - ◆ hadronic interactions
 - ◆ pion decay
 - ◆ all processes of electromagnetic showers
- ▶ 2/3 of the shower is hadronic, 1/3 electromagnetic
- ▶ depth of the shower maximum:

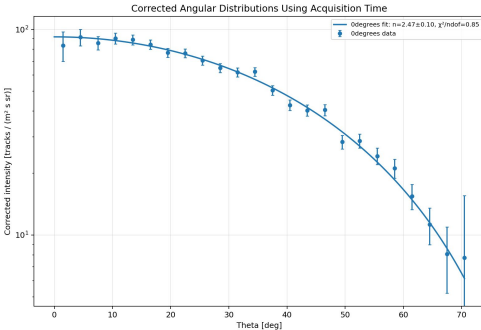
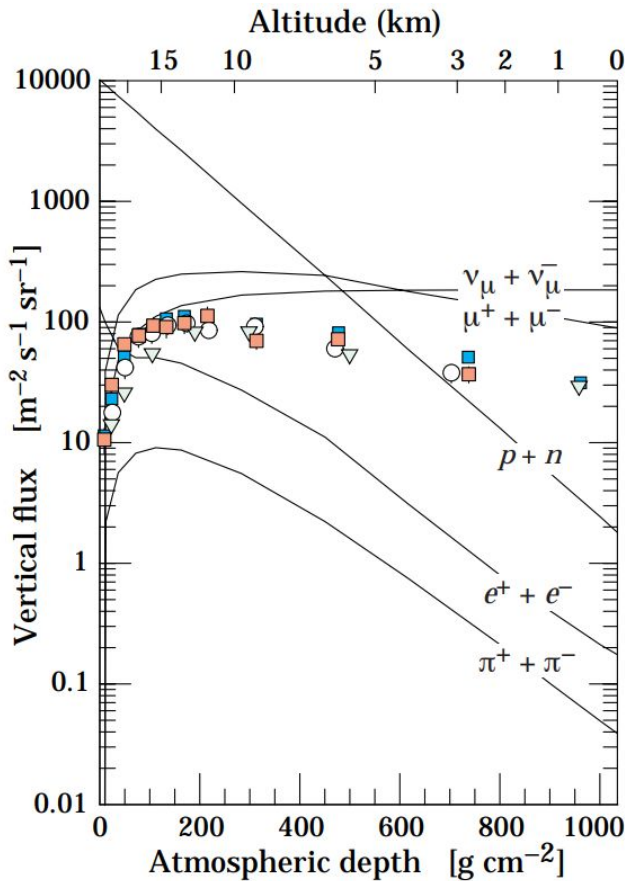
$$X_{\max} = \lambda_{\text{int}} + X_0 \log_2 \left(\frac{1}{3} \frac{E_0}{E_{\pi,c}} \right)$$

- ▶ **number of muons:**

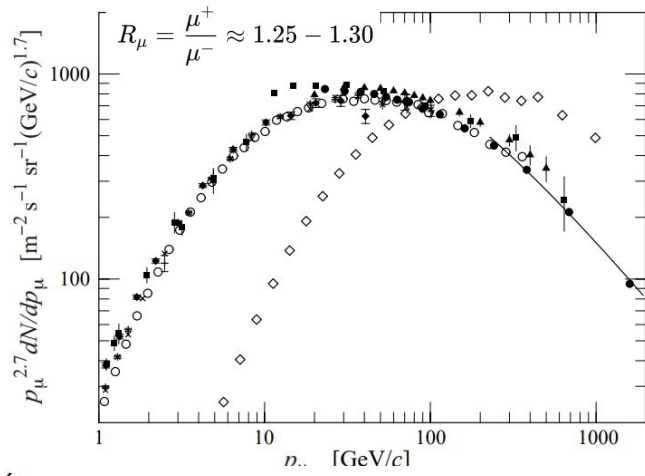
$$N_{\mu} \propto \left(\frac{E_0}{E_c} \right)^{0.9}$$



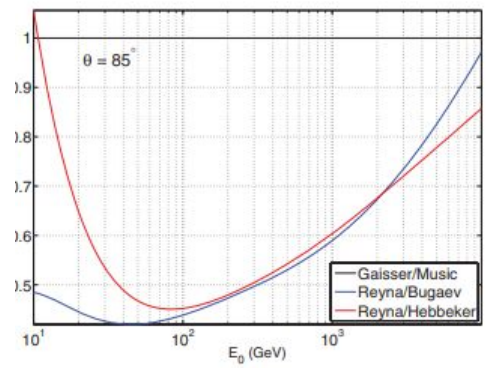
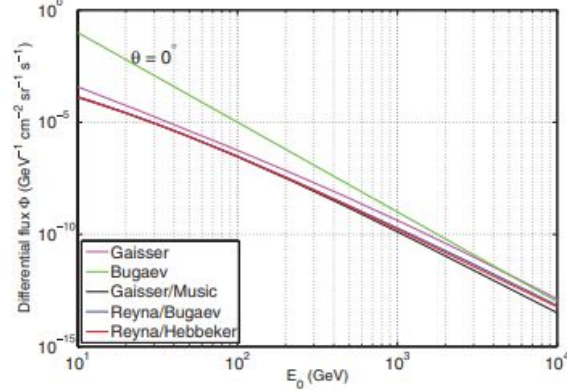
Medidas: Los observables de interés son espectros **energéticos** y **angulares**



MINOS Near Detector midió:



$$R_{\mu} = 1.266 \pm 0.001(\text{stat.})_{-0.014}^{+0.015}(\text{syst.})$$



CORSIKA: Cosmic Ray Simulation for KASCADE

KASCADE: an experiment to measure the composition of cosmic rays in Karlsruhe (Germany) 1997-2009

- EM model: EGS4 Low energy
- hadronic models: FLUKA UrQMD GHEISHA
- High energy hadronic models: QGSJET EPOS-LHC DPMJET SIBYLL

Electrons, positrons, gammas
Muons, hadrons

$$E = 10^{15} \text{ eV}, \theta = 0^\circ$$

γ

P

Fe

μ

Tuned at collider energies (TeV) and extrapolated to 10^{20} eV

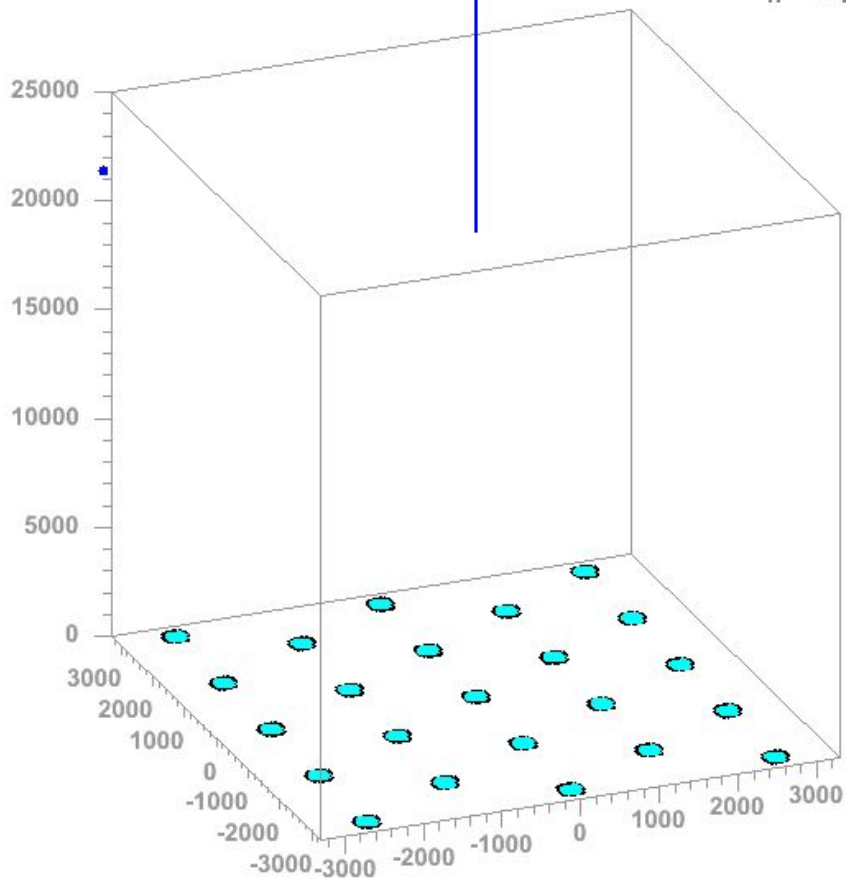
Cosmic ray simulations - CORSIKA Source: F. Schmidt, J.Knapp – <https://www-zeuthen.desy.de/~jknapp/fs/showerimages.html>

hadrons muons electrs neutrs

$0.00 \cdot 10^{-6}$ sec

Proton 10^{14} eV

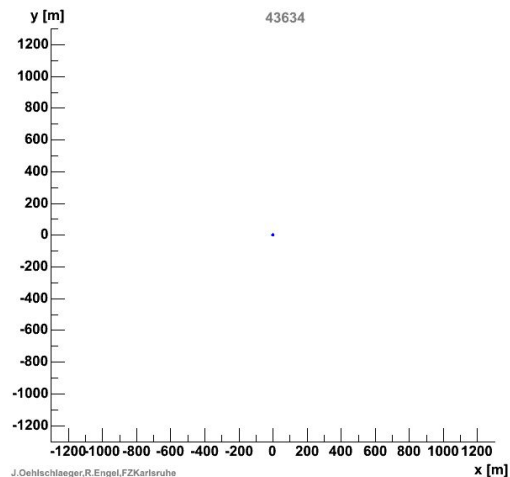
$h^{1st} = 21311$ m



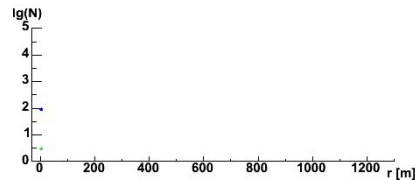
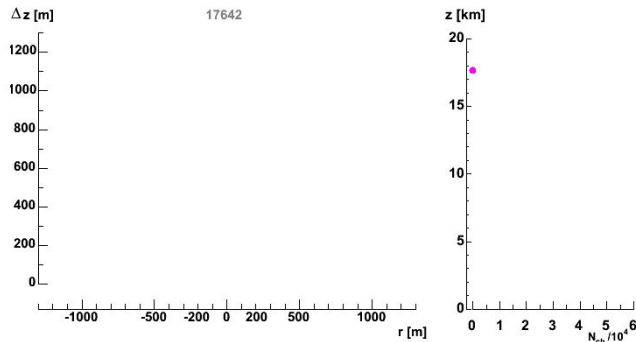
J.Oehlschlaeger,R.Engel,FZKarlsruhe

hadrons muons electrs neutrs

Iron 10^{14} eV



J.Oehlschlaeger,R.Engel,FZKarlsruhe



Proton 10^{14} eV

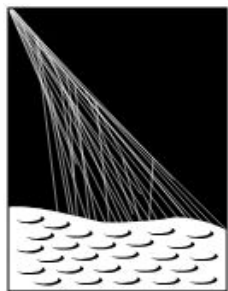
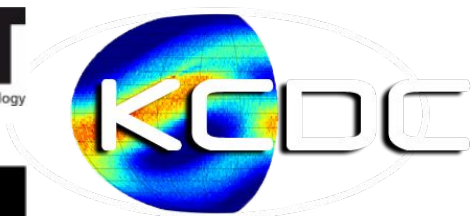
$h^{1st} = 17642$ m

hadrons muons

neutrons electrs

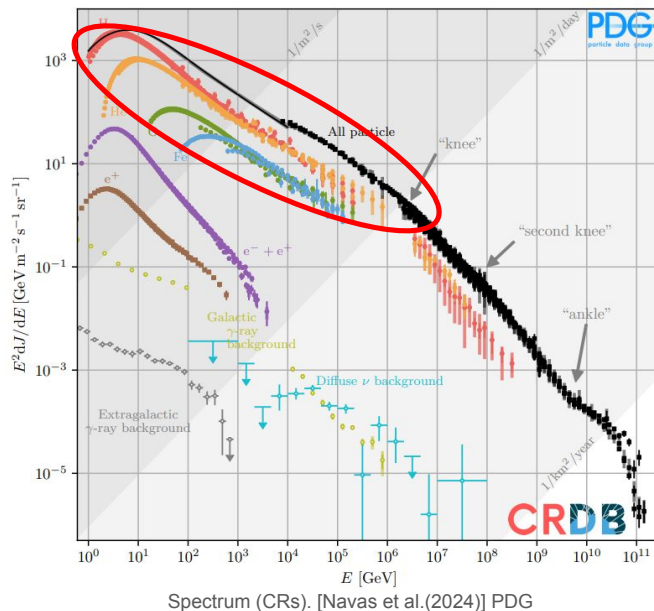
J.Oehlschlaeger,R.Engel,FZKarlsruhe

CORSIKA: Software consolidado en la comunidad científica

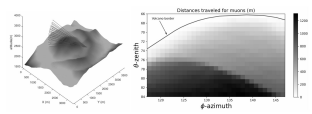


Sin embargo...

... para la muografía nos interesa el flujo de secundarios generados por los primarios



Flux Estimation: ARTI framework Monte Carlo/CORSIKA based.



Vokano	Criterion 1	Criterion 2	Criterion 3
Tarapur	Y	Y	N
Chim	Y	Y	N
Chimbal	Y	Y	N
Dona-Juana	Y	Y	N
Galena	Y	Y	Y
Marina	Y	Y	Y
Nevado del Huala	Y	Y	N
Nevado del Ruiz	Y	Y	Y
Nevado Santa Isabel	Y	Y	Y
Nevado del Edmundo	Y	Y	Y
Panajachel	Y	Y	Y
Selk'	Y	Y	N

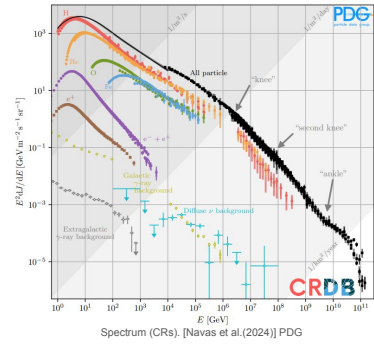
Vesga-Ramirez, A., et al (2020). Muon

15.

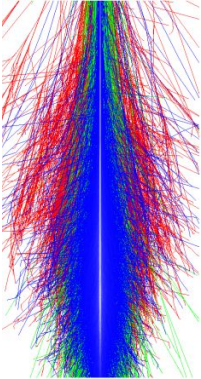
Provides the reference site-dependent muon flux. The generative model learns to reproduce it quickly.

