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Notions of Spectral Action

Short Communication (30 min.)

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Abstract

Strongly motivated by physics (relativity and quantum mechanics), Connes and Chamseddine have defined the spectral action based on spectral facts. The goal of this talk is to review the necessary tools of noncommutative geometry and its spectral approach (spectral geometry) which are behind the spectral action to be able to compute it on few examples.

In that way, first, I will present standard material of noncommutative integration theory around the notion of spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$. This means to understand the notion of differential (or pseudodifferential) operators in this context. Next, I will explain the fundamentals of heat kernel theory and its expansion as $t \to 0^+$ in terms of coefficients of the elliptic generalized laplacian operator Δ, with a method to compute the coefficients of this expansion. This coefficients will be linked, via the noncommutative integrals of powers of $|\mathcal{D}|$, with the asymptotic expansion in Λ of the spectral action $\mathrm{Tr}(f(\mathcal{D}/\Lambda))$. This action plays an essential role in physics, and, during the talk, I will relate it with the Einstein-Hilbert action in gravity and the Yang-Mills action in particle physics.

For each part of the talk, I will suggest references since this one is by no means original.

Keywords

Spectral geometry; spectral triples; noncommutative manifolds.

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