Primary flux integration from observations at 112 km.

$$\Phi(E_p, Z, A, \Omega) = j_0(Z, A) \left(\frac{E_p}{E_0} \right)^{\alpha(E_p, Z, A)}$$

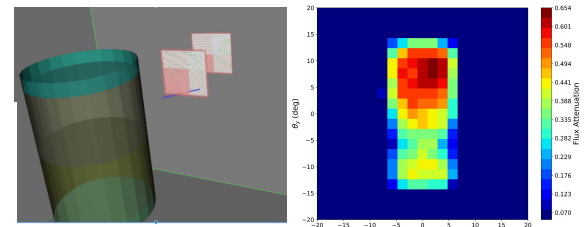
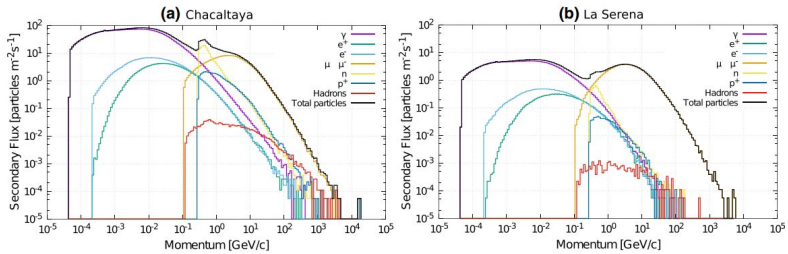


CORSIKA simulation for all the primaries particles with site depend condition



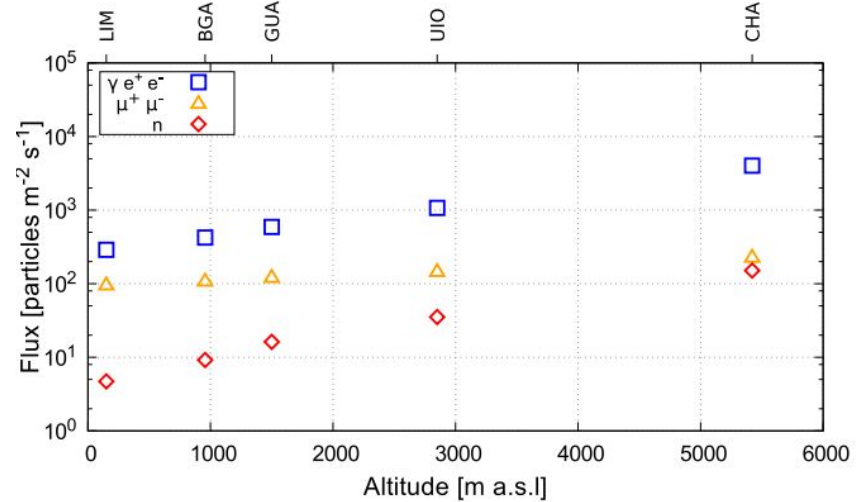
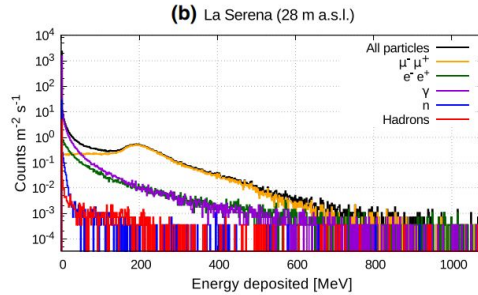
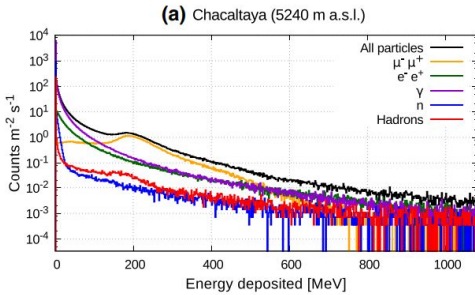
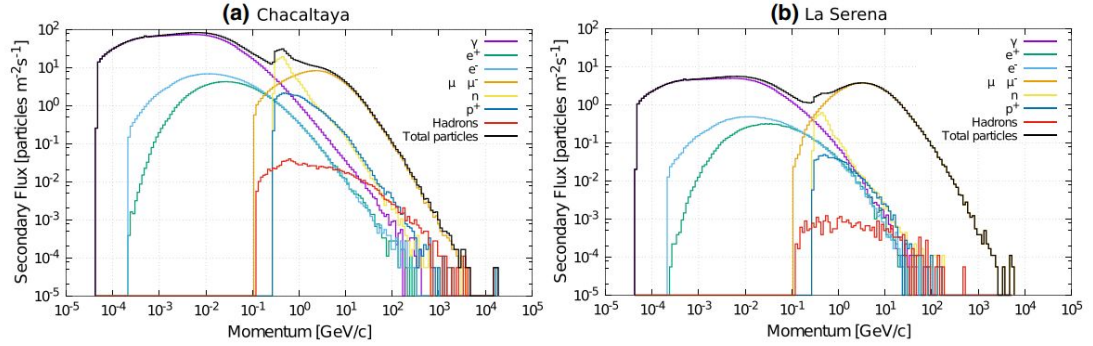
Sarmiento-Cano, & LAGO Collaboration. (2022). The ARTI framework: cosmic rays atmospheric background simulations. *The European Physical Journal C*, 82(11), 1019.

Recording of the secondary particles generated



R. A. Martínez-Rivero, et al. (2025); Muon imaging of hydrotreatment reactors. *J. Appl. Phys.*

- El flujo de partículas secundarias varía con la **altitud**, el **campo geomagnético local** y el **perfil atmosférico**.
- Por ello, cada ubicación presenta un **espectro de fondo distinto**.
- En consecuencia, el fondo secundario debe evaluarse **individualmente para cada sitio de observación**.



ARTI Simulation framework

Sarmiento-Cano, & LAGO Collaboration. (2022).
The ARTI framework: cosmic rays atmospheric background simulations. *The European Physical Journal C*, 82(11), 1019.



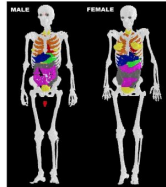
Files used by
 CORSIKA for
 simulation setup

Perl language



MAGCOS uses
 CORSIKA to produce
 a corrected flux

Root & C++



Bash interface



Initial conditions as
 energy interval,
 zenith, azimuth and
 geomagnetic field.

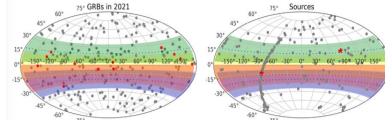
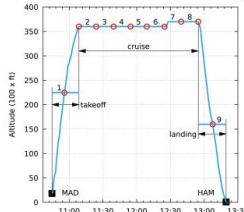
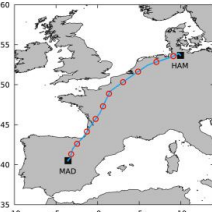
Fortran & C++



Output: Binary files
 and pre-analysis files.

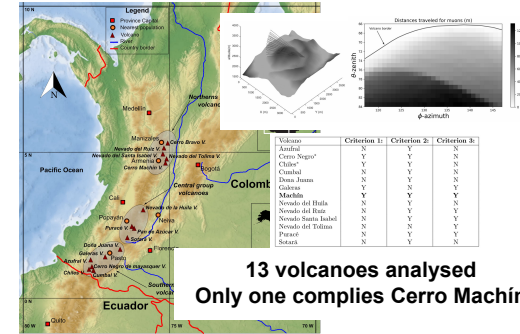


Output: Distribution of
 photoelectrons &
 Charge histogram



Sidelnik, I., & LAGO Collaboration. (2023). **The capability of water Cherenkov detectors arrays of the LAGO project to detect Gamma-Ray Burst and high energy astrophysics sources.** *Nuclear Instruments and Methods in Physics Research* 1056, 168576.

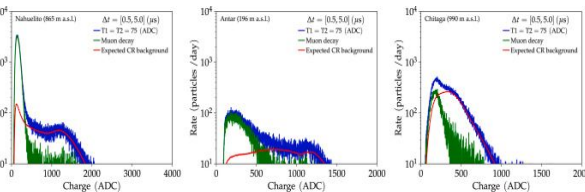
Country	Site	Altitude [m asl]	Latitude [deg]	Longitude [deg]
Sierra Negra	SNG	4,550	18.2 N	97.9 W
Chimborazo	CHI	5,000	1.5 S	78.8 W
Imata	IMA	4,600	15.9 S	71.1 W
Atacama	ATA	5,100	23.0 S	67.8 W
S. A. de los Cobres	SAC	4,500	24.2 S	66.3 W



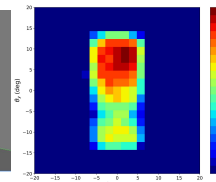
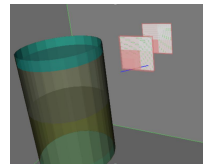
13 volcanoes analysed
Only one complies Cerro Machín

Vesga-Ramirez, A., et al (2020). **Muon Tomography sites for Colombian volcanoes.** *ANNALS OF GEOPHYSICS*, 63(6).

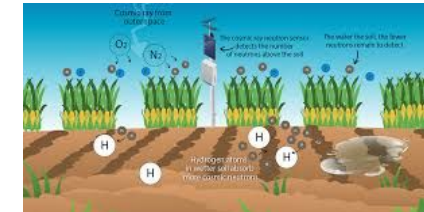
Asorey, H., Suárez-Durán, M., & Mayo-García, R. (2023). **ACORDE: A new application for estimating the dose absorbed by passengers and crews in commercial flights.** *Applied Radiation and Isotopes*, 196, 110752.



Otiniano, L., & LAGO Collaboration. (2023). **Measurement of the muon lifetime and the Michel spectrum in the LAGO water Cherenkov detectors as a tool to enhance the signal-to-noise ratio.** *Nuclear Instruments and Methods in Physics Research* 1056, 168567.



R. A. Martínez-Rivero, C. Sarmiento-Cano, D. Castillo-Morales, J. Perea-Pérez, V. G. Baldovino-Medrano, J. D. Sanabria-Gómez, L. A. Núñez; **Muon imaging of hydrotreatment reactors.** *J. Appl. Phys.* 28 December 2025; 138 (24): 244901. <https://doi.org/10.1063/5.0281667>



J. Betancourt, et al. (2025) **Enhanced water Cherenkov detector for soil moisture detection.** <https://doi.org/10.48550/arXiv.2509.08562>

Geant4 – A Simulation Toolkit

Geant 4

<http://www.geant4.org/>

S. Agostinelli et al.
Geant4: a simulation toolkit
NIM A, vol. 506, no. 3, pp. 250-303, 2003

J. Allison et al.
Geant4 Developments and Applications
IEEE Trans. Nucl. Sci., vol. 53, no. 1, pp. 270-278, 2006

MEIGA

Objeto/Flujo de salida

Interacciones y transporte en la materia

Respuesta del detector

Geometría, Interacciones, eficiencias.

Geant4

What is MEIGA

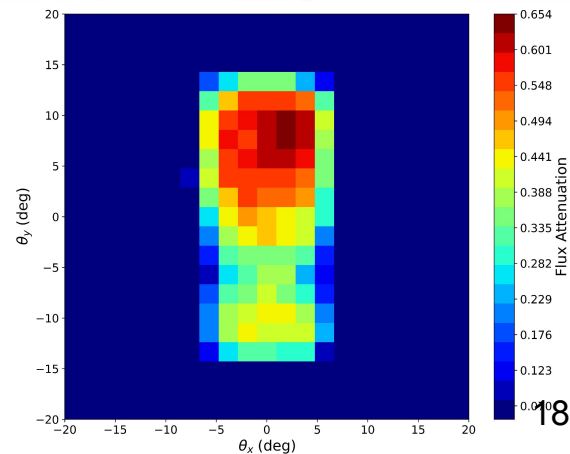
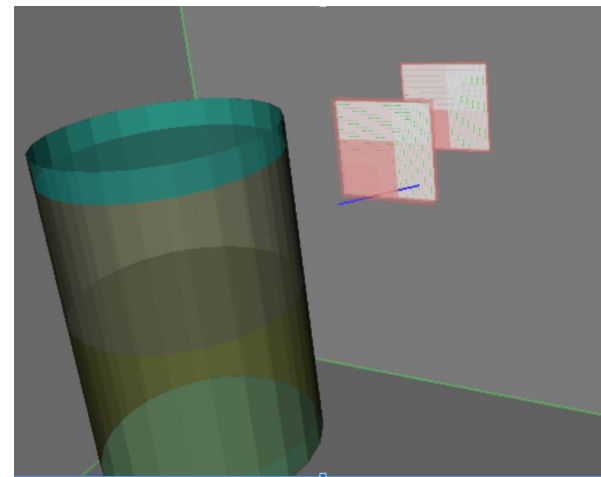
What is it?

MEIGA is a framework based on C++ classes. It combines three elements:

- Cosmic-ray flux calculation
- Particle propagation
- Detector response simulation

How does it work?

- Uses **JSON configuration files** (input/output paths, detector setup, physics lists, verbosity).
- Runs **Geant4 simulations**, injecting particles at ground level.
- Tracks particles inside the detector geometry defined by XML files.
- Collects detector response (energy deposits, SiPM signals, time traces).



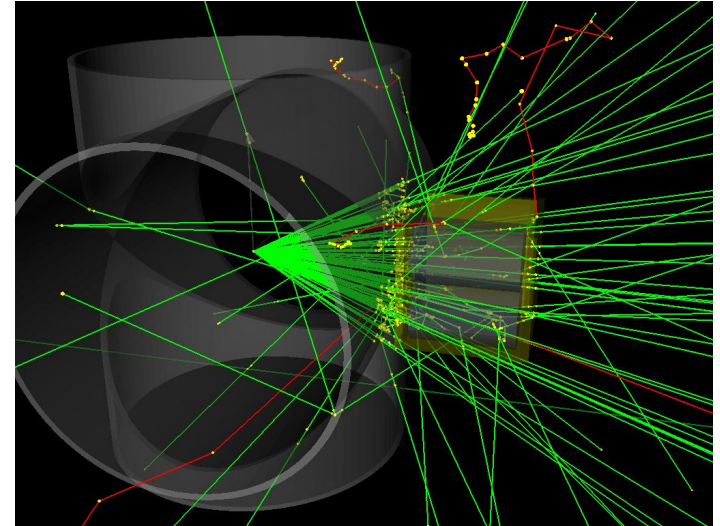
The core of MEIGA is GEANT4

- Toolkit to simulate **particle interactions with matter**.
- Uses databases, cross-sections, and physics models.
- Widely used in **HEP, medical physics, and space science**.

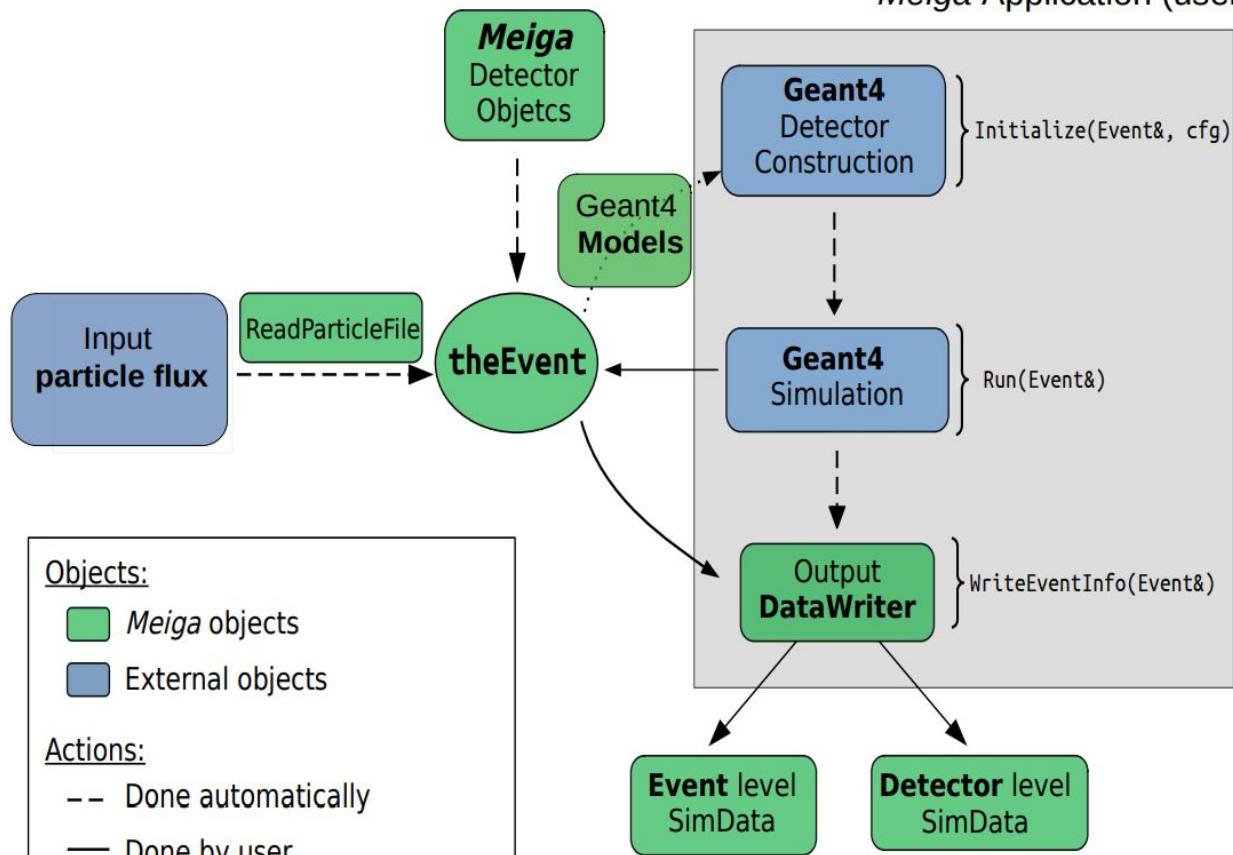


Key Features

- **Physics List**
 - Define particles
 - Define processes
- **Detector Construction**
 - Define materials
 - Build geometry
 - Set sensitive regions
- **User Action Classes**
 - Generate primary particles
 - Extract data (energy, tracks, steps)



Meiga Application (users)

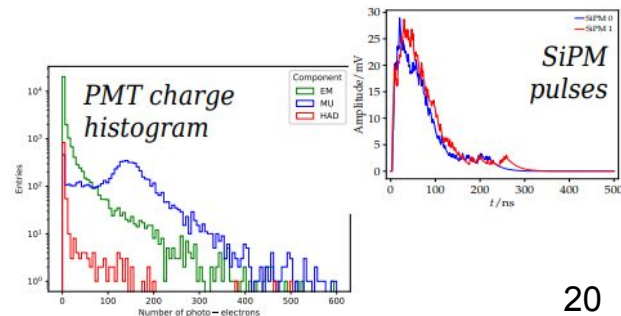
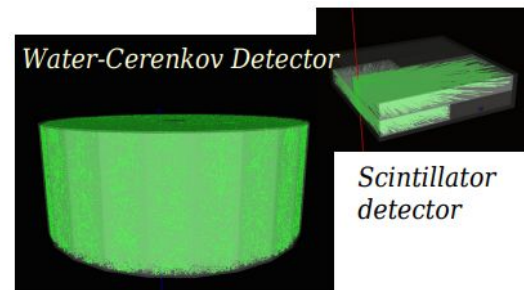
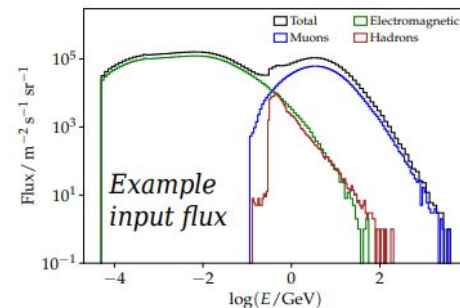


Objects:

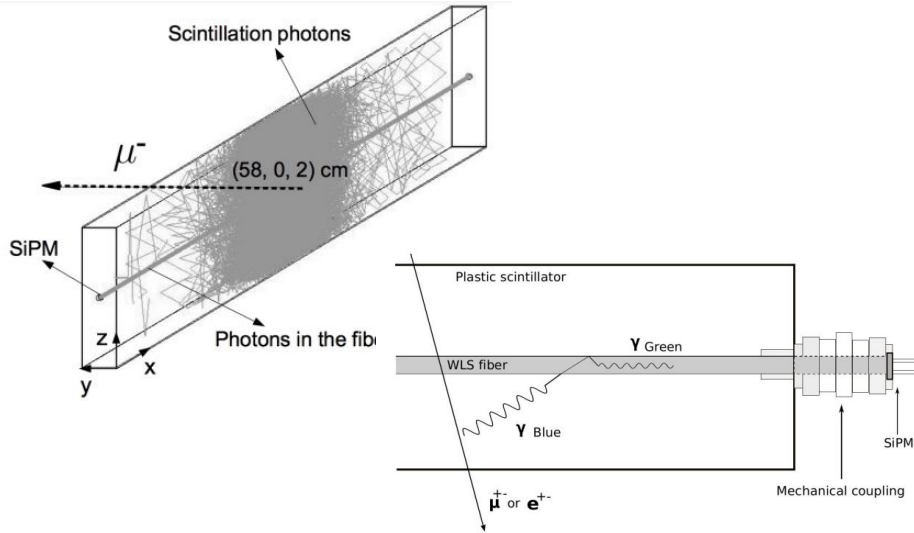
- Meiga objects
- External objects

Actions:

- Done automatically
- Done by user
- User configurable / predefined



Modelado del detector



- **Diseño geométrico:** Barras de centelleo organizadas en un arreglo **NxN** distribuidas en dos paneles perpendiculares.
- **Dimensiones de las barras:** Ancho, longitud y grosor ajustados según la simulación.
- **Carcasa externa:** Protege y afecta el comportamiento óptico de la luz emitida.

Propiedades Ópticas del Centelleador

- **Energía de los fotones:** 2.00 eV a 4.20 eV.
- **Índice de refracción:** Constante de 1.5.
- **Longitud de absorción:** 4 cm a 24 cm, dependiendo de la energía.
- **Componentes rápidas y lentas:** Simulación de respuesta temporal.

Fibras Ópticas y Detección

- **Transporte de fotones:** Fibras ópticas de PMMA con índice de refracción de 1.60.
- **Detección óptica:** Fotomultiplicadores con modelado preciso de su eficiencia cuántica.

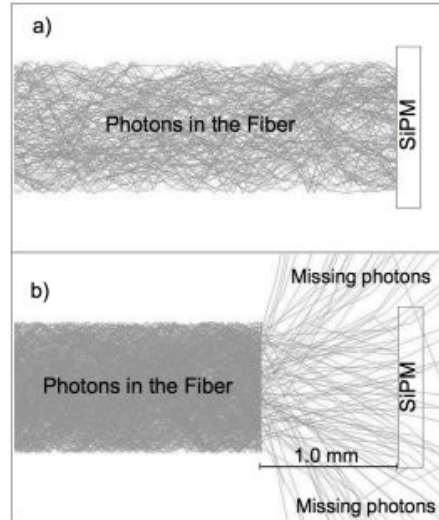
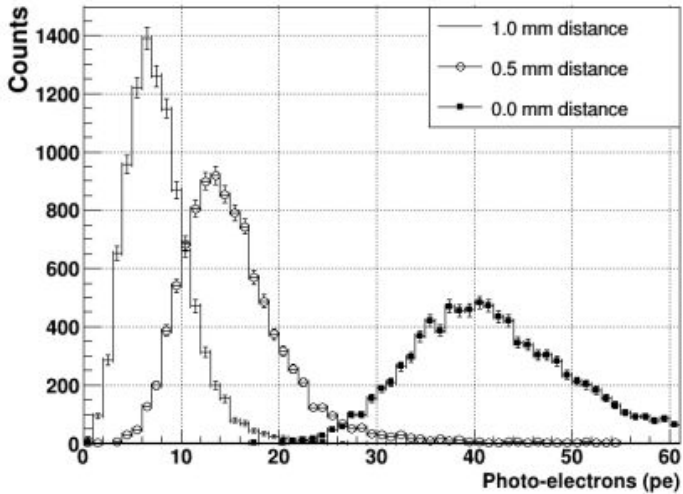
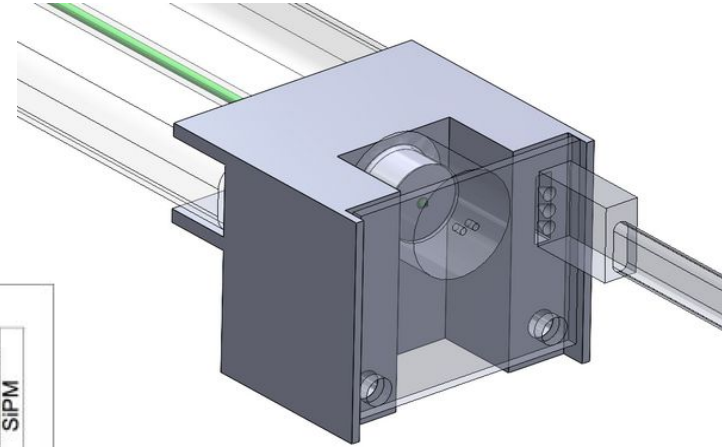
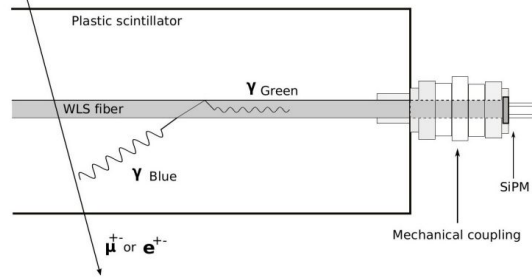
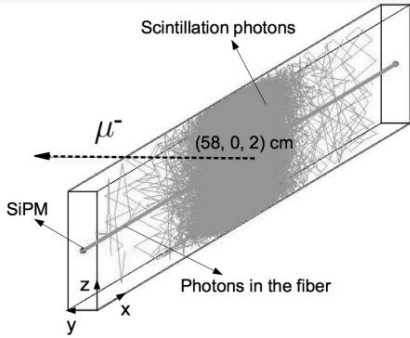
Reflexión y Dispersión

- **Rugosidad superficial:** Control de la dispersión de fotones.
- **Reflexión especular:** 20% de la luz reflejada en un lóbulo especular, sin reflexión especular exacta.

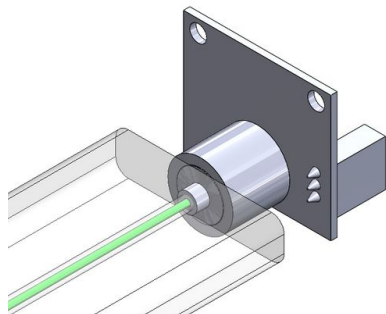
Sensor Óptico SiPM

Blindaje de Pb con densidad 11.3 g/cm³

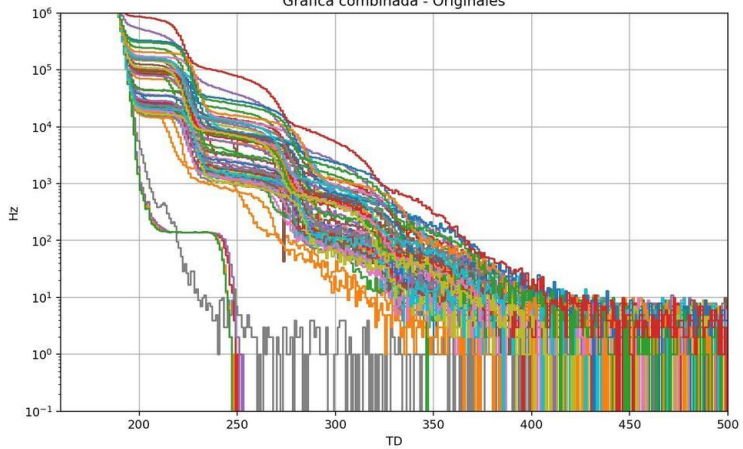
Similar para diseñar



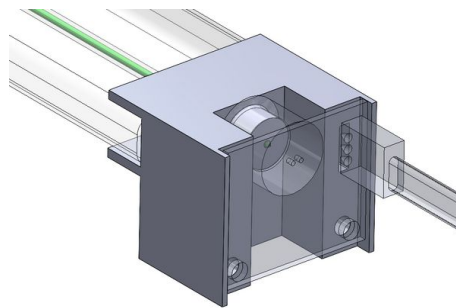
Acople MuTe 2.0



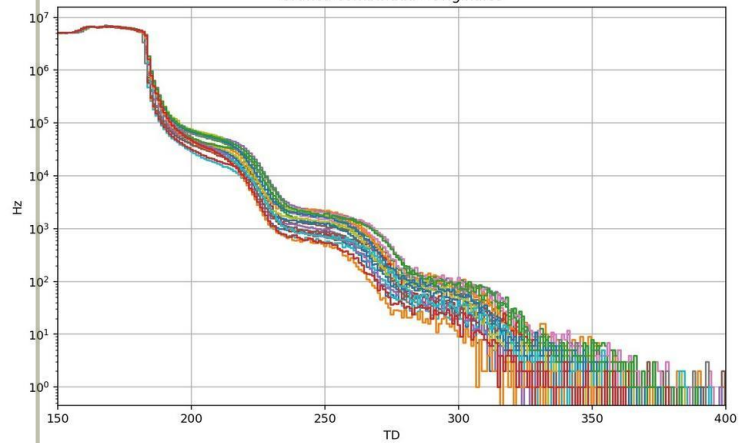
Gráfica combinada - Originales



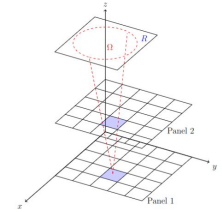
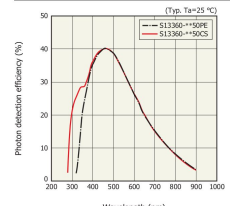
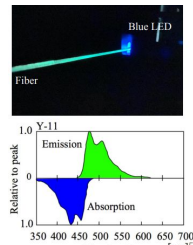
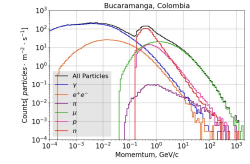
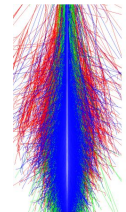
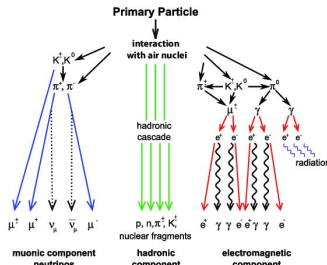
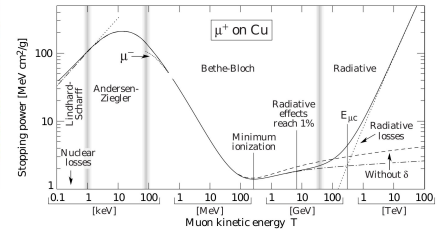
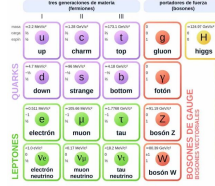
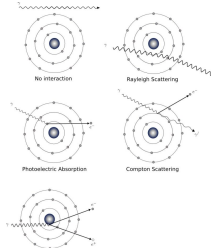
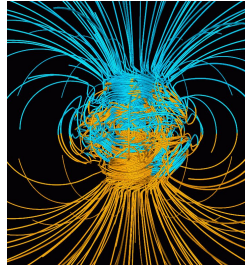
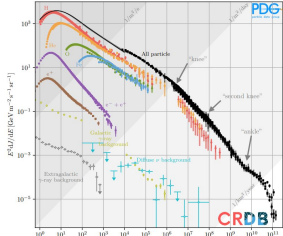
Acople
Alternativa C



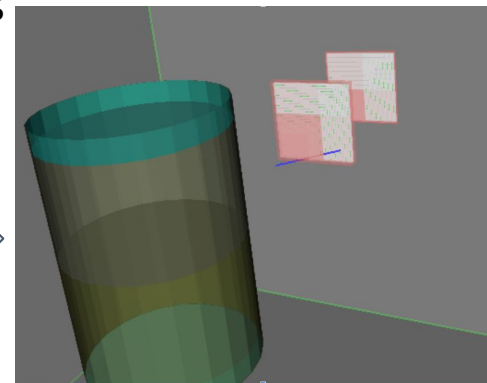
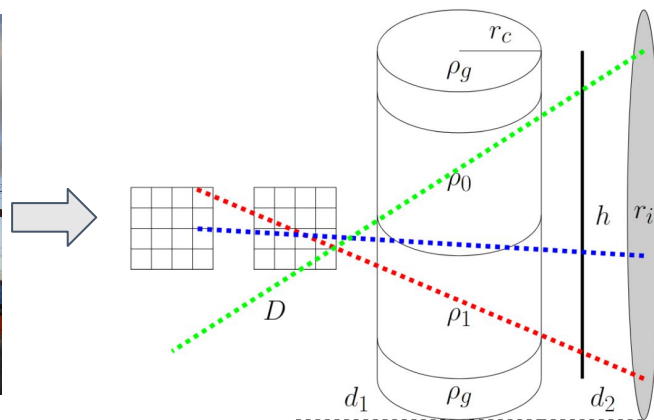
Gráfica combinada - Originales



Problema directo: Cadena de procesos = Cadena de simulaciones

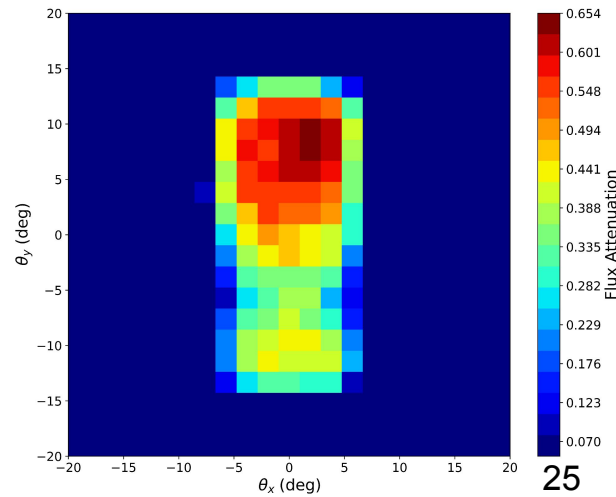
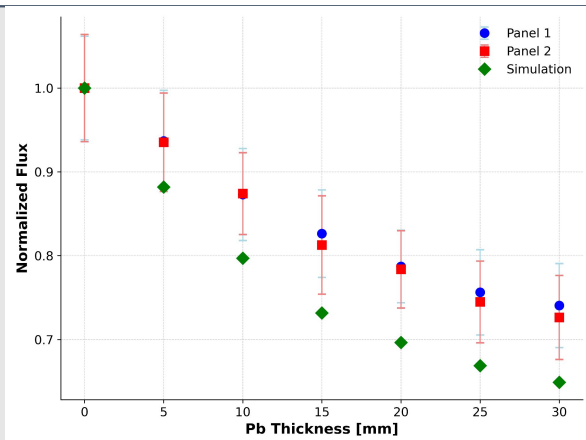


Aplicación particular: Exploring muon imaging in reactors



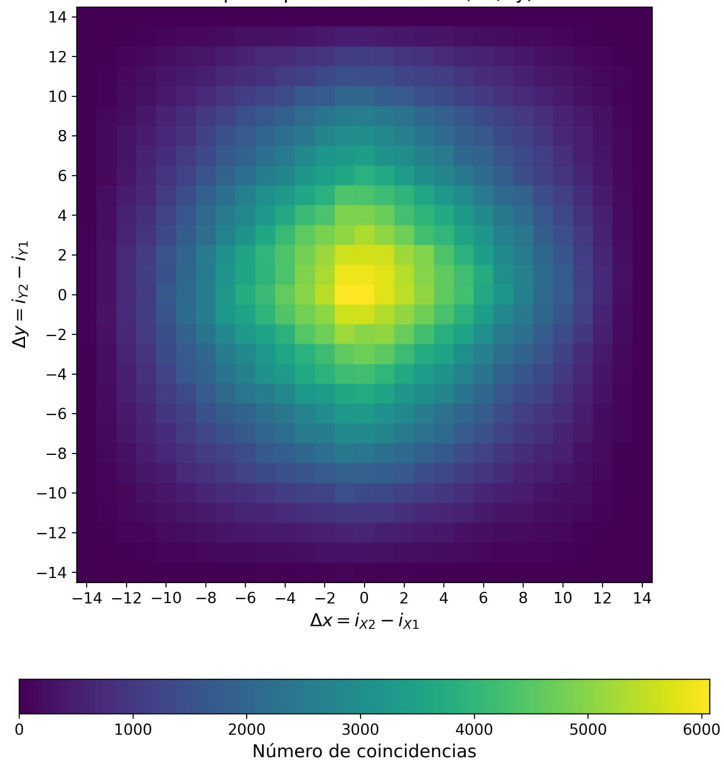
Height: 20-40 m (some reach over 50 m)
Diameter: 3-6 m

Framework validation with measurement

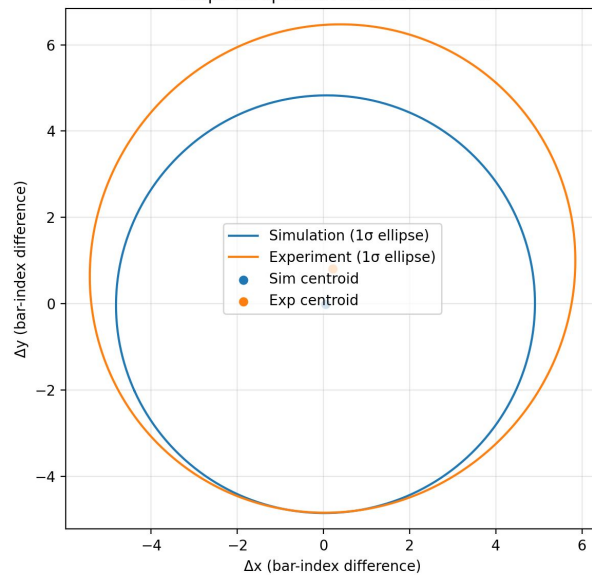


Validación reconstrucción

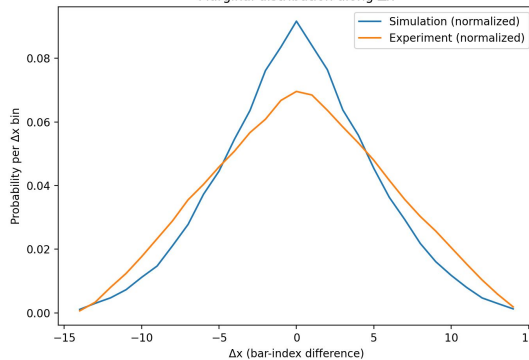
Mapa de píxeles discretos $N(\Delta x, \Delta y)$



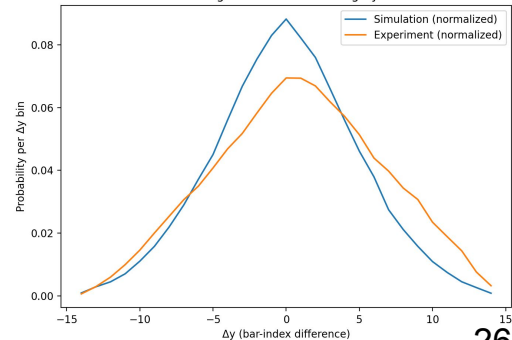
Shape comparison via 2nd moments



Marginal distribution along Δx



Marginal distribution along Δy

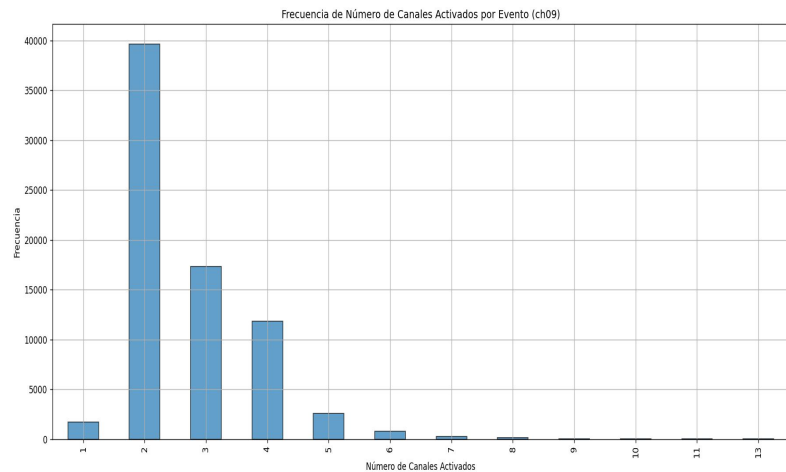


Canales activados por evento

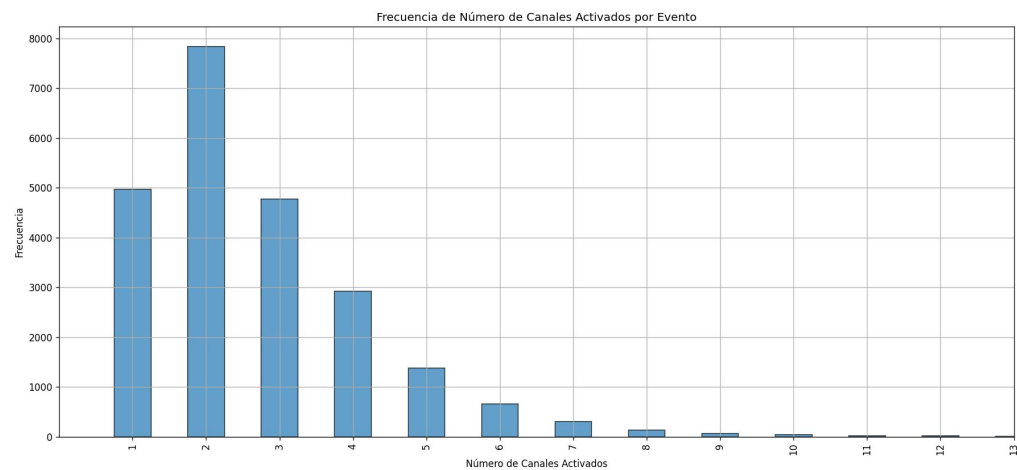
Pico en 2 barras confirma detección de trayectorias limpias.

❑ Diferencias: más eventos de 1 barra en datos experimentales.

❑ Causa principal: dark count en SiPM + partículas poco energéticas.



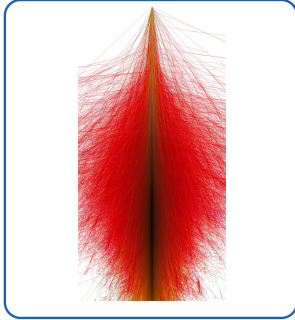
Simulación



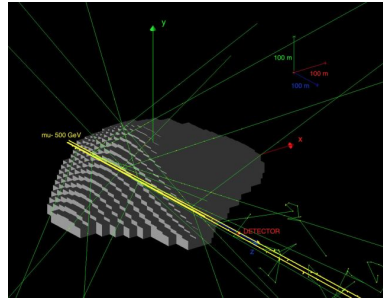
Medición

The Forward Problem in Muography simulation

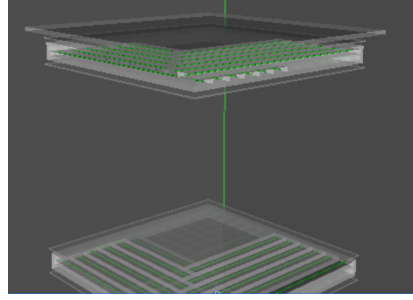
air-shower source (CORSIKA)



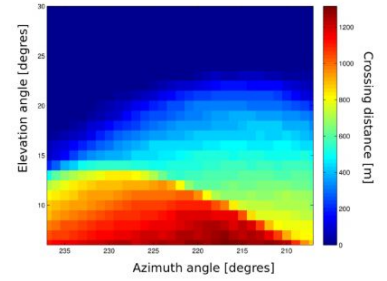
Particle interactions and transport



Detector response



Muogram

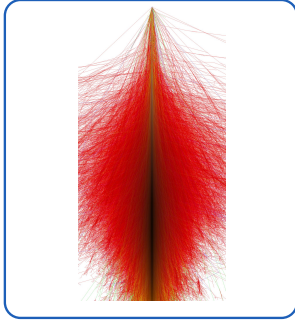


Monte Carlo is detailed but expensive

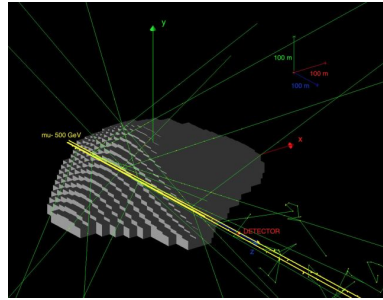


The Forward Problem in Muography simulation

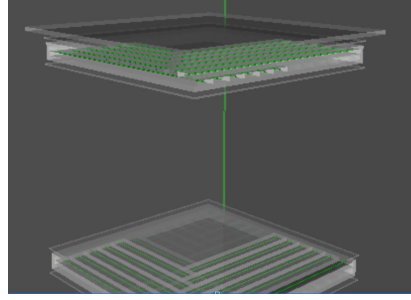
air-shower source (CORSIKA)



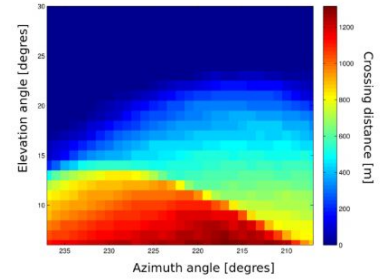
Particle interactions and transport



Detector response



Muogram



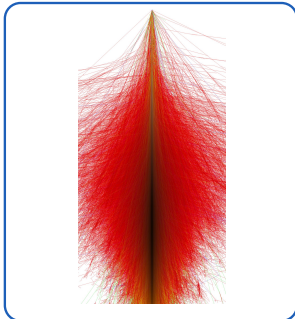
Monte Carlo is detailed but expensive



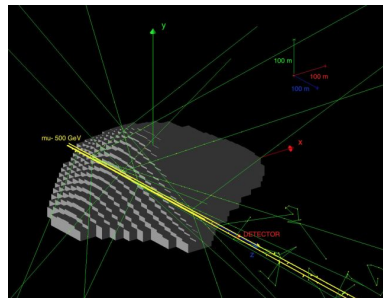
How can we keep the key physics without running a full Monte Carlo every time?

The Forward Problem in Muography simulation

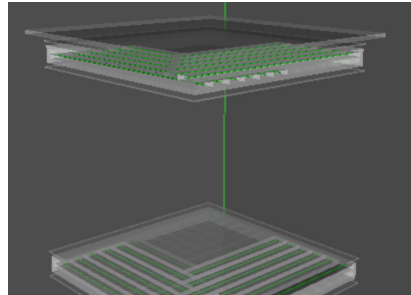
air-shower source (CORSIKA)



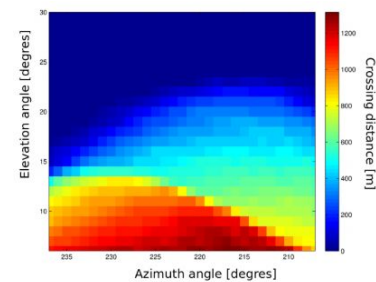
Particle interactions and transport



Detector response



Muogram

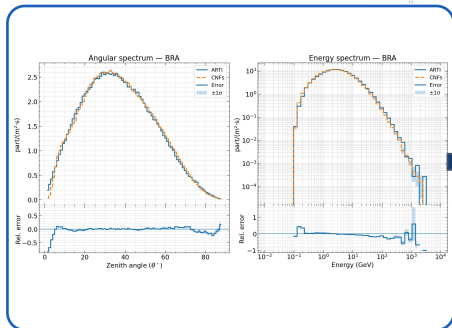


Monte Carlo is detailed in physics but it is expensive

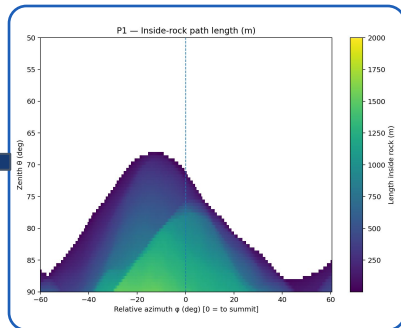


How can we keep the key physics without running a full Monte Carlo every time?

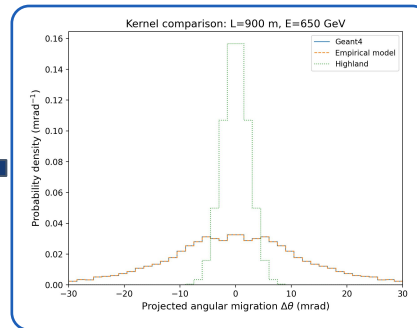
Deep learning muon flux



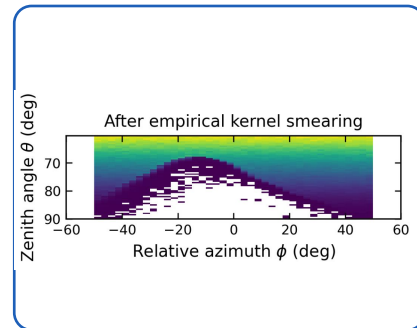
Backpropagation



Angular Scattering



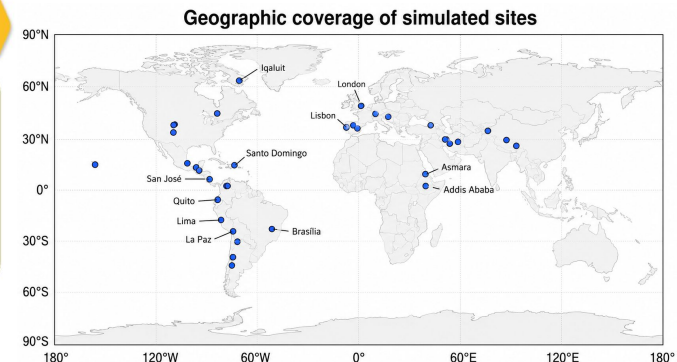
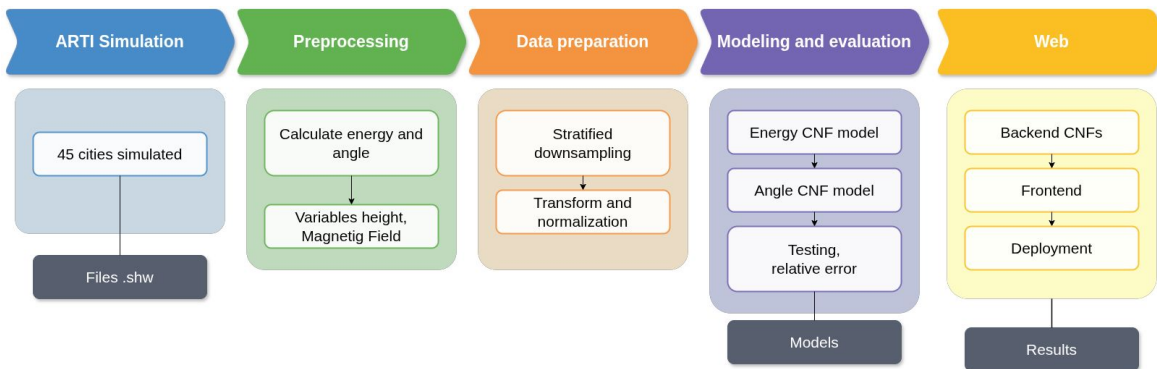
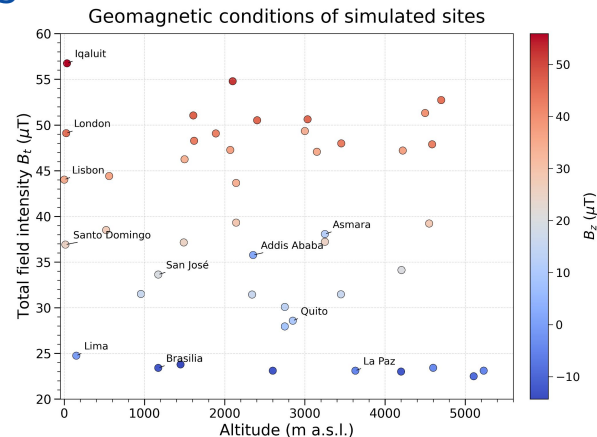
muogram



From Monte Carlo simulation to fast AI flux generation

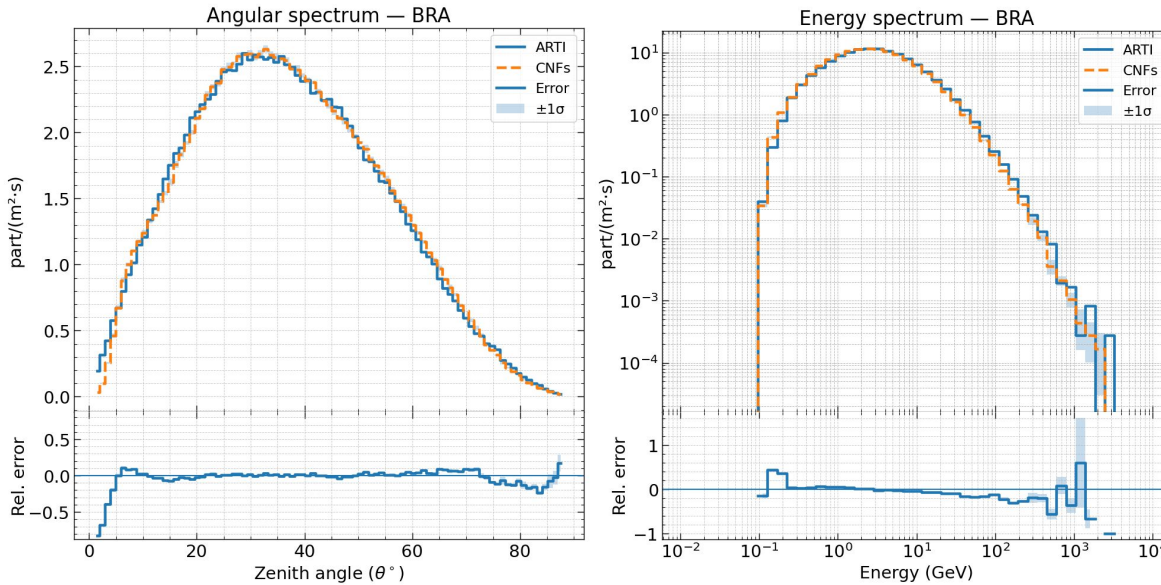
We use CORSIKA outputs as training data, so the AI model can generate muon fluxes faster.

- Monte Carlo simulation gives the reference muon-flux data.
- The training sites cover different altitude and geomagnetic conditions.
- The AI model learns site-dependent angular and energy spectra.
- The generated flux becomes the input of the forward model.

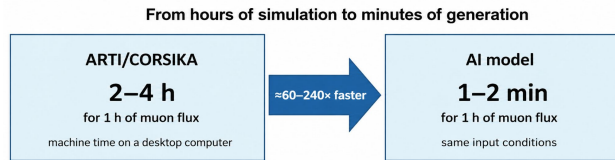


Deep learning flux generation and web deployment

We use CORSIKA outputs as training data, so the AI model can generate muon fluxes faster.



<https://muon-generator.vercel.app/>



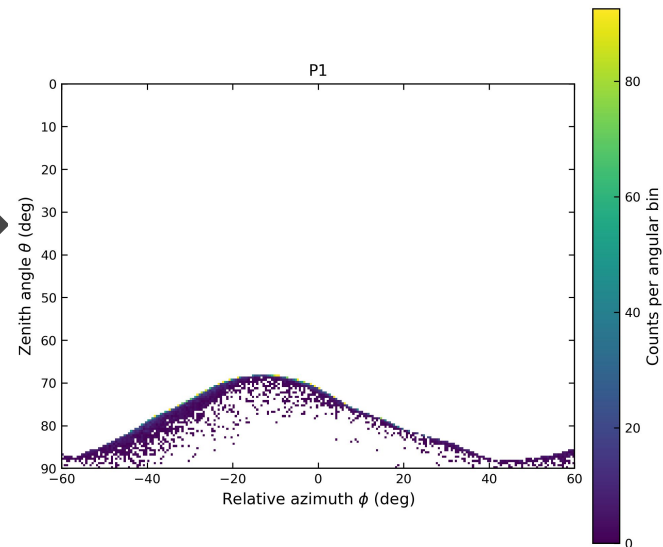
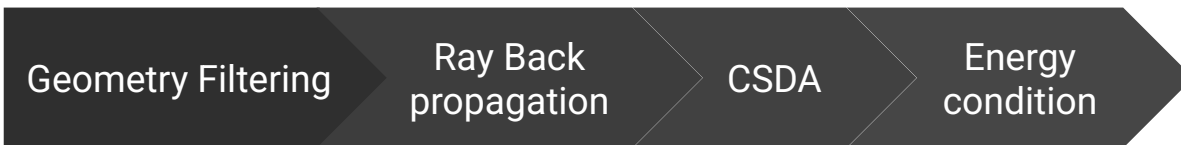
Warning: Web runs may take ~3–4 min in this pilot phase.

Now that we have the flux, we can propagate it through the volcano.

Backpropagation through the volcano

We convert detector directions into rock thickness, and then into a muon survival condition.

- Detector angles are traced through the volcano geometry.
- Each direction gives a rock thickness.
- CSDA converts rock thickness into a critical energy.
- Muons survive only if $E_\mu \geq E_{\text{crit}}(L, \rho)$

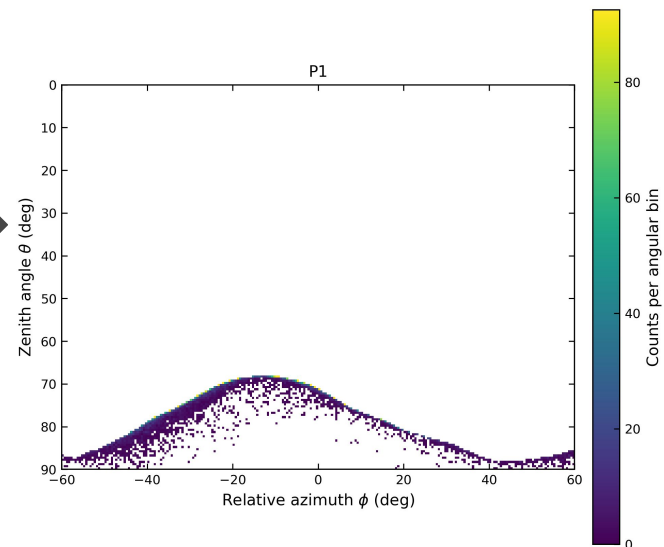
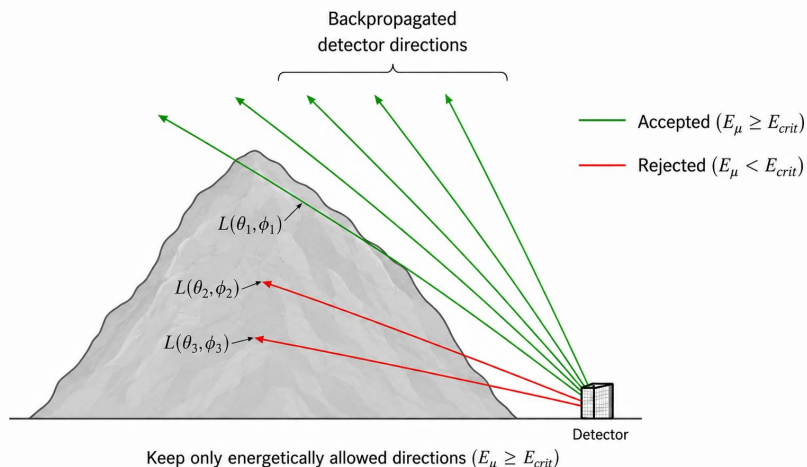
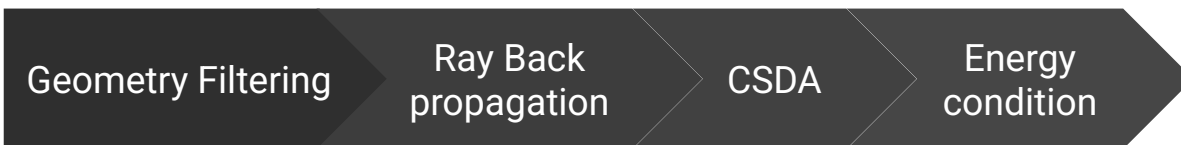


Now we know which muons survive. But they do not travel in perfect straight lines.

Backpropagation through the volcano

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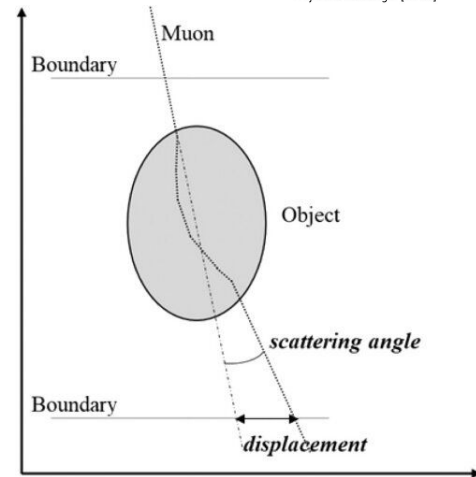
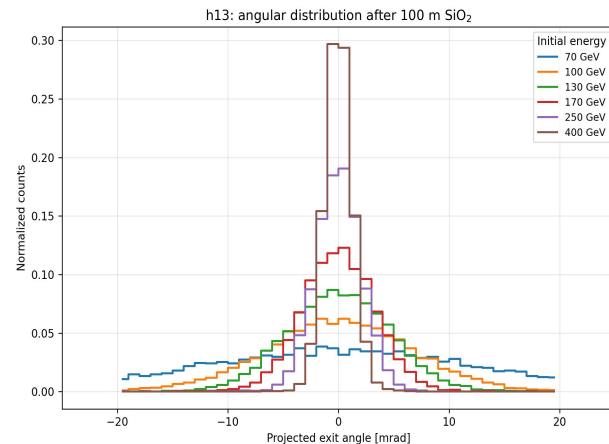
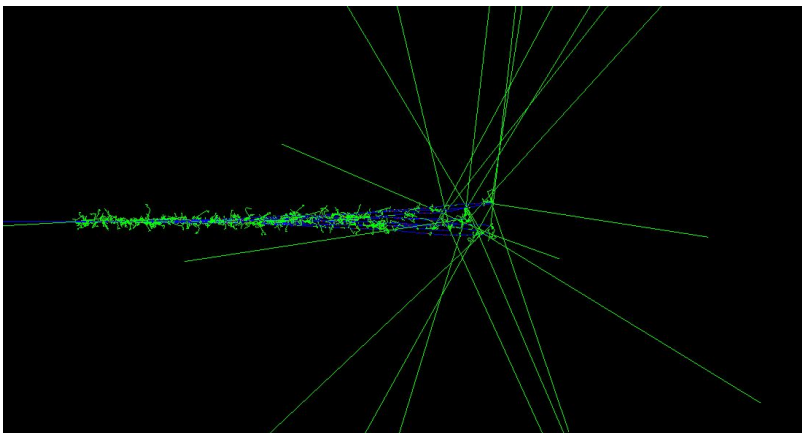
Now we know which muons survive. But they do not travel in perfect straight lines.

Why do we need a scattering kernel?

After the volcano filter, muons survive, but they do not travel in perfect straight lines.

- Straight-line propagation ignores angular scattering.
- Lynch & Dahl gives a first scattering scale.
- But it does not describe the full migration pattern.
- Geant4 gives detailed exit-angle distributions.
- These distributions are stored in an empirical kernel.

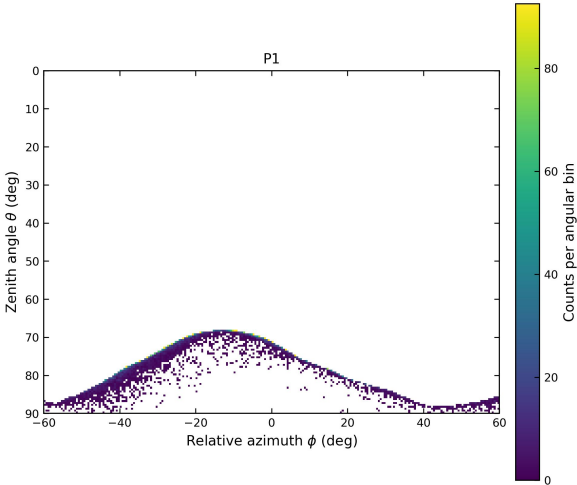
$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.088 \log_{10} \left(\frac{x z^2}{X_0 \beta^2} \right) \right]$$



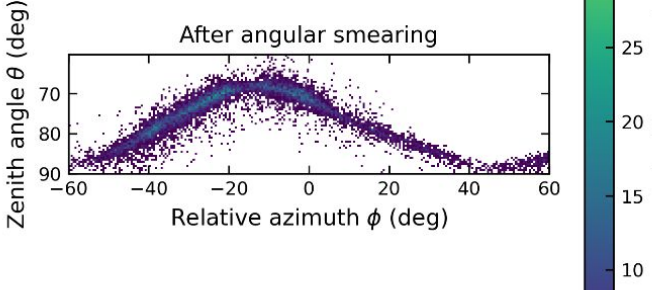
Scattering changes the final muogram

We run Geant4 once, and reuse the result many times.

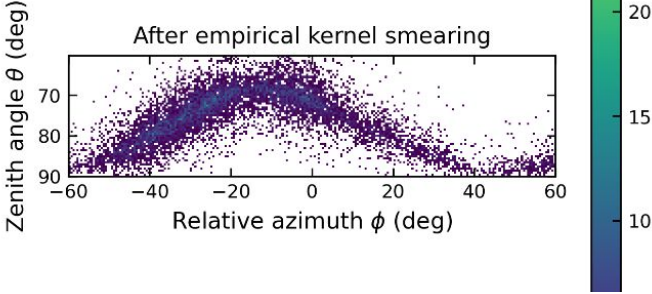
- Run Geant4 for many thickness–energy cases.
- Store the exit-angle distributions.
- Convert them into probability kernels.
- Apply the kernel to surviving muons.
- Obtain the scattered muogram.



Lynch & Dahl



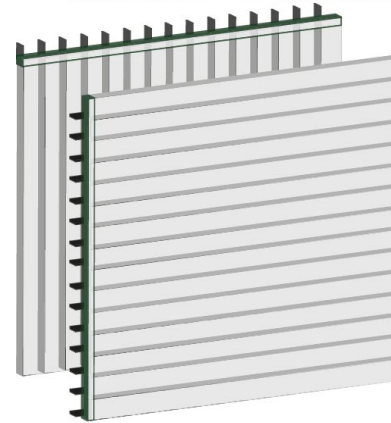
Our kernel



Next step: apply the model and compare it with real measurements.

MuTe: En una slide

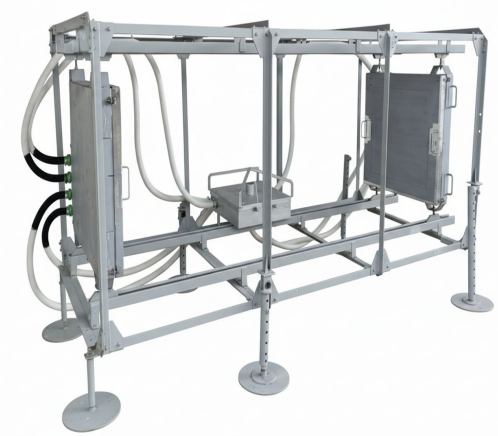
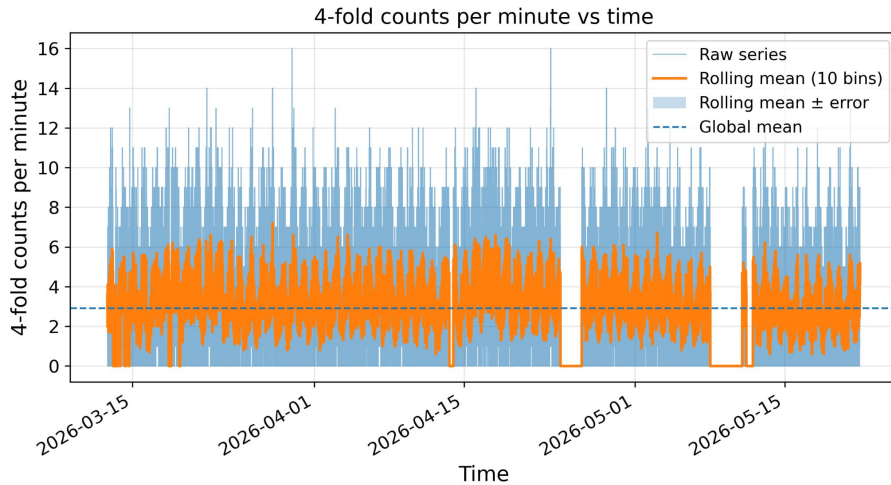
- Barras centelleadoras plásticas con lectura por fibra óptica.
- SiPM Hamamatsu.
- Electrónica CAEN FERS A5202/DT5202 de 64 canales.
- Trigger configurable: OR, mayoría y coincidencias.
- Modo de operación: OR32_AND2 para eventos correlacionados.
- Monitoreo continuo de temperatura, humedad y presión.
- Operación remota en campo mediante Starlink.
- Sistema preparado para adquisición estable y prolongada.



Real MuTe data for model comparison

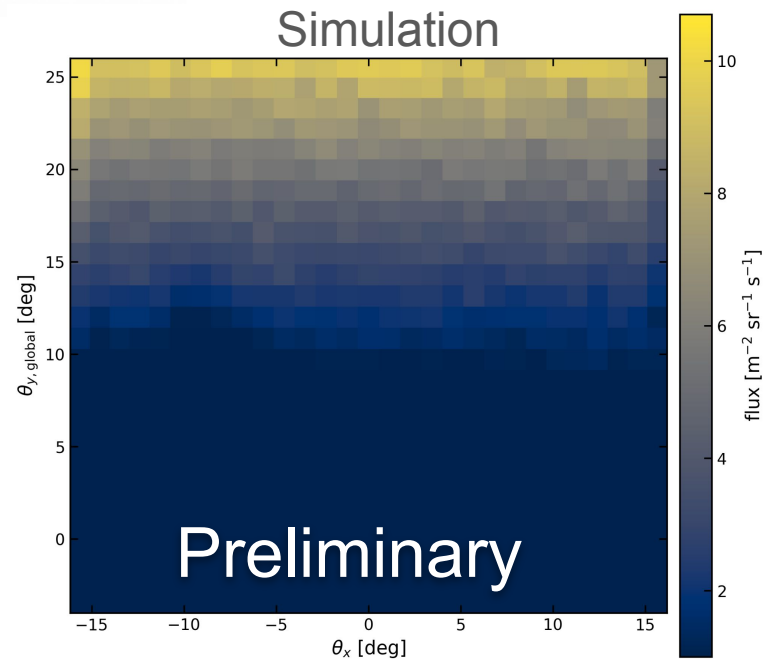
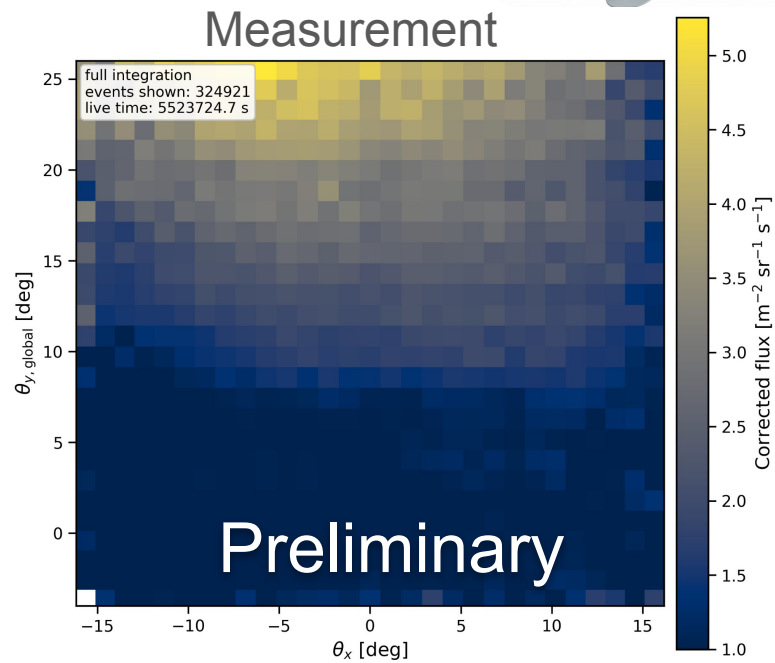
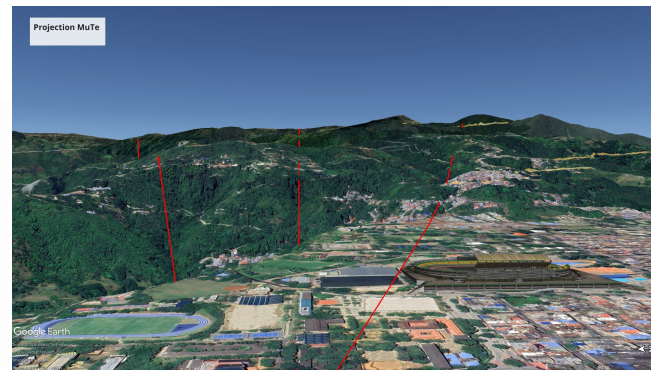
We use the current mountain-facing campaign to compare the simulated shadow with measured muon data.

- Mountain-facing MuTe campaign from the laboratory building.
- More than 100 days of accumulated muon data.
- 4-fold coincidence events are used for the measured flux.
- Large rock thickness produces a strong open-sky flux suppression.



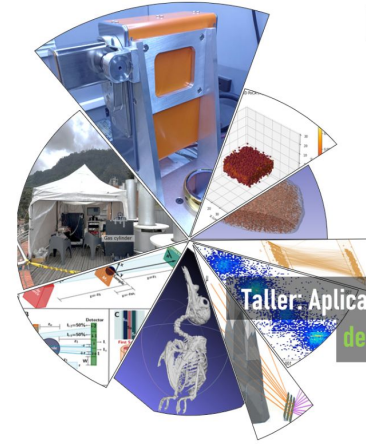
Muon shadow from the Andes: measurement and simulation

The measured mountain shadow is compared with the simulated muogram.





Gracias



2026

Taller: Aplicaciones Interdisciplinarias
de Detectores de Partículas

Junio 10 - 12.

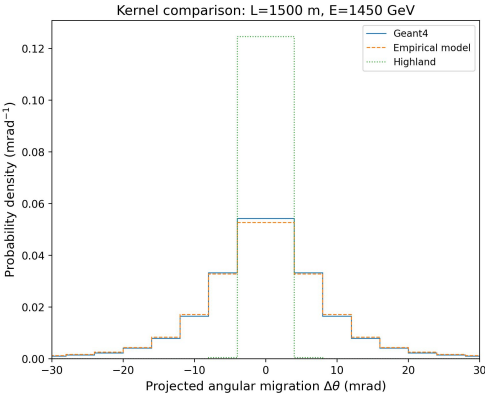
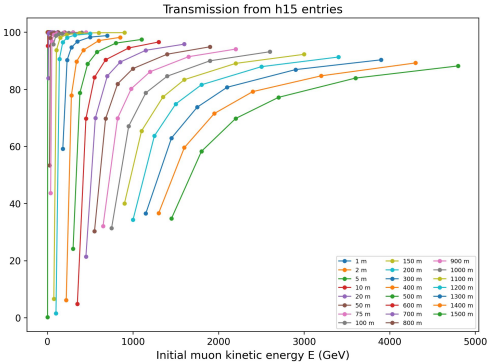
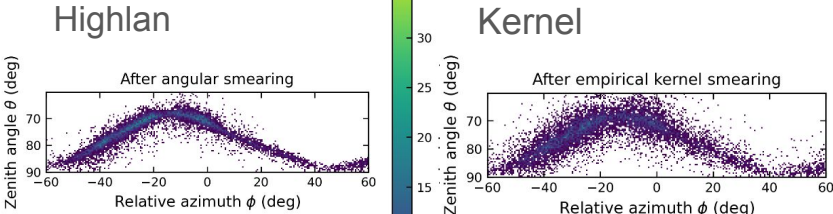
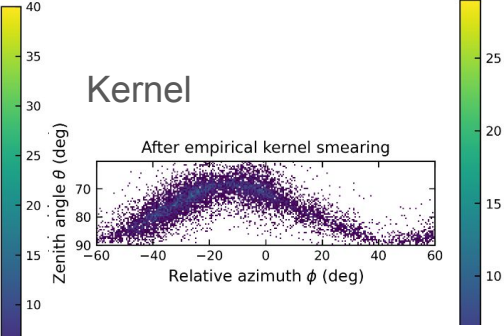
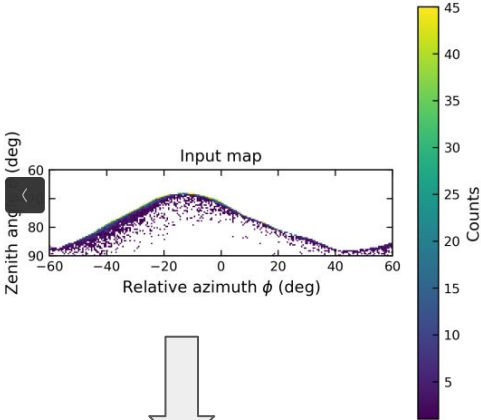


Rafael A. Martínez Rivero.
rafael2248058@correo.uis.edu.co

Scattering changes the final muogram

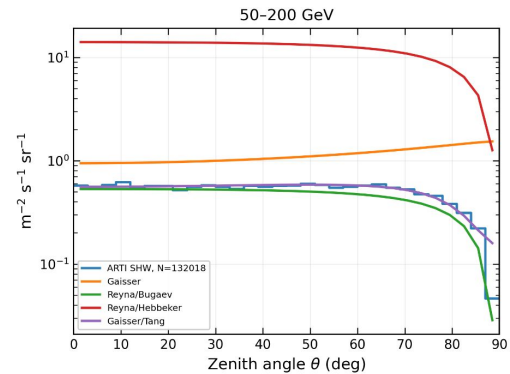
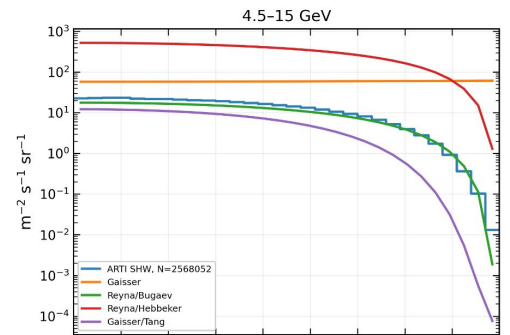
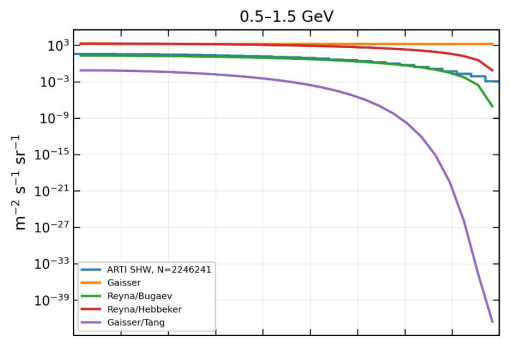
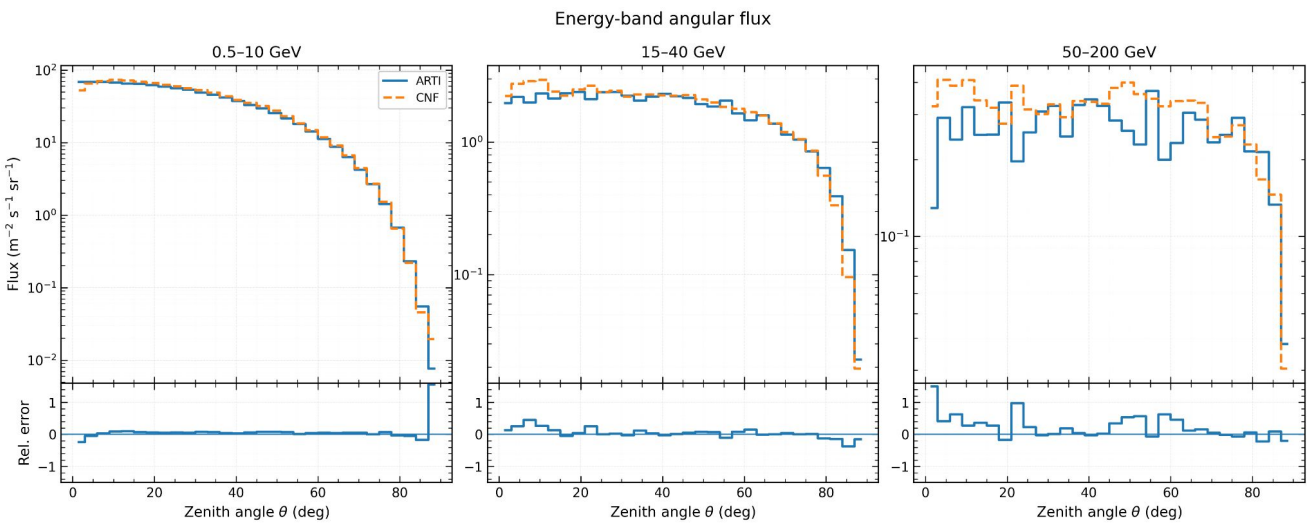
We run Geant4 once, and reuse the result many times.

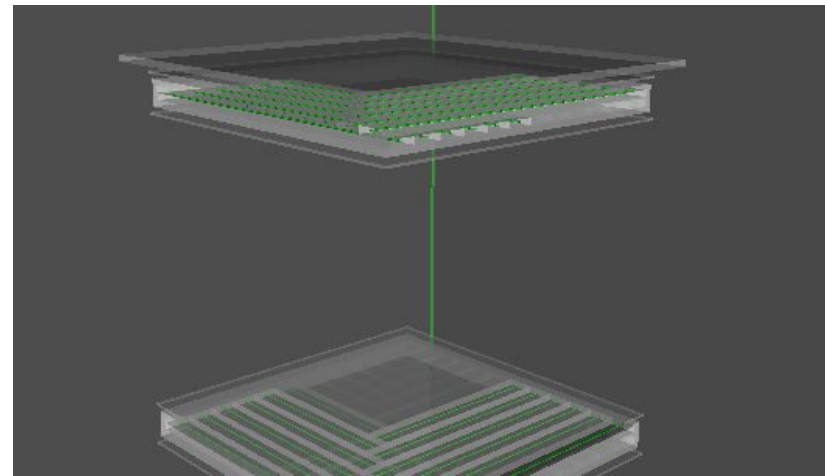
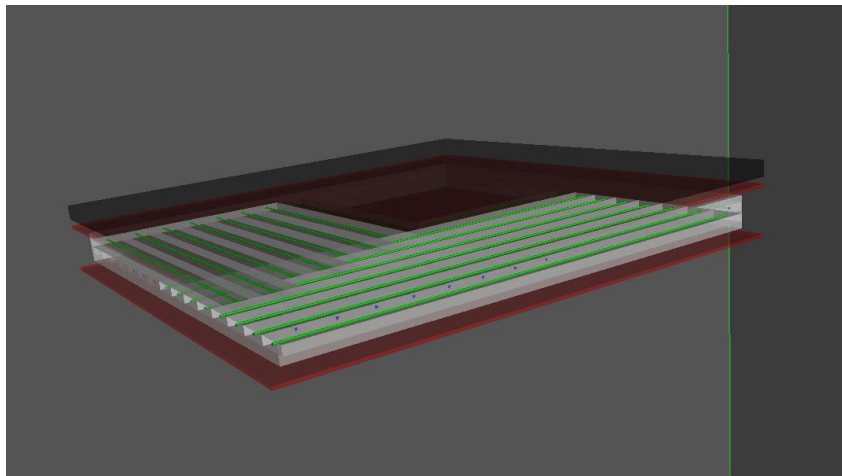
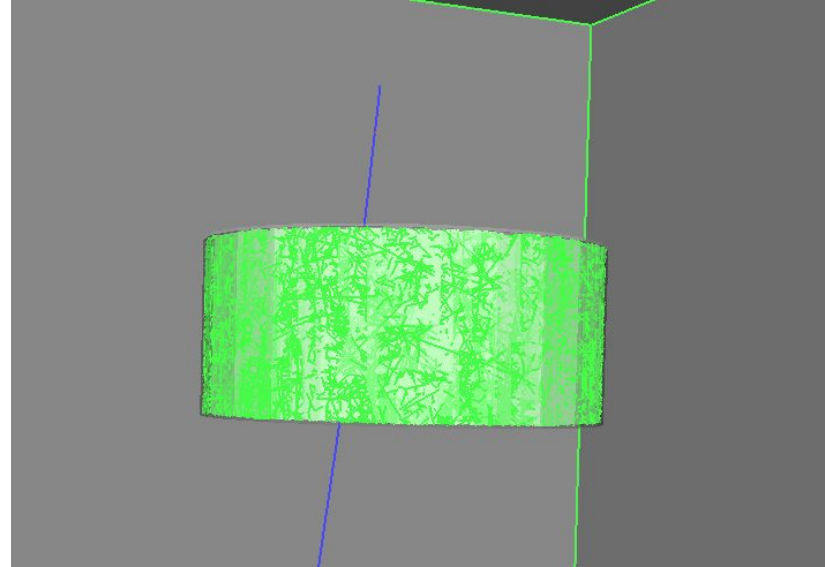
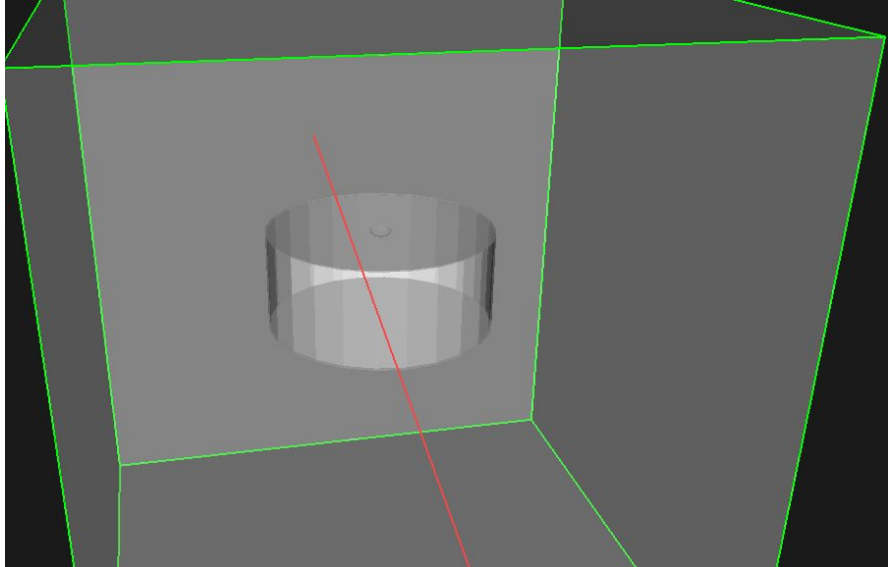
- Run Geant4 for many thickness–energy cases.
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- Apply the kernel to surviving muons.
- Obtain the scattered muogram.



Next step: apply the model and compare it with real measurements.

ARTI-Model in energy bands

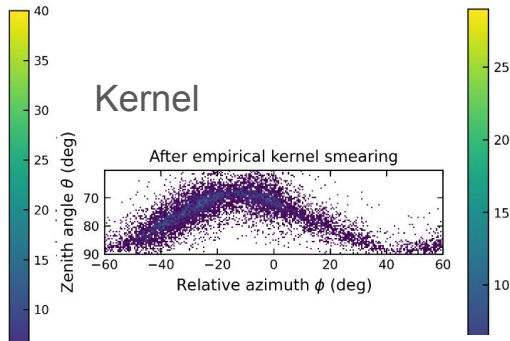
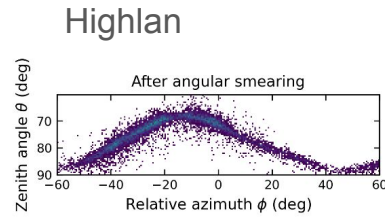
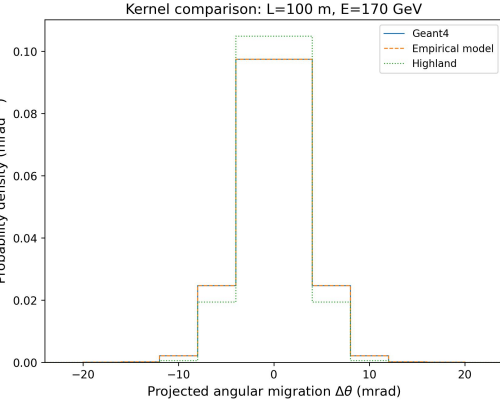
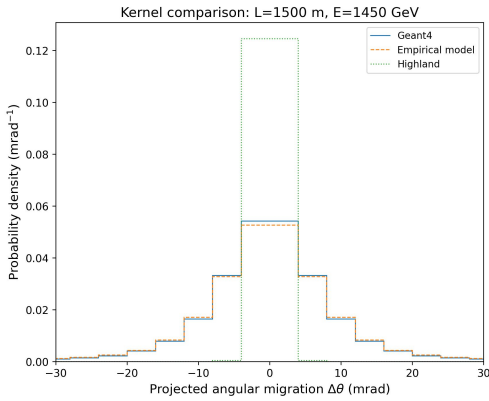
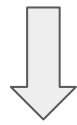
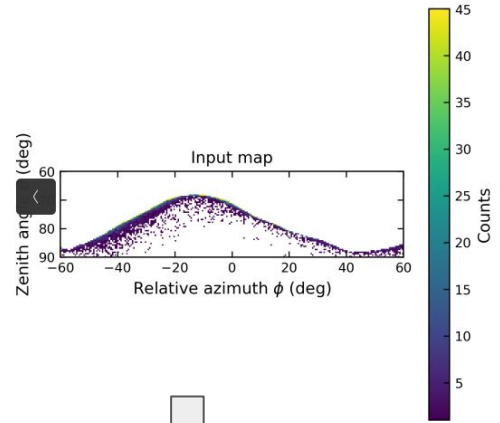


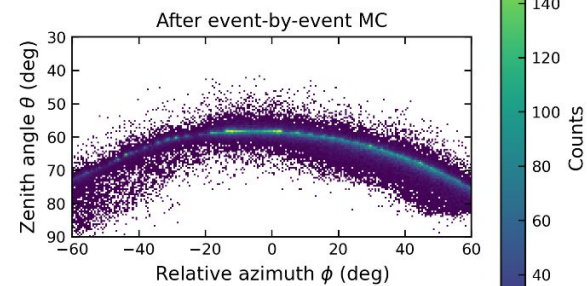
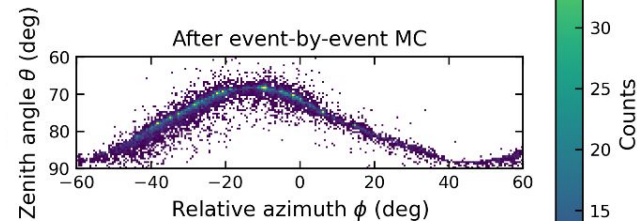
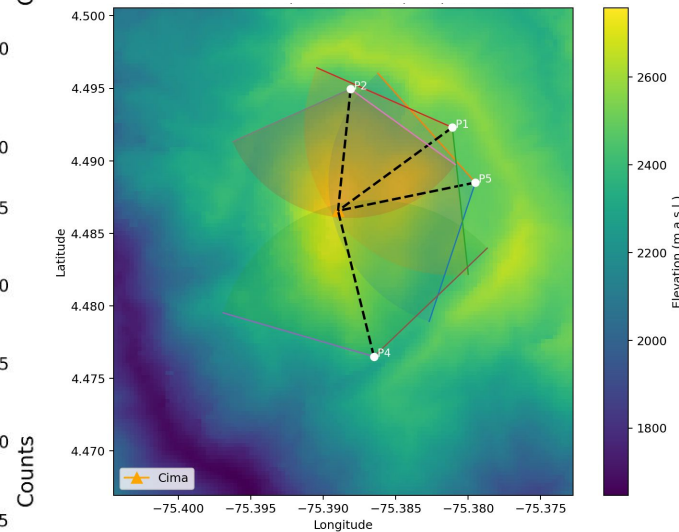
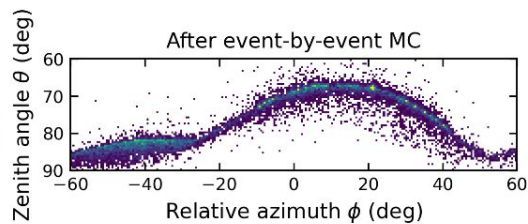
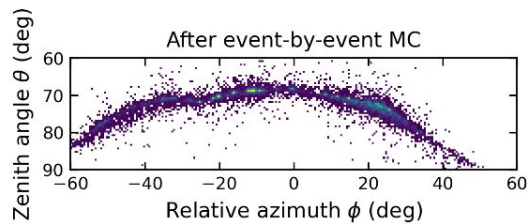


From Full MC to fast kernel of MCS: Muogram impact

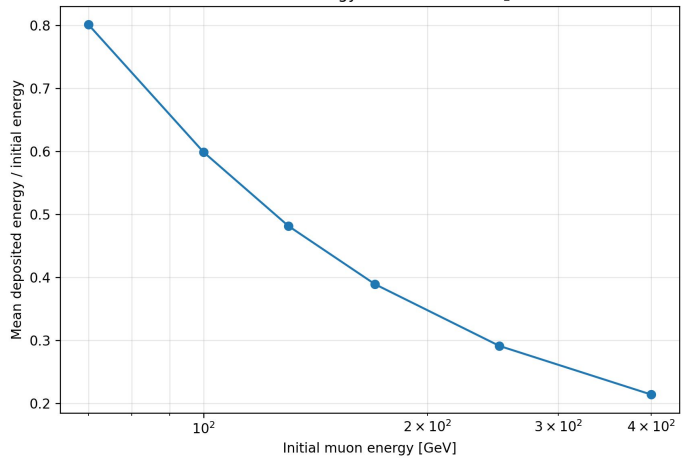
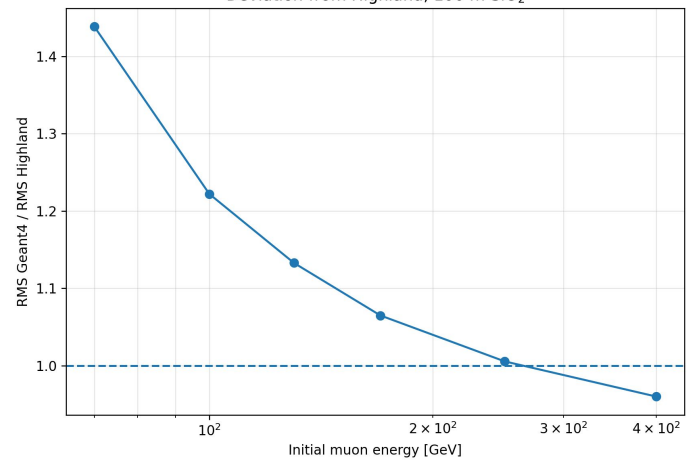
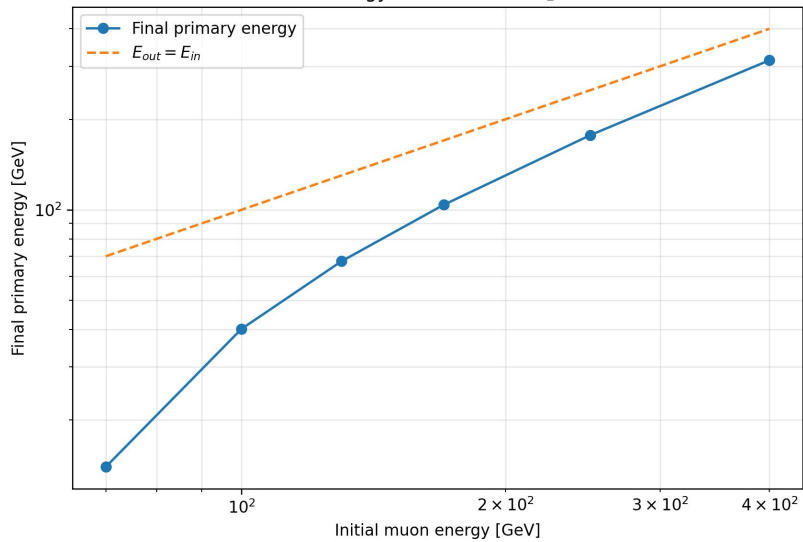
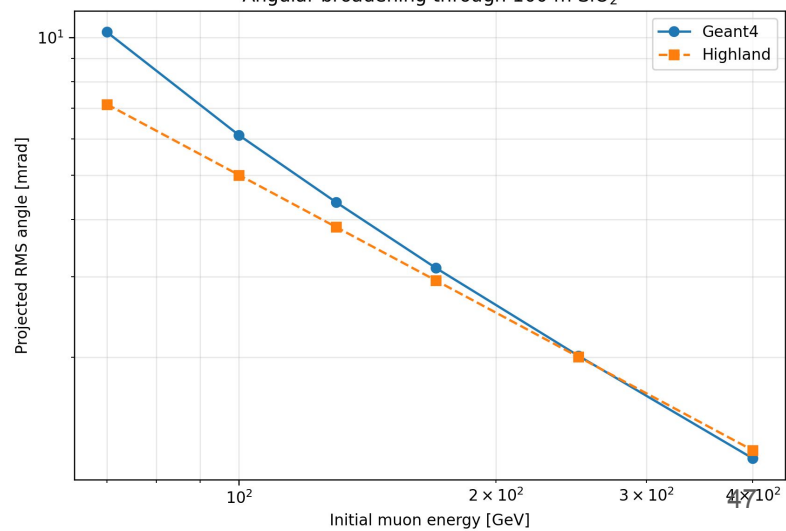
$$K(\Delta\Omega|E_\mu, L, \rho) = p(\Omega_{\text{out}} - \Omega_{\text{in}} | E_\mu, L, \rho), \int K(\Delta\Omega | E_\mu, L, \rho) d(\Delta\Omega) = 1.$$

$$N_{\text{scat}}(\Omega_{\text{rec}}) = \int dE_\mu d\Omega N_{\text{surv}}(E_\mu, \Omega) K(\Omega_{\text{rec}} - \Omega | E_\mu, L(\Omega), \rho).$$



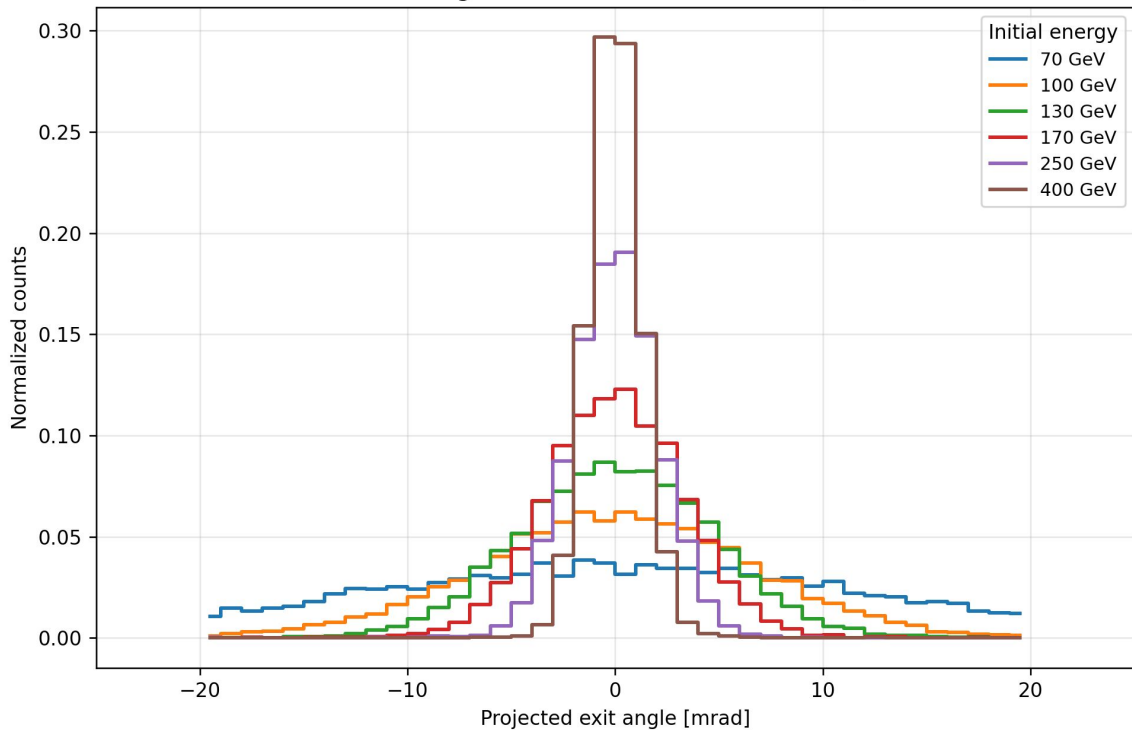


Geant4 Simulation

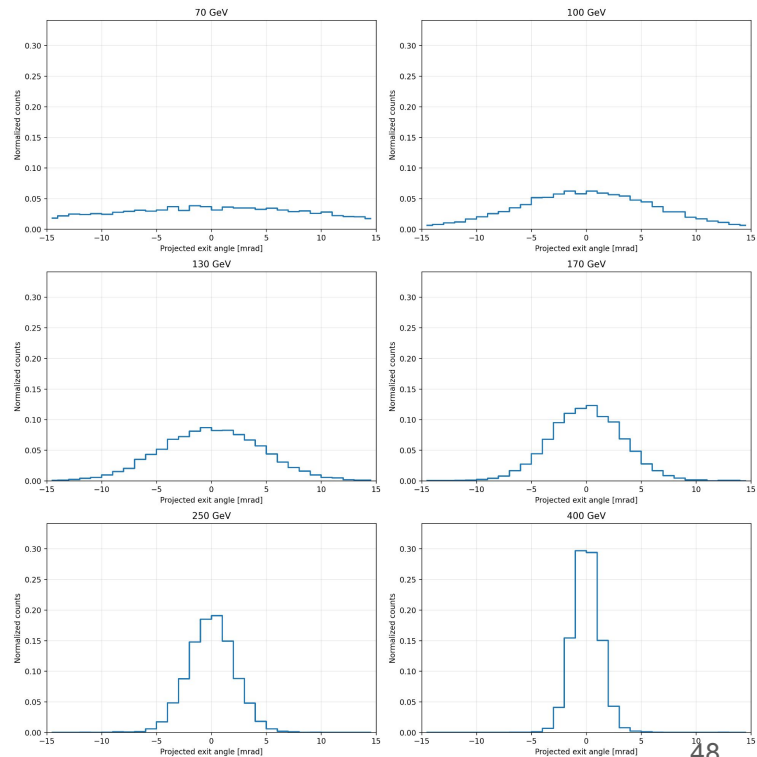
Relative energy loss in 100 m SiO₂Deviation from Highland, 100 m SiO₂Energy after 100 m SiO₂Angular broadening through 100 m SiO₂

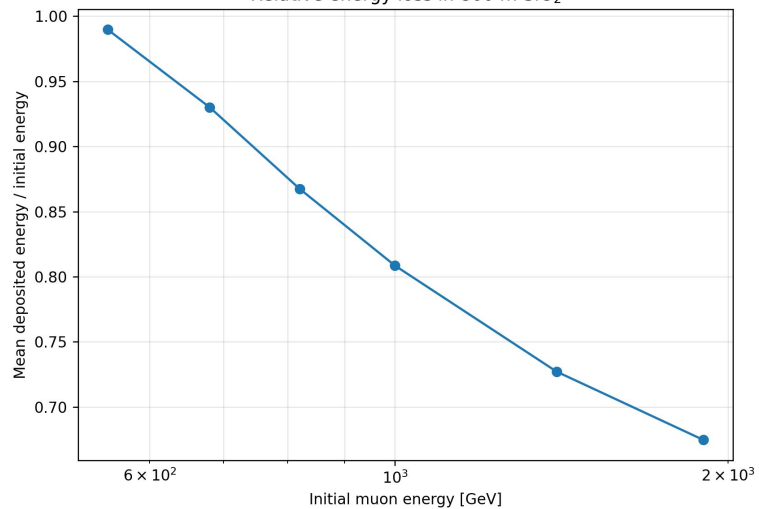
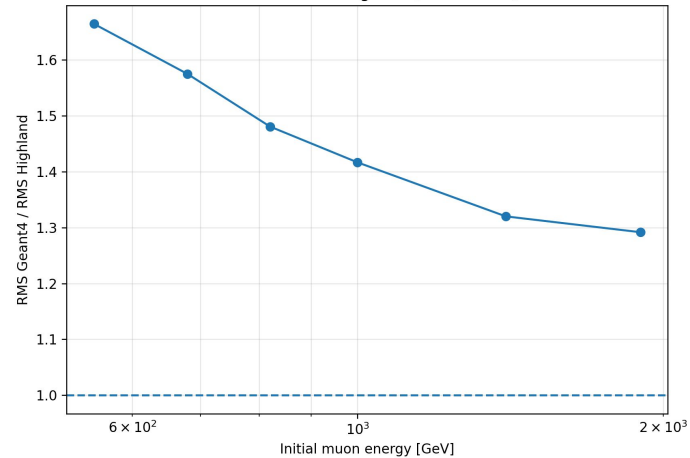
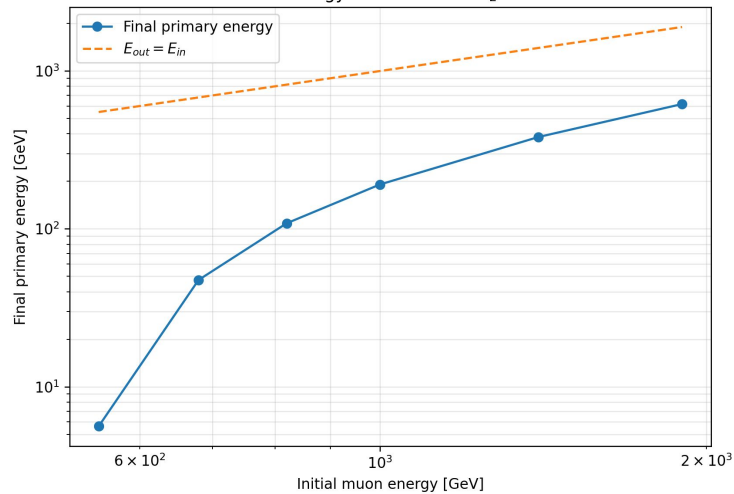
Exit-angle distribution

h13: angular distribution after 100 m SiO₂



h13: distributions at fixed thickness L=100 m



Relative energy loss in 800 m SiO₂Deviation from Highland, 800 m SiO₂Energy after 800 m SiO₂Angular broadening through 800 m SiO₂